

The discovery of a radio galaxy of at least 5 Mpc

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ABSTRACT

Context. Giant radio galaxies (GRGs, or colloquially ‘giants’) are the Universe’s largest structures generated by individual galaxies. They comprise synchrotron-radiating AGN ejecta and attain cosmological (Mpc-scale) lengths. However, the main mechanisms that drive their exceptional growth remain poorly understood.

Aims. To deduce the main mechanisms that drive a phenomenon, it is usually instructive to study extreme examples. If there exist host galaxy characteristics that are an important cause for GRG growth, then the hosts of the *largest* GRGs are likely to possess them. Similarly, if there exist particular large-scale environments that are highly conducive to GRG growth, then the *largest* GRGs are likely to reside in them. For these reasons, we aim to perform a case study of the largest GRG available.

Methods. We reprocessed the LOFAR Two-metre Sky Survey (LoTSS) DR2 by subtracting compact sources and performing multi-scale CLEAN deconvolution at 60” and 90” resolution. The resulting images constitute the most sensitive survey yet for radio galaxy lobes, whose diffuse nature and steep synchrotron spectra have allowed them to evade previous detection attempts at higher resolution and shorter wavelengths. We visually searched these images for GRGs.

Results. We discover *Alcyoneus*, a low-excitation radio galaxy with a projected proper length $l_p = 4.99 \pm 0.04$ Mpc. Its jets and lobes are all four detected at very high significance, and the SDSS-based identification of the host, at spectroscopic redshift $z_{\text{spec}} = 0.24674 \pm 6 \cdot 10^{-5}$, is unambiguous. The total luminosity density at $\nu = 144$ MHz is $L_\nu = 8 \pm 1 \cdot 10^{25}$ W Hz⁻¹, which is below-average, though near-median (percentile $45 \pm 3\%$), for GRGs. The host is an elliptical galaxy with a stellar mass $M_\star = 2.4 \pm 0.4 \cdot 10^{11} M_\odot$ and a supermassive black hole mass $M_\bullet = 4 \pm 2 \cdot 10^8 M_\odot$, both of which tend towards the lower end of their respective GRG distributions (percentiles $25 \pm 9\%$ and $23 \pm 11\%$). The host resides in a filament of the Cosmic Web. Through a new Bayesian model for radio galaxy lobes in three dimensions, we estimate the pressures in the Mpc³-scale northern and southern lobe to be $P_{\text{min},1} = 4.8 \pm 0.3 \cdot 10^{-16}$ Pa and $P_{\text{min},2} = 4.9 \pm 0.6 \cdot 10^{-16}$ Pa, respectively. The corresponding magnetic field strengths are $B_{\text{min},1} = 46 \pm 1$ pT and $B_{\text{min},2} = 46 \pm 3$ pT.

Conclusions. We have discovered what is in projection the largest known structure made by a single galaxy – a GRG with a projected proper length $l_p = 4.99 \pm 0.04$ Mpc. The true proper length is at least $l_{\text{min}} = 5.04 \pm 0.05$ Mpc. Beyond geometry, *Alcyoneus* and its host are suspiciously ordinary: the total low-frequency luminosity density, stellar mass and supermassive black hole mass are all lower than, though similar to, those of the medial GRG. Thus, very massive galaxies or central black holes are not necessary to grow large giants, and, if the observed state is representative of the source over its lifetime, neither is high radio power. A low-density environment remains a possible explanation. The source resides in a filament of the Cosmic Web, with which it might have significant thermodynamic interaction. The pressures in the lobes are the lowest hitherto found, and *Alcyoneus* therefore represents the most promising radio galaxy yet to probe the warm–hot intergalactic medium.

Key words. galaxies: active – galaxies: individual: *Alcyoneus* – galaxies: jets – intergalactic medium – radio continuum: galaxies

1. Introduction

Most galactic bulges hold a supermassive (i.e. $M_\bullet > 10^6 M_\odot$) Kerr black hole (e.g. Soltan 1982) that grows by accreting gas, dust and stars from its surroundings (Kormendy & Ho 2013). The black hole ejects a fraction of its accretion disk plasma from the host galaxy along two collimated, magnetised jets that are aligned with its rotation axis (e.g. Blandford & Rees

1974). The relativistic electrons contained herein experience Lorentz force and generate, through spiral motion, synchrotron radiation that is observed by radio telescopes. The two jets either fade gradually or end in hotspots at the end of diffuse lobes, and ultimately enrich the intergalactic medium (IGM) with cosmic rays and magnetic fields. The full luminous structure is referred to as a *radio galaxy* (RG). Members of a rare RG subpopulation attain megaparsec-scale proper (and thus comoving) lengths (e.g. Willis et al. 1974; Andernach et al. 1992; Ishwara-Chandra & Saikia 1999; Jamrozy et al. 2008; Machalski 2011; Kuźmicz et al. 2018; Dabhade et al.

* In dear memory of Pallas. If your name hadn’t been this popular with asteroid discoverers, you’d now be the giants’ giant – once again looking down at the sprawling ants below.

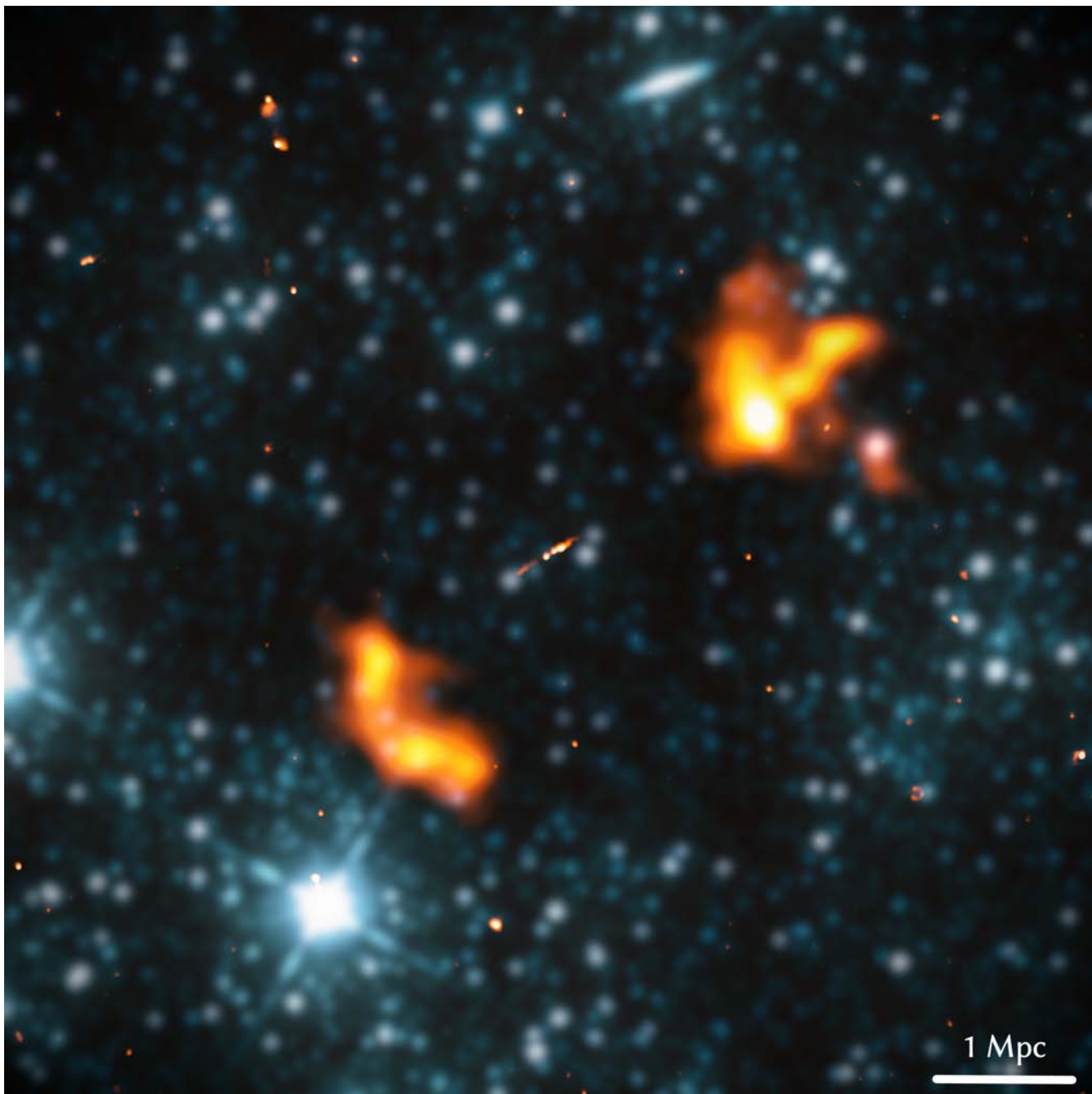


Fig. 1: **Joint radio-infrared view of Alcyoneus, a radio galaxy with a projected proper length of 5.0 Mpc.** We show a $2048'' \times 2048''$ solid angle centred around right ascension 123.590372° and declination 52.402795° . We superimpose LOFAR Two-metre Sky Survey (LoTSS) DR2 images at 144 MHz of two different resolutions ($6''$ for the core and jets, and $60''$ for the lobes) (orange), with the Wide-field Infrared Survey Explorer (WISE) image at $3.4 \mu\text{m}$ (blue). To highlight the radio emission, the infrared emission has been blurred to $0.5'$ resolution.

2020a). The *giant* radio galaxy (GRG, or colloquially ‘*giant*’) definition accommodates our limited ability to infer an RG’s true proper length from observations: an RG is called a GRG if and only if its proper length *projected onto the plane of the sky* exceeds some threshold $l_{\text{p,GRG}}$, usually chosen to be 0.7 or 1 Mpc. Because the conversion between angular length and projected proper length depends on cosmological parameters, which remain uncertain, it is not always clear whether a given observed RG satisfies the GRG definition. Currently, there are about a thousand GRGs known, the majority of which have been found in the Northern Sky. About one

hundred exceed 2 Mpc and ten exceed 3 Mpc; at 4.9 Mpc, the literature’s projectively longest is J1420-0545 (Machalski et al. 2008). As such, GRGs – and the rest of the megaparsec-scale RGs – are the largest single-galaxy-induced phenomena in the Universe. It is a key open question what physical mechanisms lead some RGs to extend for $\sim 10^2$ times their host galaxy diameter. To determine whether there exist particular host galaxy characteristics or large-scale environments that are essential for GRG growth, it is instructive to analyse the largest GRGs, since in these systems it is most likely that all major favourable growth factors are present. We thus aim to perform

a case study of the largest GRG available.

As demonstrated by Dabhade et al. (2020b)'s record sample of 225 discoveries, the Low-frequency Array (LOFAR) (van Haarlem et al. 2013) is among the most attractive contemporary instruments for finding new GRGs. This Pan-European radio interferometer features a unique combination of short baselines to provide sensitivity to large-scale emission, and long baselines to mitigate source confusion.¹ These qualities are indispensable for observational studies of GRGs, which require identifying both extended lobes and compact cores and jets. Additionally, the metre wavelengths at which the LOFAR operates allow it to detect steep-spectrum lobes far away from host galaxies. Such lobes reveal the full extent of GRGs, but are missed by decimetre observatories. Thus, in Section 2, we describe a reprocessing of the LOFAR Two-metre Sky Survey (LoTSS) Data Release 2 (DR2) aimed at revealing hitherto unknown RG lobes – among other goals. An overview of the reprocessed images, which cover thousands of square degrees, and statistics of the lengths and environments of the GRGs they have revealed, are subjects of future publications. For now, these images allow us to discover *Alcyoneus*², a 5 Mpc GRG, whose properties we determine and discuss in Section 3.

Figure 1 provides a multi-wavelength, multi-resolution view of this giant. Section 4 contains our concluding remarks.

We assume a concordance inflationary Λ CDM model with parameters \mathfrak{M} from Planck Collaboration et al. (2020); i.e. $\mathfrak{M} = (h = 0.6766, \Omega_{\text{BM},0} = 0.0490, \Omega_{\text{M},0} = 0.3111, \Omega_{\Lambda,0} = 0.6889)$, where $H_0 := h \cdot 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We define the spectral index α such that it relates to flux density F_ν at frequency ν as $F_\nu \propto \nu^\alpha$. Regarding terminology, we strictly distinguish between a *radio galaxy*, a radio-bright structure of relativistic particles and magnetic fields (consisting of a core, jets, hotspots and lobes), and the *host galaxy* that generates it.

2. Data and methods

The LoTSS, conducted by the LOFAR High-band Antennae (HBA), is a 120–168 MHz interferometric survey (Shimwell et al. 2017, 2019, in prep.) with the ultimate aim to image the full Northern Sky at resolutions of 6'', 20'', 60'' and 90''. Its central frequency $\nu_c = 144$ MHz. The latest data release – the LoTSS DR2 (Shimwell et al. in prep.) – covers 27% of the Northern Sky, split over two regions of 4178 deg² and 1457 deg²; the largest of these contains the Sloan Digital Sky Survey (SDSS) DR7 (Abazajian et al. 2009) area. By default, the LoTSS DR2 provides imagery at the 6'' and 20'' resolutions. We show these standard products in Figure 2 for the same sky region as in Figure 1. In terms of total source counts, the LoTSS DR2 is the largest radio survey carried out thus far: its catalogue contains $4.4 \cdot 10^6$ sources, most of which are considered *compact*. By contrast, the 60'' and 90'' imagery, which we discuss in more detail in Oei et al. (in prep.), is intended to reveal *extended* structures in the low-frequency radio sky, such

¹ Source confusion is an instrumental limitation that arises when the resolution of an image is low compared to the sky density of statistically significant sources. It causes angularly adjacent, but physically unrelated sources to blend together, making it hard or even impossible to distinguish them (e.g. Condon et al. 2012).

² Alcyoneus was the son of Ouranos, the Greek primordial god of the sky. According to Ps.-Apollodorus, he was also one of the greatest of the *Gigantes* (Giants), and a challenger to Heracles during the Gigantomachy – the battle between the Giants and the Olympian gods for supremacy over the Cosmos. The poet Pindar described him as ‘huge as a mountain’, fighting by hurling rocks at his foes.

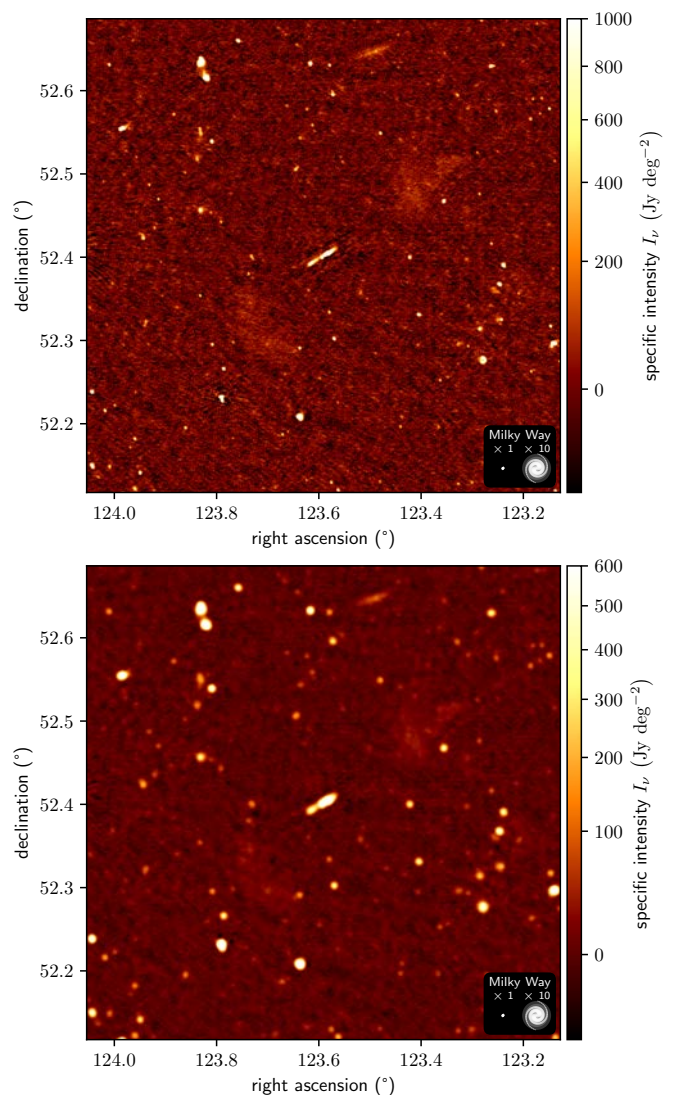


Fig. 2: **Alcyoneus’ lobes are easily overlooked in the LoTSS DR2 at its standard resolutions.** We show images at central frequency $\nu_c = 144$ MHz and resolutions $\theta_{\text{FWHM}} = 6''$ (top) and $\theta_{\text{FWHM}} = 20''$ (bottom), centred around host galaxy J081421.68+522410.0.

as giant radio galaxies, supernova remnants in the Milky Way, radio halos and shocks in galaxy clusters, and – potentially – accretion shocks or volume-filling emission from filaments of the Cosmic Web. To avoid the source confusion limit at these resolutions, following van Weeren et al. (2021), we used DDFacet (Tasse et al. 2018) to predict visibilities corresponding to the 20'' LoTSS DR2 sky model and subtracted these from the data, before imaging at 60'' and 90'' with WSClean IDG (Offringa et al. 2014; van der Tol et al. 2018). We used -0.5 Briggs weighting and multiscale CLEAN (Offringa & Smirnov 2017), with -multiscale-scales 0, 4, 8, 16, 32, 64. Importantly, we did not impose an inner (u, v) -cut. We imaged each pointing separately, then combined the partially overlapping images into a mosaic by calculating, for each direction, a beam-weighted average.

Finally, we visually searched the LoTSS DR2 for GRGs, primarily at 6'' and 60'' using the Hierarchical Progressive Survey (HiPS) system in *Aladin Desktop 11.0* (Bonnarel et al. 2000).

3. Results and discussion

3.1. Radio morphology and interpretation

During our LoTSS DR2 search, we identified a three-component radio structure of total angular length $\phi = 20.8'$, visible at all ($6''$, $20''$, $60''$ and $90''$) resolutions. **Figure 2** provides a sense of our data quality; it shows that the outer components are barely discernible in the LoTSS DR2 at its standard $6''$ and $20''$ resolutions. Meanwhile, **Figure 1** shows the outer components at $60''$, and the top panel of **Figure 9** shows them at $90''$; at these resolutions, they lie firmly above the noise. Compared with the outer structures, the central structure is bright and elongated, with a $155''$ major axis and a $20''$ minor axis. The outer structures lie along the major axis at similar distances from the central structure, are diffuse and amorphous, and feature specific intensity maxima along this axis.

In the arcminute-scale vicinity of the outer structures, the DESI Legacy Imaging Surveys (Dey et al. 2019) DR9 does not reveal galaxy overdensities or low-redshift spiral galaxies, the ROSAT All-sky Survey (RASS) (Voges et al. 1999) does not show X-ray brightness above the noise, and there is no Planck Sunyaev-Zeldovich catalogue 2 (PSZ2) (Planck Collaboration et al. 2016) source nearby. The outer structures therefore cannot be supernova remnants in low-redshift spiral galaxies or radio relics and radio halos in galaxy clusters. Instead, the outer structures presumably represent radio galaxy emission. The radio-optical overlays in **Figure 3**'s top and bottom panel show that it is improbable that each outer structure is a radio galaxy of its own, given the lack of significant $6''$ radio emission (solid light green contours) around host galaxy candidates suggested by the morphology of the $60''$ radio emission (translucent white contours). For these reasons, we interpret the central (jet-like) structure and the outer (lobe-like) structures as components of the *same* radio galaxy.

Subsequent analysis – presented below – demonstrates that this radio galaxy is the largest hitherto discovered, with a projected proper length of 5.0 Mpc. We dub this GRG *Alcyoneus*.

3.2. Host galaxy identification

Based on the middle panel of **Figure 3** and an SDSS DR12 (Alam et al. 2015) spectrum, we identify a source at a J2000 right ascension of 123.590372° , a declination of 52.402795° and a spectroscopic redshift of $z_{\text{spec}} = 0.24674 \pm 6 \cdot 10^{-5}$ as *Alcyoneus*' host. Like most GRG hosts, this source, with SDSS DR12 name J081421.68+522410.0, is an elliptical galaxy³ without a quasar. From optical contours, we find that the galaxy's minor axis makes a $\sim 20^\circ$ angle with *Alcyoneus*' jet axis.

In **Figure 4**, we further explore the connection between J081421.68+522410.0 and *Alcyoneus*' radio core and jets. From top to bottom, we show the LoTSS DR2 at $6''$, the Very Large Array Sky Survey (VLASS) (Lacy et al. 2020) at $2.2''$, and the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS) DR1 (Chambers et al. 2016) *i*-band. Two facts confirm that the host identification is highly certain. First, for both the LoTSS DR2 at $6''$ and the VLASS at $2.2''$, the angular separation between J081421.68+522410.0 and the arc connecting *Alcyoneus*' two innermost jet features is subarcsecond. Moreover, the alleged host galaxy is the brightest Pan-STARRS DR1 *i*-band

³ Based on the SDSS morphology, Kuminski & Shamir (2016) calculate a probability of 89% that the galaxy is an elliptical.

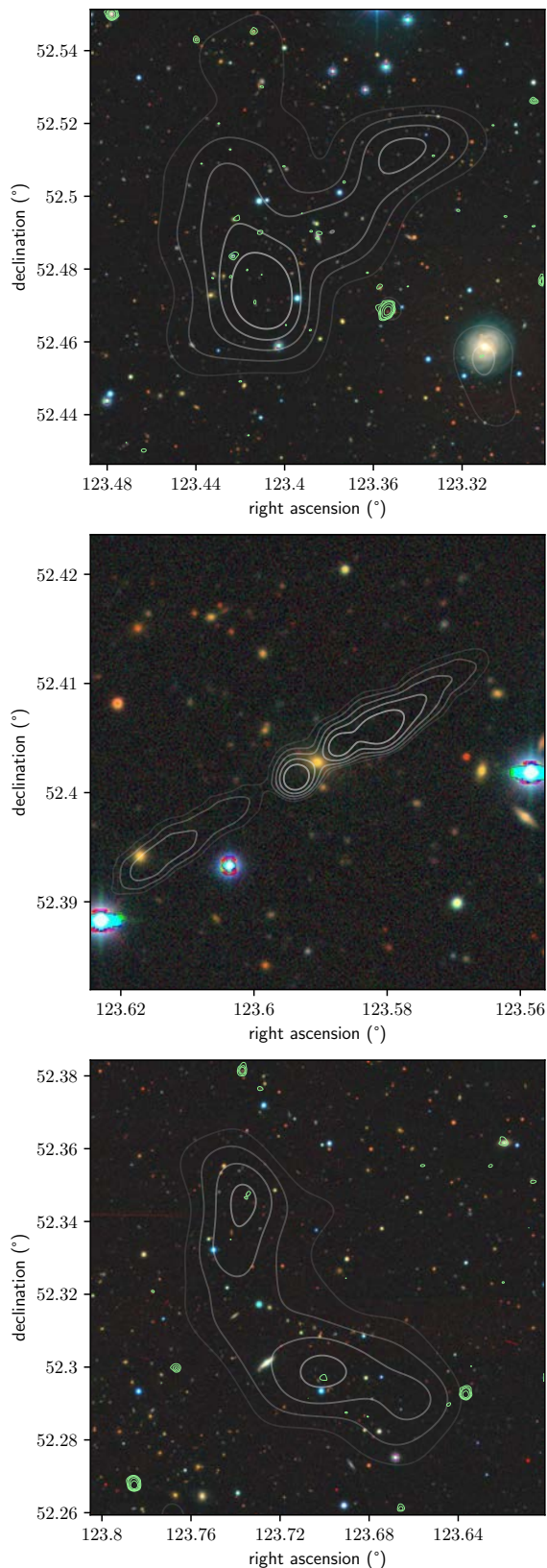


Fig. 3: Joint radio-optical views show that Figure 1's outer structures are best interpreted as a pair of radio galaxy lobes fed by central jets. On top of DESI Legacy Imaging Surveys DR9 (*g, r, z*)-imagery, we show the LoTSS DR2 at various resolutions through contours at multiples of σ , where σ is the image noise at the relevant resolution. The top and bottom panel show translucent white $60''$ contours at $3, 5, 7, 9, 11\sigma$ and solid light green $6''$ contours at $4, 7, 10, 20, 40\sigma$. The central panel shows translucent white $6''$ contours at $5, 10, 20, 40, 80\sigma$.

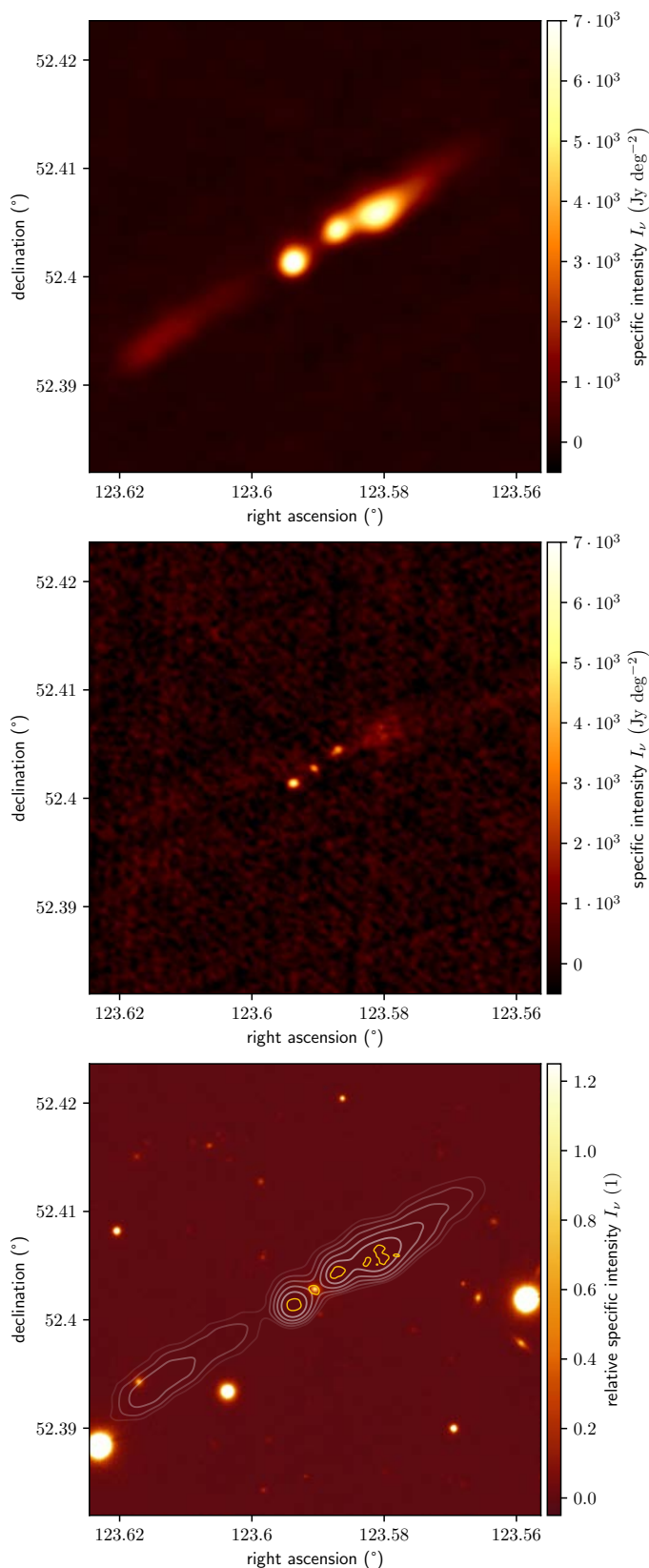


Fig. 4: The SDSS DR12 source J081421.68+522410.0 is Alcyoneus’ host galaxy. The panels cover a $2.5' \times 2.5'$ region around J081421.68+522410.0, an elliptical galaxy with spectroscopic redshift $z_{\text{spec}} = 0.24674 \pm 6 \cdot 10^{-5}$. From top to bottom, we show the LoTSS DR2 6'', the VLASS 2.2'', and the Pan-STARRS DR1 *i*-band – relative to the peak specific intensity of J081421.68+522410.0 – with LoTSS contours (white) as in **Figure 3** and a VLASS contour (gold) at 5σ .

source within a radius of $45''$ of the central VLASS image component.

3.3. Radiative- or jet-mode active galactic nucleus

Current understanding (e.g. Heckman & Best 2014) suggests that the population of active galactic nuclei (AGN) exhibits a dichotomy: AGN seem to be either radiative-mode AGN, which generate high-excitation radio galaxies (HERGs), or jet-mode AGN, which generate low-excitation radio galaxies (LERGs). Is Alcyoneus a HERG or a LERG? The SDSS spectrum of the host features very weak emission lines; indeed, the star formation rate (SFR) is just $1.6 \cdot 10^{-2} M_{\odot} \text{ yr}^{-1}$ (Chang et al. 2015). Following the classification rule of Best & Heckman (2012); Best et al. (2014); Pracy et al. (2016); Williams et al. (2018) based on the strength and equivalent width of the OIII 5007 Å line, we conclude that Alcyoneus is a LERG. Moreover, the WISE photometry (Cutri et al. 2012) at $11.6 \mu\text{m}$ and $22.1 \mu\text{m}$ is below the instrumental detection limit. Following the classification rule of Gürkan et al. (2014) based on the $22.1 \mu\text{m}$ luminosity density, we affirm that Alcyoneus is a LERG. Through automated classification, Best & Heckman (2012) came to the same conclusion. Being a jet-mode AGN, the supermassive black hole (SMBH) in the centre of Alcyoneus’ host galaxy presumably accretes at an efficiency below 1% of the Eddington limit, and is fueled mainly by slowly cooling hot gas.

3.4. Projected proper length

We calculate Alcyoneus’ projected proper length l_p through its angular length ϕ and spectroscopic redshift z_{spec} . We formally determine $\phi = 20.8' \pm 0.15'$ from the compact-source-subtracted $90''$ image (top panel of **Figure 9**) by selecting the largest great-circle distance between all possible pairs of pixels with a specific intensity higher than three sigma-clipped standard deviations above the sigma-clipped median. We find $l_p = 4.99 \pm 0.04$ Mpc; this makes Alcyoneus the projectively largest radio galaxy known. For methodology details, and for a probabilistic comparison between the projected proper lengths of Alcyoneus and J1420-0545, see **Appendix A**.

3.5. Radio luminosity densities and kinetic jet powers

From the LoTSS DR2 $6''$ image (top panel of **Figure 4**), we measure that two northern jet local maxima occur at angular distances of $9.2 \pm 0.2''$ and $23.7 \pm 0.2''$ from the host, or at projected proper distances of 36.8 ± 0.8 kpc and 94.8 ± 0.8 kpc. Two southern jet local maxima occur at angular distances of $8.8 \pm 0.2''$ and $62.5 \pm 0.2''$ from the host, or at projected proper distances of 35.2 ± 0.8 kpc and 249.9 ± 0.8 kpc.

At the central observing frequency of $\nu_c = 144$ MHz, the northern jet has a flux density $F_{\nu} = 193 \pm 20$ mJy, the southern jet has $F_{\nu} = 110 \pm 12$ mJy, whilst the northern lobe has $F_{\nu} = 63 \pm 7$ mJy and the southern lobe has $F_{\nu} = 44 \pm 5$ mJy. To minimise contamination from fore- and background galaxies, we determined the lobe flux densities from the compact-source-subtracted $90''$ image. The flux density uncertainties are dominated by the 10% flux scale uncertainty inherent to the LoTSS DR2 (Shimwell et al. in prep.). The host galaxy flux density is relatively weak, and the corresponding emission has, at $\nu_c = 144$ MHz and $6''$ resolution, no clear angular separation from the inner jets’ emission; we have therefore not determined it.

Due to cosmological redshifting, the conversion between flux

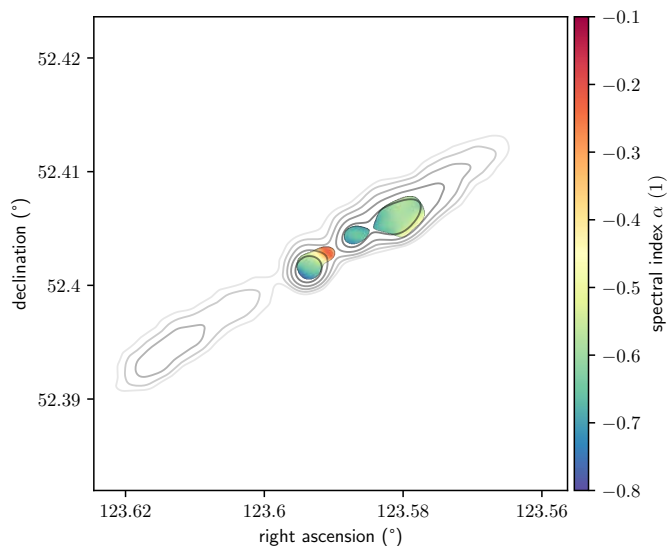


Fig. 5: **The LoTSS–VLASS spectral index map reveals Alcyoneus’ flat-spectrum core and steeper-spectrum jets.** We show all directions where both the LoTSS and VLASS image have at least 5σ significance. In black, we overlay the same LoTSS contours as in **Figures 3 and 4**. The core spectral index is $\alpha = -0.25 \pm 0.1$ and the combined inner jet spectral index is $\alpha = -0.65 \pm 0.1$.

density and luminosity density depends on the spectral indices α of Alcyoneus’ luminous components. We estimate the spectral indices of the core and jets from the LoTSS DR2 $6''$ and VLASS $2.2''$ images. After convolving the VLASS image with a Gaussian to the common resolution of $6''$, we calculate the mean spectral index between LoTSS’ $\nu_c = 144$ MHz and VLASS’ $\nu_c = 2.99$ GHz. Using only directions for which both images have a significance of at least 5σ , we deduce a core spectral index $\alpha = -0.25 \pm 0.1$ and a combined inner jet spectral index $\alpha = -0.65 \pm 0.1$. The spectral index uncertainties are dominated by the LoTSS DR2 and VLASS flux scale uncertainties. We show the full spectral index map in **Figure 5**. We have not determined the spectral index of the lobes, as they are only detected in the LoTSS imagery.

The luminosity densities of the northern and southern jet at *rest-frame* frequency $\nu = 144$ MHz are $L_\nu = (3.6 \pm 0.4) \cdot 10^{25}$ W Hz $^{-1}$ and $L_\nu = (2.0 \pm 0.2) \cdot 10^{25}$ W Hz $^{-1}$, respectively. Following [Dabhade et al. \(2020a\)](#), we estimate the kinetic power of the jets from their luminosity densities and the results of the simulation-based analytical model of [Hardcastle \(2018\)](#). We find $Q_{\text{jet},1} = 1.2 \pm 0.1 \cdot 10^{36}$ W and $Q_{\text{jet},2} = 6.6 \pm 0.7 \cdot 10^{35}$ W, so that the total kinetic jet power is $Q_{\text{jets}} := Q_{\text{jet},1} + Q_{\text{jet},2} = 1.9 \pm 0.2 \cdot 10^{36}$ W. Interestingly, this total kinetic jet power is *lower* than the average $Q_{\text{jets}} = 3.7 \cdot 10^{36}$ W, and close to the median $Q_{\text{jets}} = 2.2 \cdot 10^{36}$ W, for low-excitation giant radio galaxies (LEGRGs) in the redshift range $0.18 < z < 0.43$ ([Dabhade et al. 2020a](#)).

Because the lobe spectral indices are unknown, we present luminosity densities for several possible values of α in **Table 1**.⁴ (Because of electron ageing, α will decrease further away from the core.)

⁴ The inferred luminosity densities have a cosmology-dependence; our results are $\sim 6\%$ higher than for modern high- H_0 cosmologies.

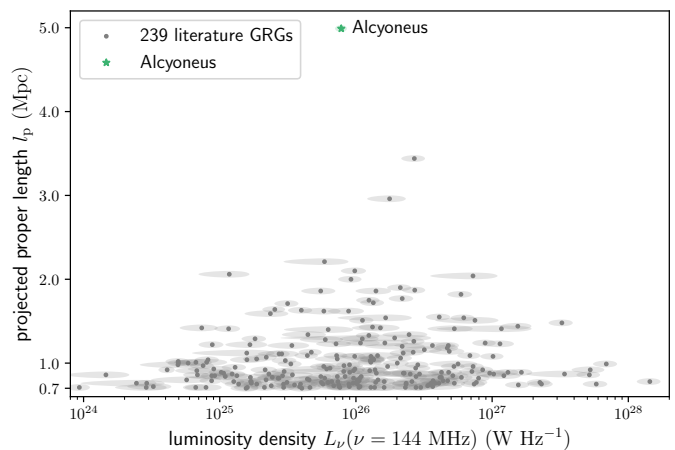


Fig. 6: **Alcyoneus has a low-frequency luminosity density typical for GRGs.** We explore the relation between GRG projected proper length l_p and total luminosity density L_ν at *rest-frame* frequency $\nu = 144$ MHz. Total luminosity densities include contributions from all available radio galaxy components (i.e. the core, jets, hotspots and lobes). Literature GRGs are from [Dabhade et al. \(2020b\)](#), and are marked with grey disks, while Alcyoneus is marked with a green star. Translucent ellipses indicate -1 to $+1$ standard deviation uncertainties. Alcyoneus has a typical luminosity density (percentile $45 \pm 3\%$).

Table 1: Luminosity densities L_ν (in 10^{24} W Hz $^{-1}$) of Alcyoneus’ lobes for three potential spectral indices α at *rest-frame* frequency $\nu = 144$ MHz, assuming a [Planck Collaboration et al. \(2020\)](#) cosmology.

	$\alpha = -0.8$	$\alpha = -1.2$	$\alpha = -1.6$
Northern lobe	12 ± 1	13 ± 1	14 ± 1
Southern lobe	8.3 ± 0.8	9.0 ± 0.9	9.9 ± 1

Assuming $\alpha = -1.2$, Alcyoneus total luminosity density at $\nu = 144$ MHz is $L_\nu = 7.8 \pm 0.8 \cdot 10^{25}$ W Hz $^{-1}$. In **Figure 6**, we compare this estimate to other GRGs’ total luminosity density at the same frequency, as found by [Dabhade et al. \(2020b\)](#) through the LoTSS DR1 ([Shimwell et al. 2019](#)). Interestingly, Alcyoneus is not particularly luminous: it has a low-frequency luminosity density typical for the currently known GRG population (percentile $45 \pm 3\%$).

3.6. True proper length: relativistic beaming

Following [Hardcastle et al. \(1998a\)](#), we simultaneously constrain Alcyoneus’ jet speed u and inclination angle θ from the jets’ flux density asymmetry: the northern-to-southern jet flux density ratio $J = 1.78 \pm 0.3$.⁵ We assume that the jets propagate with identical speeds u in exactly opposing directions (making angles with the line-of-sight θ and $\theta + 180^\circ$), and have statistically identical relativistic electron populations, so that they have a common synchrotron spectral index α . Using $\alpha = -0.65 \pm 0.1$ as before, and

$$\beta := \frac{u}{c}; \quad \beta \cos \theta = \frac{J^{\frac{1}{2-\alpha}} - 1}{J^{\frac{1}{2-\alpha}} + 1}, \quad (1)$$

⁵ Because J is obtained through division of two independent normal random variables (RVs) with non-zero mean, J is an RV with an uncorrelated noncentral normal ratio distribution.

we find $\beta \cos \theta = 0.106 \pm 0.03$. Because $\cos \theta \leq 1$, β is bounded from below by $\beta_{\min} = 0.106 \pm 0.03$.

From detailed modelling of ten Fanaroff–Riley (FR) I radio galaxies (which have jet luminosities comparable to Alcyoneus’), Laing & Bridle (2014) deduced that initial jet speeds are roughly $\beta = 0.8$, which decrease until roughly $0.6 r_0$, with r_0 being the recollimation distance. Most of Laing & Bridle (2014)’s ten recollimation distances are between 5 and 15 kpc, with the largest being that of NGC 315: $r_0 = 35$ kpc. Because the local specific intensity maxima in Alcyoneus’ jets closest to the host occur at *projected* proper distances of 36.8 ± 0.8 kpc and 35.2 ± 0.8 kpc, the true proper distances must be even larger. We conclude that the observed jet emission presumably comes from a region further from the host than r_0 , so that the initial stage of jet deceleration – in which the jet speed is typically reduced by several tens of percents of c – must already be completed. Thus, $\beta_{\max} = 0.8$ is a safe upper bound.

Taking $\beta_{\max} = 0.8$, θ is bounded from above by $\theta_{\max} = 82.4 \pm 2^\circ$ ($\theta \in [0, 90^\circ]$), or bounded from below by $180^\circ - \theta_{\max} = 97.6 \pm 2^\circ$ ($\theta \in [90^\circ, 180^\circ]$).⁶ If we model Alcyoneus’ geometry as a line segment, and assume no jet reorientation, Alcyoneus’ true proper length l and projected proper length l_p relate as

$$l = \frac{l_p}{\sin \theta}; \quad l \geq l_{\min} = \frac{l_p}{\sin \theta_{\max}}. \quad (2)$$

We bound l from below: $l_{\min} = 5.04 \pm 0.05$ Mpc. A *triangular* prior on β between β_{\min} and β_{\max} with the mode at β_{\max} induces a skewed prior on l ; the 90% credible interval is $l \in [5.0 \text{ Mpc}, 5.5 \text{ Mpc}]$, with the mean and median being 5.2 Mpc and 5.1 Mpc, respectively. A *flat* prior on β between β_{\min} and β_{\max} also induces a skewed prior on l ; the 90% credible interval is $l \in [5.0 \text{ Mpc}, 7.1 \text{ Mpc}]$, with the mean and median being 5.6 Mpc and 5.1 Mpc, respectively. The median of l seems particularly well determined, as it is insensitive to variations of the prior on β .

In **Appendix B**, we explore the inclination angle conditions under which Alcyoneus has the largest true proper length of all known (> 4 Mpc) GRGs.

3.7. Stellar and supermassive black hole mass

Does a galaxy or its central black hole need to be massive in order to generate a GRG?

Alcyoneus’ host has a stellar mass $M_\star = 2.4 \pm 0.4 \cdot 10^{11} M_\odot$ (Chang et al. 2015). We test whether or not this is a typical stellar mass among the total known GRG population. We assemble a literature catalogue of 1013 GRGs by merging the compendium of Dabhade et al. (2020a), which is complete up to April 2020, with the GRGs discovered in Galvin et al. (2020), Ishwara-Chandra et al. (2020), Tang et al. (2020), Bassani et al. (2021), Brügggen et al. (2021), Delhaize et al. (2021), Masini et al. (2021), Kuźmicz & Jamroz (2021), Andernach et al. (2021) and Mahato et al. (2021). We collect stellar masses with uncertainties from Chang et al. (2015), which are based on SDSS and WISE photometry, and from Salim et al. (2018), which are based on GALEX, SDSS and WISE photometry. We give precedence to the stellar masses by Salim et al. (2018) when both are available. We obtain stellar masses for 151 previously known GRGs. The typical stellar mass range is $10^{11} - 10^{12} M_\odot$, the median

⁶ Taking $\beta_{\max} = 1$ instead, θ is bounded from above by $\theta_{\max} = 83.9 \pm 2^\circ$ ($\theta \in [0, 90^\circ]$), or bounded from below by $180^\circ - \theta_{\max} = 96.1 \pm 2^\circ$ ($\theta \in [90^\circ, 180^\circ]$).

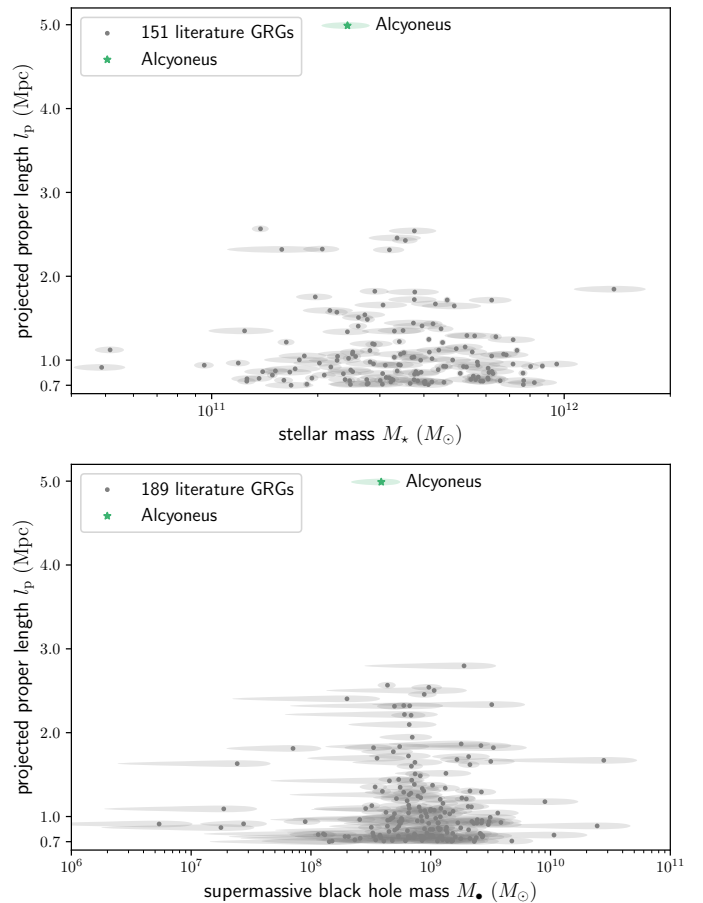


Fig. 7: Alcyoneus’ host has a lower stellar and supermassive black hole mass than most GRG hosts. We explore the relations between GRG projected proper length l_p and host galaxy stellar mass M_\star (top panel) or host galaxy supermassive black hole mass M_\bullet (bottom panel). Our methods allow determining these properties for a small proportion of all literature GRGs only. Literature GRGs are marked with grey disks, while Alcyoneus is marked with a green star. Translucent ellipses indicate -1 to $+1$ standard deviation uncertainties. Alcyoneus’ host has a fairly typical – though below-average – stellar mass (percentile $25 \pm 9\%$) and supermassive black hole mass (percentile $23 \pm 11\%$).

$M_\star = 3.5 \cdot 10^{11} M_\odot$ and the mean $M_\star = 3.8 \cdot 10^{11} M_\odot$. Strikingly, the top panel of **Figure 7** illustrates that Alcyoneus’ host has a fairly *low* (percentile $25 \pm 9\%$) stellar mass compared with the currently known population of GRG hosts.

For the GRGs in our literature catalogue, we also estimate SMBH masses via the M-sigma relation. We collect SDSS DR12 stellar velocity dispersions with uncertainties (Alam et al. 2015), and apply the M-sigma relation of **Equation 7** in Kormendy & Ho (2013). Alcyoneus’ host has a SMBH mass $M_\bullet = 3.9 \pm 1.7 \cdot 10^8 M_\odot$. We obtain SMBH masses for 189 previously known GRGs. The typical SMBH mass range is $10^8 - 10^{10} M_\odot$, the median $M_\bullet = 7.9 \cdot 10^8 M_\odot$ and the mean $M_\bullet = 1.5 \cdot 10^9 M_\odot$. Strikingly, the bottom panel of **Figure 7** illustrates that Alcyoneus’ host has a fairly *low* (percentile $23 \pm 11\%$) SMBH mass compared with the currently known population of GRG hosts. We note that Alcyoneus is the only GRG with $l_p > 3$ Mpc whose host’s stellar mass is known through Chang et al. (2015) or Salim et al. (2018), and whose host’s SMBH mass can be estimated

through its SDSS DR12 velocity dispersion. These data allow us to state confidently that exceptionally high stellar or SMBH masses are not necessary to generate 5-Mpc-scale GRGs.

3.8. Surrounding large-scale structure

Several approaches to large-scale structure (LSS) classification, such as the T-web scheme (Hahn et al. 2007), partition the modern Universe into galaxy clusters, filaments, sheets and voids. In this section, we determine Alcyoneus’ most likely environment type.

We conduct a tentative quantitative analysis using the SDSS DR7 spectroscopic galaxy sample (Abazajian et al. 2009). Does Alcyoneus’ host have fewer, about equal or more galactic neighbours in SDSS DR7 than a randomly drawn galaxy of similar r -band luminosity density and redshift? Let $r(z)$ be the comoving radial distance corresponding to cosmological redshift z . We consider a spherical shell with the observer at the centre, inner radius $\max\{r(z = z_{\text{spec}}) - r_0, 0\}$ and outer radius $r(z = z_{\text{spec}}) + r_0$. We approximate Alcyoneus’ cosmological redshift with z_{spec} and choose $r_0 = 25$ Mpc. As all galaxies in the spherical shell have a similar distance to the observer (i.e. distances are at most $2r_0$ different), the SDSS DR7 galaxy number density *completeness* must also be similar throughout the spherical shell.⁷ For each enclosed galaxy with an r -band luminosity density between $1 - \delta$ and $1 + \delta$ times that of Alcyoneus’ host, we count the number of SDSS DR7 galaxies $N_{<R}(R)$ within a sphere of comoving radius R around it – regardless of luminosity density, and excluding itself. Alcyoneus’ host has an SDSS r -band apparent magnitude $m_r = 18.20$; the corresponding luminosity density is $L_\nu(\lambda_c = 623.1 \text{ nm}) = 3.75 \cdot 10^{22} \text{ W Hz}^{-1}$. We choose $\delta = 0.25$; this yields 9,358 such enclosed galaxies.

In **Figure 8**, we show the distribution of $N_{<R}(R)$ for $R = 5$ Mpc and $R = 10$ Mpc. We verify that the distributions are insensitive to reasonable changes in r_0 and δ . Note that there is no SDSS DR7 galaxy within a comoving distance of 5 Mpc from Alcyoneus’ host. The nearest such galaxy, J081323.49+524856.1, occurs at a comoving distance of 7.9 Mpc: the nearest $\sim 2,000 \text{ Mpc}^3$ of comoving space are free of galactic neighbours with $L_\nu(\lambda_c) > 5.57 \cdot 10^{22} \text{ W Hz}^{-1}$.⁸ In the same way as in **Section 3.1**, we verify that the DESI Legacy Imaging Surveys DR9, RASS and PSZ2 do not contain evidence for a galaxy cluster in the direction of Alcyoneus’ host. The nearest galaxy cluster, according to the SDSS-III cluster catalogue of Wen et al. (2012), instead lies $24'$ away at right ascension 123.19926° , declination 52.72468° and photometric redshift $z_{\text{ph}} = 0.2488$. It has an $R_{200} = 1.1$ Mpc and, according to the DESI cluster catalogue of Zou et al. (2021), a total mass $M = 2.2 \cdot 10^{14} M_\odot$. The comoving distance between the cluster and Alcyoneus’ host is 11 Mpc. All in all, we conclude that Alcyoneus does not reside in a galaxy cluster. Meanwhile, there are five SDSS DR7 galaxies within a comoving distance of 10 Mpc from Alcyoneus’ host: this makes it implausible that Alcyoneus lies in a void. Finally, one could interpret $N_{<R}(R)$ as a proxy for the LSS total matter density around a galaxy. For $R = 10$ Mpc, just 17% of galaxies in the shell with a similar luminosity density as Alcyoneus’ host have a higher LSS total matter density. Being on the high end of the

⁷ For $r_0 = 25$ Mpc, this is a good approximation, because the shell is cosmologically thin: $2r_0 = 50$ Mpc roughly amounts to the length of a *single* Cosmic Web filament.

⁸ This is the luminosity density that corresponds to the SDSS r -band apparent magnitude completeness limit $m_r = 17.77$ (Strauss et al. 2002).

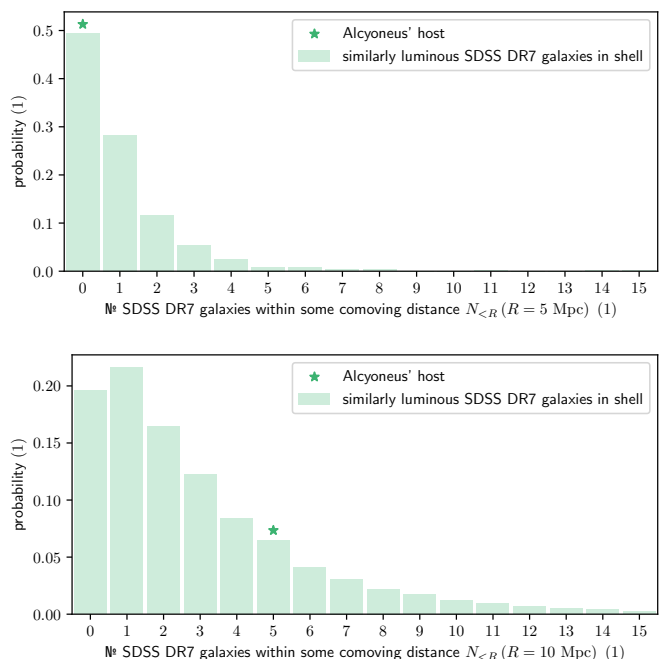


Fig. 8: Like most galaxies of similar r -band luminosity density and redshift, Alcyoneus’ host has no galactic neighbours in SDSS DR7 within 5 Mpc. However, within 10 Mpc, Alcyoneus’ host has more neighbours than most similar galaxies. For all 9,358 SDSS DR7 galaxies with an r -band luminosity density between 75% and 125% that of Alcyoneus’ host and a comoving radial distance that differs at most $r_0 = 25$ Mpc from Alcyoneus’, we count the number of SDSS DR7 galaxies $N_{<R}(R)$ within a sphere of comoving radius $R = 5$ Mpc (top panel) and $R = 10$ Mpc (bottom panel). The top panel indicates that Alcyoneus does not inhabit a galaxy cluster; the bottom panel indicates that Alcyoneus does not inhabit a void.

density distribution, but lying outside a cluster, Alcyoneus most probably inhabits a filament of the Cosmic Web.

3.9. Proper lobe volumes

We determine the proper volumes of Alcyoneus’ lobes with a new Bayesian model. The model describes the lobes through a pair of doubly truncated, optically thin cones, each of which has a spatially constant and isotropic monochromatic emission coefficient (MEC) (Rybicki & Lightman 1986). We allow the 3D orientations and opening angles of the cones to differ, as the lobes may traverse their way through differently pressured parts of the warm-hot intergalactic medium (WHIM): e.g. the medium near the filament axis, and the medium near the surrounding voids. By adopting a *spatially constant* MEC, we neglect electron density and magnetic field inhomogeneities as well as spectral-ageing gradients; by adopting an *isotropic* MEC, we assume non-relativistic velocities within the lobe so that beaming effects are negligible. Numerically, we first generate the GRG’s 3D MEC field over a cubical voxel grid, and then calculate the corresponding model image through projection, including expansion-related cosmological effects. Before comparison with the observed image, we convolve the model image with a Gaussian kernel to the appropriate resolution. We exploit the approximately Gaussian LoTSS DR2 image noise to formu-

late the likelihood, and assume a flat prior distribution over the parameters. Using a Metropolis–Hastings (MH) Markov chain Monte Carlo (MCMC), we sample from the posterior distribution.⁹

In the top panel of **Figure 9**, we show the LoTSS DR2 compact-source-subtracted 90'' image of Alcyoneus. The central region has been excluded from source subtraction, and hence Alcyoneus' core and jets remain. (However, when we run our MH MCMC on this image, we do mask this central region.) In the middle panel, we show the highest-likelihood (and thus maximum a posteriori (MAP)) model image *before* convolution. In the bottom panel, we show the same model image convolved to 90'' resolution, with 2σ and 3σ contours of the observed image overlaid. We provide the full parameter set that corresponds with this model in **Table C.1**.

The posterior mean, calculated through the MH MCMC samples after burn-in, suggests the following geometry. The northern lobe has an opening angle $\gamma_1 = 10 \pm 1^\circ$, and the cone truncates at an inner distance $d_{i,1} = 2.6 \pm 0.2$ Mpc and at an outer distance $d_{o,1} = 4.0 \pm 0.2$ Mpc from the host galaxy. The southern lobe has a larger opening angle $\gamma_2 = 26 \pm 2^\circ$, but its cone truncates at smaller distances of $d_{i,2} = 1.5 \pm 0.1$ Mpc and $d_{o,2} = 2.0 \pm 0.1$ Mpc from the host galaxy. These parameters fix the proper volumes of Alcyoneus' northern and southern lobes. We find $V_1 = 1.5 \pm 0.2$ Mpc³ and $V_2 = 1.0 \pm 0.2$ Mpc³, respectively (see **Equation C.15**).¹⁰

How are the lobes oriented? **Figure 1** provides a visual hint that the lobes are subtly non-coaxial. The posterior indicates that the position angles of the northern and southern lobes are $\varphi_1 = 307 \pm 1^\circ$ and $\varphi_2 = 139 \pm 2^\circ$, respectively. The position angle difference is thus $\Delta\varphi = 168 \pm 2^\circ$: although close to $\Delta\varphi = 180^\circ$, we can reject coaxiality with high significance. Interestingly, the posterior also constrains the angles that the lobe axes make with the plane of the sky: $|\theta_1 - 90^\circ| = 51 \pm 2^\circ$ and $|\theta_2 - 90^\circ| = 18 \pm 7^\circ$. Again, the uncertainties imply that the lobes are probably *not* coaxial. We stress that these inclination angle results are tentative only. Future model extensions should explore how sensitive they are to the assumed lobe geometry (by testing other shapes than just truncated cones, such as ellipsoids).

One way to validate the model is to compare the observed lobe flux densities of **Section 3.5** to the predicted lobe flux densities. According to the posterior, the MECs of the northern and southern lobes are $j_{v,1} = 17 \pm 2$ Jy deg⁻² Mpc⁻¹ and $j_{v,2} = 18 \pm 3$ Jy deg⁻² Mpc⁻¹. Combining MECs and volumes, we predict northern and southern lobe flux densities $F_{v,1}(v_c) = 63 \pm 4$ mJy and $F_{v,2}(v_c) = 45 \pm 5$ mJy (see **Equation C.16**). We find excellent agreement: the relative differences with the observed results are 0% and 2%, respectively.

⁹ For a detailed description of the model parameters, the MH MCMC and formulae for derived quantities, see **Appendix C**.

¹⁰ As a sanity check, we compare our results to those from a less rigorous, though simpler ellipsoid-based method of estimating volumes. By fitting ellipses to **Figure 9**'s top panel image, one obtains a semi-minor and semi-major axis; the half-diameter along the ellipsoid's third dimension is assumed to be their mean. This method suggests a northern lobe volume $V_1 = 1.4 \pm 0.3$ Mpc³ and a southern lobe volume $V_2 = 1.1 \pm 0.3$ Mpc³. These results agree well with our Bayesian model results. (If the half-diameter along the third dimension is instead treated as an RV with a uniform distribution between the semi-minor axis and the semi-major axis, the estimates remain the same.)

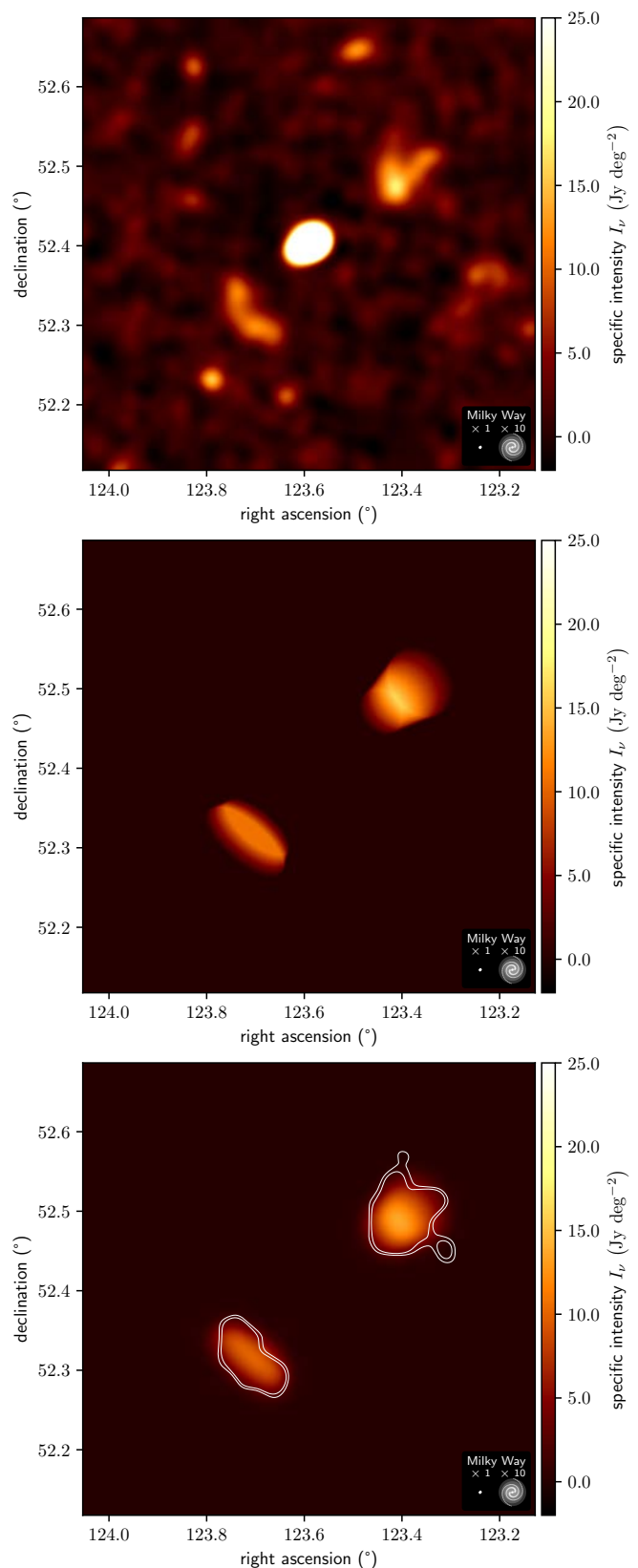


Fig. 9: Alcyoneus' lobe volumes can be estimated by comparing the observed radio image to modelled radio images. Top: LoTSS DR2 compact-source-subtracted 90'' image of Alcyoneus. For scale, we show the stellar Milky Way disk (diameter: 50 kpc) and a 10 times inflated version; the spiral galaxy shape follows [Ringermacher & Mead \(2009\)](#). **Middle:** Highest-likelihood model image. **Bottom:** The same model image convolved to 90'' resolution, with 2σ and 3σ contours of the observed image overlaid.

3.10. Lobe pressures and the local WHIM

From Alcyoneus' lobe flux densities and volumes, we can infer lobe pressures and magnetic field strengths. We calculate these through `pysynch`¹¹ (Hardcastle et al. 1998b), which uses the formulae first proposed by Myers & Spangler (1985) and re-examined by Beck & Krause (2005). Following the notation of Hardcastle et al. (1998b), we assume that the electron energy distribution is a power law in Lorentz factor γ with $\gamma_{\min} = 10$, $\gamma_{\max} = 10^4$ and exponent $p = -2$; we also assume that the kinetic energy density of protons is vanishingly small compared with that of electrons ($\kappa = 0$), and that the plasma filling factor is unity ($\phi = 1$). Assuming the minimum-energy condition (Burbidge 1956), we find minimum-energy pressures $P_{\min,1} = 4.8 \pm 0.3 \cdot 10^{-16}$ Pa and $P_{\min,2} = 4.9 \pm 0.6 \cdot 10^{-16}$ Pa for the northern and southern lobes, respectively. The corresponding minimum-energy magnetic field strengths are $B_{\min,1} = 46 \pm 1$ pT and $B_{\min,2} = 46 \pm 3$ pT. Assuming the equipartition condition (Pacholczyk 1970), we find equipartition pressures $P_{\text{eq},1} = 4.9 \pm 0.3 \cdot 10^{-16}$ Pa and $P_{\text{eq},2} = 4.9 \pm 0.6 \cdot 10^{-16}$ Pa for the northern and southern lobes, respectively. The corresponding equipartition magnetic field strengths are $B_{\text{eq},1} = 43 \pm 2$ pT and $B_{\text{eq},2} = 43 \pm 2$ pT. The minimum-energy and equipartition results do not differ significantly.

From pressures and volumes, we estimate the internal energy of the lobes $E = 3PV$. We find $E_{\min,1} = 6.2 \pm 0.5 \cdot 10^{52}$ J, $E_{\min,2} = 4.3 \pm 0.6 \cdot 10^{52}$ J, $E_{\text{eq},1} = 6.3 \pm 0.5 \cdot 10^{52}$ J and $E_{\text{eq},2} = 4.4 \pm 0.6 \cdot 10^{52}$ J. Next, we can bound the ages of the lobes from below by neglecting synchrotron losses, and assuming that the jets have been injecting energy in the lobes continuously at the currently observed kinetic jet powers. Using $\Delta t = EQ_{\text{jet}}^{-1}$, we find $\Delta t_{\min,1} = 1.7 \pm 0.2$ Gyr, $\Delta t_{\min,2} = 2.1 \pm 0.4$ Gyr, and identical results when assuming the equipartition condition. Finally, we can obtain a rough estimate of the average expansion speed of the radio galaxy during its lifetime $u = l_p(\Delta t)^{-1}$. We find $u = 2.6 \pm 0.3 \cdot 10^3$ km s⁻¹, or about 1% of the speed of light.

Several other authors (Andernach et al. 1992; Lacy et al. 1993; Subrahmanyan et al. 1996; Parma et al. 1996; Mack et al. 1998; Schoenmakers et al. 1998, 2000; Ishwara-Chandra & Saikia 1999; Lara et al. 2000; Machalski & Jamroz 2000; Machalski et al. 2001; Saripalli et al. 2002; Jamroz 2005; Subrahmanyan et al. 2006, 2008; Saikia et al. 2006; Machalski et al. 2006, 2007, 2008; Safouris et al. 2009; Malarecki et al. 2013; Tamhane et al. 2015; Sebastian et al. 2018; Heesen et al. 2018; Cantwell et al. 2020) have estimated the minimum-energy or equipartition pressure of the lobes of GRGs embedded in non-cluster environments (i.e. in voids, sheets or filaments of the Cosmic Web). We compare Alcyoneus to the other 151 GRGs with known lobe pressures in the top panel of Figure 10.¹² Alcyoneus reaffirms the negative correlation between length and lobe pressure (Jamroz & Machalski 2002; Machalski & Jamroz 2006), and has the lowest lobe pressures found thus far. Alcyoneus' lobe pressure is in fact so low, that it is comparable to the pressure in dense and hot parts of the WHIM: for a baryonic matter (BM) density

¹¹ The `pysynch` code is publicly available online: <https://github.com/mhardcastle/pysynch>.

¹² We have included all publications that provide pressures, energy densities or magnetic field strengths. Note that some authors assume $\gamma_{\min} = 1$, we assume $\gamma_{\min} = 10$ and Malarecki et al. (2013) assume $\gamma_{\min} = 10^3$. If possible, angular lengths were updated using the LoTSS DR2 at 6" and redshift estimates were updated using the SDSS DR12. All projected proper lengths have been recalculated using our Planck Collaboration et al. (2020) cosmology. When authors provided pressures for both lobes, we have taken the average.

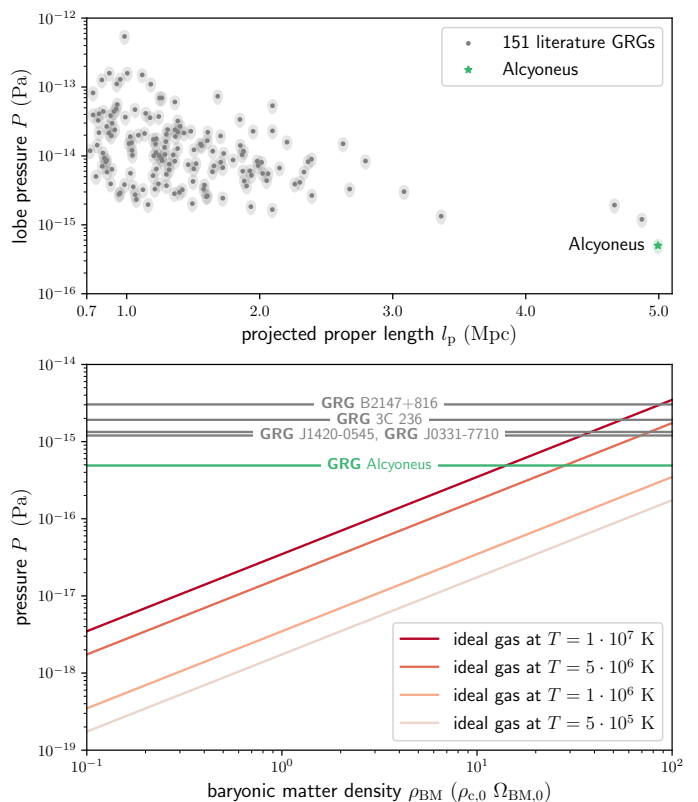


Fig. 10: Of all GRGs with known lobe pressures, Alcyoneus is the most plausible candidate for pressure equilibrium with the WHIM. In the top panel, we explore the relation between length and lobe pressure for Alcyoneus and 151 literature GRGs. In the bottom panel, we compare the lobe pressure of Alcyoneus (green line) with the lobe pressures of the largest four similarly analysed GRGs (grey lines) and with WHIM pressures (red lines).

$\rho_{\text{WHIM}} = 10 \rho_{c,0} \Omega_{\text{BM},0}$ and $T_{\text{WHIM}} = 10^7$ K, $P_{\text{WHIM}} = 4 \cdot 10^{-16}$ Pa. Here, $\rho_{c,0}$ is today's critical density, so that $\rho_{c,0} \Omega_{\text{BM},0}$ is today's mean baryon density. See the bottom panel of Figure 10 for a more extensive comparison between P_{\min} (green line) and P_{WHIM} (red lines). For comparison, we also show the lobe pressures of the four other thus-analysed GRGs with $l_p > 3$ Mpc (grey lines). These are J1420-0545 of $l_p = 4.9$ Mpc (Machalski et al. 2008), 3C 236 of $l_p = 4.7$ Mpc (Schoenmakers et al. 2000), J0331-7710 of $l_p = 3.4$ Mpc (Malarecki et al. 2013) and B2147+816 of $l_p = 3.1$ Mpc (Schoenmakers et al. 2000).

Although proposed as probes of WHIM thermodynamics for decades, the bottom panel of Figure 10 demonstrates that even the largest non-cluster literature GRGs are unlikely to be in pressure equilibrium with their environment. Relying on results from the Overwhelmingly Large Simulations (OWLS) (Schaye et al. 2010), Malarecki et al. (2013) point out that baryon densities $\rho_{\text{BM}} > 50 \rho_{c,0} \Omega_{\text{BM},0}$, which are necessary for pressure equilibrium in these GRGs (see the intersection of grey and red lines in the bottom panel of Figure 10), occur in only 1% of the WHIM's volume. By contrast, Alcyoneus can be in pressure equilibrium with the WHIM at baryon densities $\rho_{\text{BM}} \sim 20 \rho_{c,0} \Omega_{\text{BM},0}$, and thus represents the most promising intergalactic barometer of its kind yet.¹³

¹³ At Alcyoneus' redshift, this density amounts to a baryon overdensity of ~ 10 .

Why do most, if not all, observed non-cluster GRGs have over-pressured lobes? The top panel of **Figure 10** suggests that GRGs must grow to several Mpc to approach WHIM pressures in their lobes, and such GRGs are rare. However, the primary reason is the limited surface brightness sensitivity of all past and current surveys. Alcyoneus' lobes are visible in the LoTSS, but not in the NRAO VLA Sky Survey (NVSS) (Condon et al. 1998) or in the Westerbork Northern Sky Survey (WENSS) (Rengelink et al. 1997). Their pressure approaches that of the bulk of the WHIM within an order of magnitude. Lobes with even lower pressure must be less luminous or more voluminous, and thus will have even lower surface brightness. It is therefore probable that most GRG lobes that are in true pressure equilibrium with the WHIM still lie hidden in the radio sky.

4. Conclusion

1. We reprocess the LoTSS DR2, the latest version of the LOFAR's Northern Sky survey at 144 MHz, by subtracting angularly compact sources and imaging at 60'' and 90'' resolution. The resulting images (Oei et al. in prep.) allow us to explore a new sensitivity regime for radio galaxy lobes, and thus represent promising data to search for unknown GRGs of large angular length. We present a sample in forthcoming work.
2. We discover the first 5 Mpc GRG, which we dub *Alcyoneus*. The projected proper length is $l_p = 4.99 \pm 0.04$ Mpc, while the true proper length is at least $l_{\min} = 5.04 \pm 0.05$ Mpc. We confidently associate the $20.8' \pm 0.15'$ radio structure to an elliptical galaxy with a jet-mode AGN detected in the DESI Legacy Imaging Surveys DR9: the SDSS DR12 source J081421.68+522410.0 at J2000 right ascension 123.590372°, declination 52.402795° and spectroscopic redshift $0.24674 \pm 6 \cdot 10^{-5}$.
3. Alcyoneus has a total luminosity density at $\nu = 144$ MHz of $L_\nu = 8 \pm 1 \cdot 10^{25}$ W Hz⁻¹, which is typical for GRGs (percentile 45 ± 3%). Alcyoneus' host has a fairly low stellar mass and SMBH mass compared with other GRG hosts (percentiles 25±9% and 23±11%). This implies that – within the GRG population – no strong positive correlation between radio galaxy length and (instantaneous) low-frequency radio power, stellar mass or SMBH mass can exist.
4. The surrounding sky as imaged by the LoTSS, DESI Legacy Imaging Surveys, RASS and PSZ suggests that Alcyoneus does *not* inhabit a galaxy cluster. According to an SDSS-III cluster catalogue, the nearest cluster occurs at a comoving distance of 11 Mpc. A local galaxy number density count suggests that Alcyoneus instead inhabits a filament of the Cosmic Web. A low-density environment therefore remains a possible explanation for Alcyoneus' formidable size.
5. We develop a new Bayesian model that parametrises in three dimensions a pair of arbitrarily oriented, optically thin, doubly truncated conical radio galaxy lobes with constant monochromatic emission coefficient. We then generate the corresponding specific intensity function, taking into account cosmic expansion, and compare it to data assuming Gaussian image noise. We use Metropolis–Hastings Markov chain Monte Carlo to optimise the parameters, and thus determine northern and southern lobe volumes of 1.5 ± 0.2 Mpc³ and 1.0 ± 0.2 Mpc³, respectively. In total, the lobes have an internal energy of $\sim 10^{53}$ J, expelled from the host galaxy over a Gyr-scale period. The lobe pressures are $4.8 \pm 0.3 \cdot 10^{-16}$ Pa and $4.9 \pm 0.6 \cdot 10^{-16}$ Pa, respectively;

these are the lowest measured in radio galaxies yet. Nevertheless, the lobe pressures still exceed a large range of plausible WHIM pressures. Most likely, the lobes are still expanding – and Alcyoneus' struggle for supremacy of the Cosmos continues.

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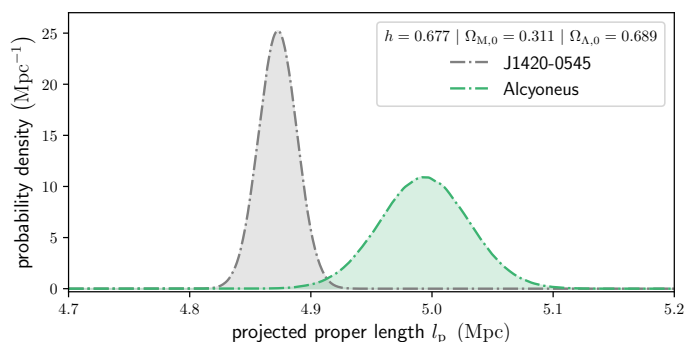


Fig. A.1: **Alcyoneus’ projected proper length just exceeds that of J1420-0545.** The probability that Alcyoneus (green) has a larger projected proper length than J1420-0545 (grey) (Machalski et al. 2008) is 99.9%. For both GRGs, we take into account uncertainty in angular length and spectroscopic redshift, as well as the possibility of peculiar motion along the line of sight.

Appendix A: J1420-0545 comparison

We verify that Alcyoneus is the largest known radio galaxy (RG) in projection by comparing it with J1420-0545 (Machalski et al. 2008), the literature’s record holder.

The angular lengths of Alcyoneus and J1420-0545 are $\phi = 20.8' \pm 0.15'$ and $\phi = 17.4' \pm 0.05'$, respectively. For J1420-0545, we adopt the angular length reported by Machalski et al. (2008) because it lies outside the LoTSS DR2 coverage. The spectroscopic redshifts of Alcyoneus and J1420-0545 are $z_{\text{spec}} = 0.24674 \pm 6 \cdot 10^{-5}$ and $z_{\text{spec}} = 0.3067 \pm 5 \cdot 10^{-4}$, respectively. For both giants, we assume the peculiar velocity along the line of sight u_p to be a Gaussian random variable (RV) with mean 0 and standard deviation 100 km s^{-1} , similar to conditions in low-mass galaxy clusters.

Equations A.1 describe how to calculate the cosmological redshift RV z via the peculiar velocity redshift RV z_p :

$$\beta_p := \frac{u_p}{c}; \quad z_p = \sqrt{\frac{1 + \beta_p}{1 - \beta_p}} - 1; \quad z = \frac{1 + z_{\text{spec}}}{1 + z_p} - 1. \quad (\text{A.1})$$

Here, c is the speed of light *in vacuo*. Finally, we calculate the projected proper length RV $l_p = r_\phi(z, \mathfrak{M}) \cdot \phi$. Here, r_ϕ is the angular diameter distance RV, which depends on cosmological model parameters \mathfrak{M} . Propagating the uncertainties in angular length ϕ , spectroscopic redshift z_{spec} and peculiar velocity along the line of sight u_p through Monte Carlo simulation, the projected proper lengths of Alcyoneus and J1420-0545 are $l_p = 4.99 \pm 0.04 \text{ Mpc}$ and $l_p = 4.87 \pm 0.02 \text{ Mpc}$, respectively.

We show the two projected proper length distributions in **Figure** A.1. The probability that Alcyoneus has the largest projected proper length is 99.9%. This result is insensitive to plausible changes in cosmological parameters; for example, the high- H_0 (i.e. $H_0 > 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$) cosmology with $\mathfrak{M} = (h = 0.7020, \Omega_{\text{BM},0} = 0.0455, \Omega_{\text{M},0} = 0.2720, \Omega_{\Lambda,0} = 0.7280)$ yields a probability of 99.8%.

Appendix B: Inclination angle comparison

Under what conditions is Alcyoneus not only the largest GRG in the plane of the sky, but also in three dimensions? To answer this question, we compare Alcyoneus to the five previously known GRGs with projected proper lengths above 4 Mpc,

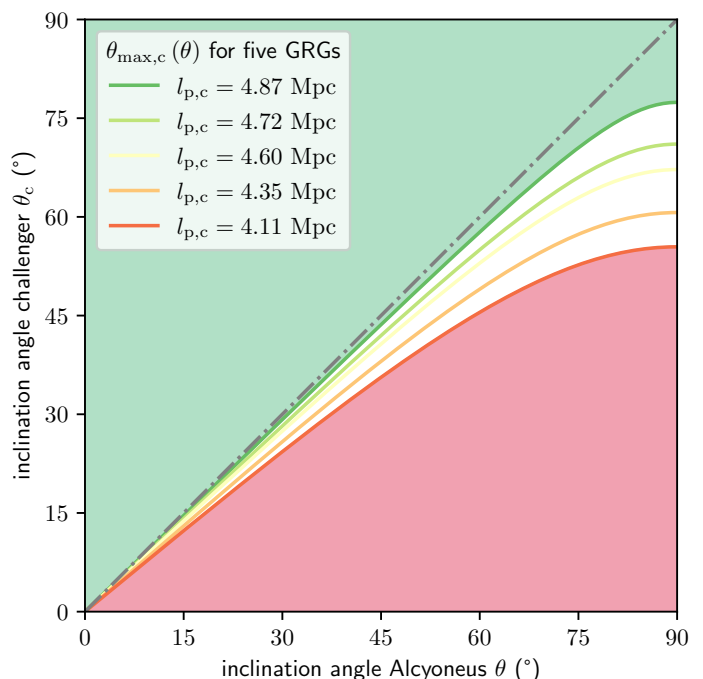


Fig. A.2: **When is Alcyoneus not only the largest GRG in the plane of the sky, but also in three dimensions?** Alcyoneus’ inclination angle θ is not well determined, and therefore the full range of possibilities is shown on the horizontal axis. To surpass Alcyoneus in true proper length, a challenger must have an inclination angle (vertical axis) of at most Alcyoneus’ (grey dotted line). More specifically, as a function of θ , we show the *maximum* inclination angle for which challengers with a projected proper length $l_{p,c} > 4 \text{ Mpc}$ trump Alcyoneus (coloured curves). The shaded areas of parameter space represent regimes with a particularly straightforward interpretation. One can imagine populating the graph with five points (located along the same vertical line), representing the ground-truth inclination angles of Alcyoneus and its five challengers. If *any* of these points fall in the red-shaded area, Alcyoneus is not the largest GRG in 3D. If *all* points fall in the green-shaded area, Alcyoneus is the largest GRG in 3D.

which we dub *challengers*. A challenger surpasses Alcyoneus in true proper length when

$$l_c > l, \quad \text{or} \quad \frac{l_{p,c}}{\sin \theta_c} > \frac{l_p}{\sin \theta}, \quad \text{or} \quad \sin \theta_c < \frac{l_{p,c}}{l_p} \sin \theta, \quad (\text{B.1})$$

where l_c , $l_{p,c}$ and θ_c are the challenger’s true proper length, projected proper length and inclination angle, respectively. Because the arcsine is a monotonically increasing function, a challenger surpasses Alcyoneus if its inclination angle obeys

$$\theta_c < \theta_{\text{max},c}(\theta), \quad \text{where} \quad \theta_{\text{max},c}(\theta) := \arcsin\left(\frac{l_{p,c}}{l_p} \sin \theta\right). \quad (\text{B.2})$$

In **Figure** A.2, we show $\theta_{\text{max},c}(\theta)$ for the five challengers with $l_{p,c} \in \{4.11 \text{ Mpc}, 4.35 \text{ Mpc}, 4.60 \text{ Mpc}, 4.72 \text{ Mpc}, 4.87 \text{ Mpc}\}$ (coloured curves). Alcyoneus is least likely to be the longest GRG in 3D when its true proper length equals its projected proper length; i.e. when $\theta = 90^\circ$. The challengers then surpass Alcyoneus in true proper length when their inclination angles are less than 55° , 61° , 67° , 71° and 77° , respectively. For $\theta < 90^\circ$, the conditions are more stringent.

The third and fourth largest challengers, whose respective SDSS DR12 host names are J100601.73+345410.5 and J093139.03+320400.1, harbour quasars in their host galaxies. If small inclination angles distinguish quasars from non-quasar AGN, as proposed by the unification model (e.g. Hardcastle & Croston 2020), these two challengers may well be the longest radio galaxies in three dimensions.

Appendix C: Lobe volumes with truncated double cone model

Appendix C.1: Synopsis

We build a Metropolis–Hastings Markov chain Monte Carlo (MH MCMC) model, similar in spirit to the model of Boxelaar et al. (2021) for galaxy cluster halos, in order to formalise the determination of RG lobe volumes from a radio image. To this end, we introduce a parametrisation of a pair of 3D radio galaxy lobes, and explore the corresponding parameter space via the Metropolis algorithm.¹⁴ For each parameter tuple encountered during exploration, we first calculate the monochromatic emission coefficient (MEC) function of the lobes on a uniform 3D grid representing a proper (rather than comoving) cubical volume. The RG is assumed to be far enough from the observer that the conversion to a 2D image through ray tracing simplifies to summing up the cube’s voxels along one dimension, and applying a cosmological attenuation factor. This factor depends on the galaxy’s cosmological redshift, which is a hyperparameter. We blur the model image to the resolution of the observed image, which is also a hyperparameter. Next, we calculate the likelihood that the observed image is a noisy version of the proposed model image. The imaged sky region is divided into patches with a solid angle equal to the PSF solid angle; the noise per patch is then assumed to be an independent Gaussian RV. These RVs have zero mean and share the same variance, which is another hyperparameter – typically obtained from the observed image. We choose a uniform prior over the full physically realisable part of parameter space. The resulting posterior, which contains both geometric and radiative parameters, allows one to calculate probability distributions for many interesting quantities, such as the RG’s lobe volumes and inclination angle. The inferences depend weakly on cosmological parameters \mathfrak{M} . Furthermore, their reliability depends significantly on the validity of the model assumptions.

Appendix C.2: Model

Appendix C.2.1: Geometry

We model each lobe in 3D with a truncated right circular cone with apex $O \in \mathbb{R}^3$, central axis unit vector $\hat{a} \in \mathbb{S}^2$ and opening angle $\gamma \in [0, \frac{\pi}{2}]$, as in Figure 9. The lobes share the same O , which is the RG host location. Each central axis unit vector can be parametrised through a position angle $\varphi \in [0, 2\pi)$ and an inclination angle $\theta \in [0, \pi]$. Each cone is truncated twice, through planes that intersect the cone perpendicularly to its central axis. Thus, each truncation is parametrised by the distance from the apex to the point where the plane intersects the central axis. The two inner ($d_{i,1}, d_{i,2} \in \mathbb{R}_{\geq 0}$) and two outer ($d_{o,1}, d_{o,2} \in \mathbb{R}_{\geq 0}$) truncation distances are parameters that we allow to vary independently, with the only constraint that each *inner* truncation

¹⁴ The more general Metropolis–Hastings variant need not be considered, as we work with a symmetric proposal distribution.

distance cannot exceed the corresponding *outer* truncation distance.

Appendix C.2.2: Radiative processes

The radiative formulation of our model is among the simplest possible. The radio emission from the lobes is synchrotron radiation. We consider the lobes to be perfectly optically thin: we neglect synchrotron self-absorption. The proper MEC is assumed spatially constant throughout a lobe, though possibly different among lobes; this leads to parameters $j_{\nu,1}, j_{\nu,2} \in \mathbb{R}_{\geq 0}$. The relationship between the specific intensity I_ν (in direction \hat{r} at central frequency ν_c) and the MEC j_ν (in direction \hat{r} at cosmological redshift z and rest-frame frequency $\nu = \nu_c (1 + z)$) is

$$I_\nu(\hat{r}, \nu_c) = \int_0^\infty \frac{j_\nu(\hat{r}, z(l), \nu_c(1+z(l)))}{(1+z(l))^3} dl \approx \frac{j_\nu(\nu) \Delta l(\hat{r})}{(1+z)^3}, \quad (\text{C.1})$$

where l represents proper length. The approximation is valid for a lobe with a spatially constant MEC that is small enough to assume a constant redshift for it. $\Delta l(\hat{r})$ is the proper length of the line of sight through the lobe in direction \hat{r} . The inferred MECs $j_{\nu,1}(\nu), j_{\nu,2}(\nu)$ thus correspond to *rest-frame* frequency ν .

Appendix C.3: Proposal distribution

In order to explore the posterior distribution on the parameter space, we follow the Metropolis algorithm. The Metropolis algorithm assumes a symmetric proposal distribution.

Appendix C.3.1: Radio galaxy axis direction

To propose a new RG axis direction given the current one whilst satisfying the symmetry assumption, we perform a trick. We populate the unit sphere with $N \in \mathbb{N}_{\geq 1}$ points (interpreted as directions) drawn from a uniform distribution. Of these N directions, the proposed axis direction is taken to be the one closest to the current axis direction (in the great-circle distance sense). Note that this approach evidently satisfies the criterion that proposing the new direction given the old one is equally likely as proposing the old direction given the new one. Also note that the distribution of the angular distance between current and proposed axis directions is determined solely by N .

In the following paragraphs, we first review how to perform uniform sampling of the unit two-sphere. More explicitly than in Scott & Tout (1989), we then derive the distribution of the angular distance between a reference point and the nearest of N uniformly drawn other points. The result is a continuous univariate distribution with a single parameter N and finite support $(0, \pi)$. Finally, we present the mode, median and maximum likelihood estimator of N . As far as we know, these properties are new to the literature.

Uniform sampling of \mathbb{S}^2 Let us place a number of points uniformly on the celestial sphere \mathbb{S}^2 . The spherical coordinates of such points are given by the RVs (Φ, Θ) , where Φ denotes position angle and Θ denotes inclination angle. As all position angles are equally likely, the distribution of Φ is uniform: $\Phi \sim \mathbb{U}[0, 2\pi)$. In order to effect a uniform number density, the probability that a point lies within a rectangle of width $d\varphi$ and height $d\theta$ in the (φ, θ) -plane equals the ratio of the solid angle of

the corresponding sky patch and the sphere's total solid angle:

$$\mathbb{P}(\varphi \leq \Phi < \varphi + d\varphi, \theta \leq \Theta < \theta + d\theta) = \frac{\sin \theta d\varphi d\theta}{4\pi}. \quad (\text{C.2})$$

The probability that the inclination angle is found somewhere in the interval $[\theta, \theta + d\theta]$, regardless of the position angle, is therefore

$$\begin{aligned} \mathbb{P}(\theta \leq \Theta < \theta + d\theta) &= dF_{\Theta}(\theta) = f_{\Theta}(\theta)d\theta \\ &= \int_0^{2\pi} \frac{\sin \theta d\theta}{4\pi} d\varphi = \frac{1}{2} \sin \theta d\theta, \end{aligned} \quad (\text{C.3})$$

where F_{Θ} is the cumulative distribution function (CDF) of Θ , and f_{Θ} the associated probability density function (PDF). So,

$$f_{\Theta}(\theta) = \frac{1}{2} \sin \theta; \quad F_{\Theta}(\theta) := \int_0^{\theta} f_{\Theta}(\theta') d\theta' = \frac{1 - \cos \theta}{2}. \quad (\text{C.4})$$

Nearest-neighbour angular distance distribution Pick a reference point and stochastically introduce N other points in above fashion, which we dub its *neighbours*. We now derive the PDF of the angular distance to the nearest neighbour (NNAD). Let $(\varphi_{\text{ref}}, \theta_{\text{ref}})$ be the coordinates of the reference point and let (φ, θ) be the coordinates of one of the neighbours. Without loss of generality, due to spherical symmetry, we can choose to place the reference point in the direction towards the observer: $\theta_{\text{ref}} = 0$. (Note that φ_{ref} is meaningless in this case.) The angular distance between two points on \mathbb{S}^2 is given by the great-circle distance ξ . For our choice of reference point, we immediately see that $\xi(\varphi_{\text{ref}}, \theta_{\text{ref}}, \varphi, \theta) = \theta$. Because θ is a realisation of Θ , ξ too can be regarded as a realisation of an RV, which we call Ξ . Evidently, the PDF $f_{\Xi}(\xi) = f_{\Theta}(\xi)$ and the CDF $F_{\Xi}(\xi) = F_{\Theta}(\xi)$. Now consider the generation of N points, whose angular distances to the reference point are the RVs $\{\Xi_i\} := \{\Xi_1, \dots, \Xi_N\}$. The NNAD RV M is the *minimum* of this set: $M := \min\{\Xi_i\}$. What are the CDF F_M and PDF f_M of M ?

$$\begin{aligned} F_M(\mu) &:= \mathbb{P}(M \leq \mu) = \mathbb{P}(\text{minimum of } \{\Xi_i\} \leq \mu) \\ &= \mathbb{P}(\text{at least one of the set } \{\Xi_i\} \leq \mu) \\ &= 1 - \mathbb{P}(\text{none of the set } \{\Xi_i\} \leq \mu) \\ &= 1 - \mathbb{P}(\text{all of the set } \{\Xi_i\} > \mu). \end{aligned} \quad (\text{C.5})$$

Because the $\{\Xi_i\}$ are independent and identically distributed,

$$\begin{aligned} F_M(\mu) &= 1 - \prod_{i=1}^N \mathbb{P}(\Xi_i > \mu) \\ &= 1 - \mathbb{P}^N(\Xi > \mu) = 1 - (1 - F_{\Xi}(\mu))^N. \end{aligned} \quad (\text{C.6})$$

By substitution, the application of a trigonometric identity and differentiation to μ , we obtain the CDF and PDF of M :

$$F_M(\mu) = 1 - \cos^{2N}\left(\frac{\mu}{2}\right); \quad f_M(\mu) = N \sin\left(\frac{\mu}{2}\right) \cos^{2N-1}\left(\frac{\mu}{2}\right). \quad (\text{C.7})$$

In **Figure C.1**, we show this PDF for various values of N . The mode of M (i.e. the most probable NNAD), μ_{mode} , is the solution to $\frac{df_M}{d\mu}(\mu_{\text{mode}}) = 0$. The median of M , μ_{median} , is the solution to $F_M(\mu_{\text{median}}) = \frac{1}{2}$. Hence,

$$\mu_{\text{mode}} = \arccos\left(1 - \frac{1}{N}\right); \quad \mu_{\text{median}} = \arccos\left(2^{1-\frac{1}{N}} - 1\right). \quad (\text{C.8})$$

As common sense dictates, both equal $\frac{\pi}{2}$ for $N = 1$ and tend to 0 as $N \rightarrow \infty$. We find the mean of M through integration by parts:

$$\begin{aligned} \mathbb{E}[M] &:= \int_0^{\pi} \mu f_M(\mu) d\mu = \int_0^{\pi} \mu dF_M(\mu) \\ &= \left[\mu F_M(\mu) \right]_0^{\pi} - \int_0^{\pi} F_M(\mu) d\mu \\ &= \int_0^{\pi} \cos^{2N}\left(\frac{\mu}{2}\right) d\mu = 2 \int_0^{\frac{\pi}{2}} \cos^{2N}(\mu) d\mu. \end{aligned} \quad (\text{C.9})$$

Again via integration by parts,

$$\mathbb{E}[M] = \pi \prod_{k=1}^N \frac{2k-1}{2k} = \frac{\pi}{2^{2N}} \binom{2N}{N}. \quad (\text{C.10})$$

Maximum likelihood estimation A typical application is the estimation of N in the PDF $f_M(\mu | N)$ (**Equation C.7**) using data. Let us assume we have measured k NNADs, denoted by $\{\mu_1, \dots, \mu_k\}$. Let the joint PDF or *likelihood* be

$$\begin{aligned} \mathcal{L}(N) &:= \prod_{i=1}^k f_M(\mu_i | N) \\ &= \left(\frac{N}{2^N}\right)^k \prod_{i=1}^k \sin \mu_i (\cos \mu_i + 1)^{N-1}. \end{aligned} \quad (\text{C.11})$$

To find N_{MLE} , we look for the value of N that maximises $\mathcal{L}(N)$. To simplify the algebra, we could however equally well maximise a k -th of the natural logarithm of the likelihood, or the *average log-likelihood* $\hat{l} := k^{-1} \ln \mathcal{L}(N)$, because the logarithm is a monotonically increasing function:

$$\begin{aligned} \hat{l}(N) &:= \frac{1}{k} \ln \mathcal{L}(N) = \ln N - N \ln 2 \\ &\quad + \frac{1}{k} \sum_{i=1}^k \ln \sin \mu_i + (N-1) \ln(\cos \mu_i + 1). \end{aligned} \quad (\text{C.12})$$

We find N_{MLE} by solving $\frac{d\hat{l}}{dN}(N_{\text{MLE}}) = 0$. This leads to

$$N_{\text{MLE}} = \left(\ln 2 - \frac{1}{k} \sum_{i=1}^k \ln(\cos \mu_i + 1) \right)^{-1}. \quad (\text{C.13})$$

An easy limit to evaluate is the case when $\mu_1, \dots, \mu_k \rightarrow 0$. In such case, $\cos \mu_i \rightarrow 1$, and so $\frac{1}{k} \sum_{i=1}^k \ln(\cos \mu_i + 1) \rightarrow \ln 2$. Then, $N_{\text{MLE}} \rightarrow (0_+)^{-1} \rightarrow \infty$. This is expected behaviour: when all measured NNADs approach 0, the number of points distributed on the sphere must be approaching infinity.

Appendix C.3.2: Other parameters

The other proposal parameters are each drawn from independent normal distributions centred around the current parameter values. These proposal distributions are evidently symmetric, but have support over the full real line, so that forbidden parameter values can in principle be proposed. As a remedy, we set the prior probability density of the proposed parameter set to 0 when the proposed opening angle is negative or exceeds $\frac{\pi}{2}$ rad, at least one of the proposed MECs is negative, or when at least one of the proposed inner truncation distances is negative or

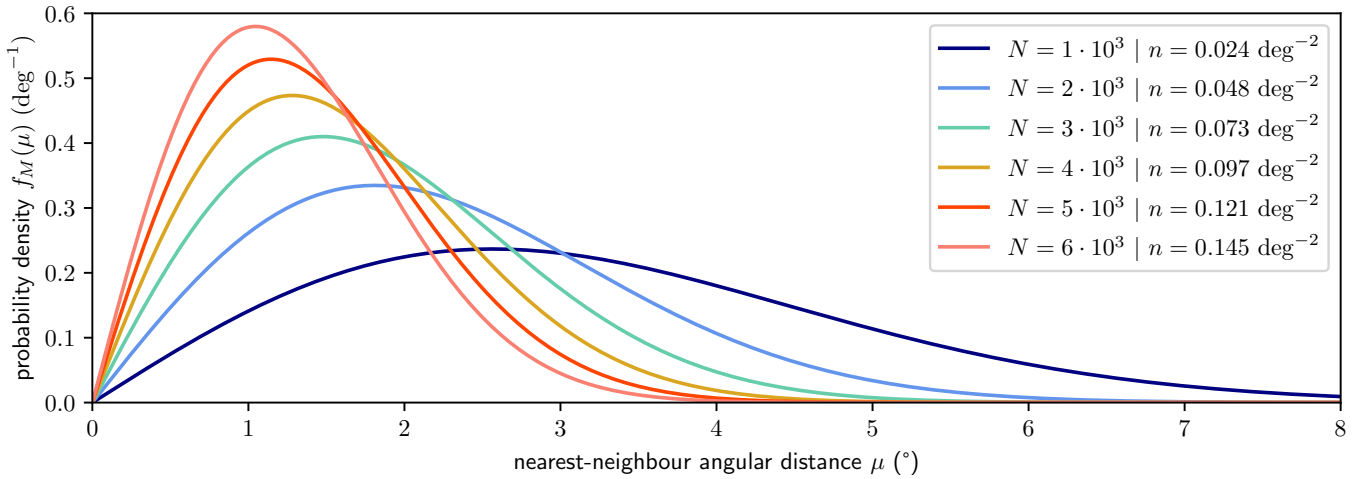


Fig. C.1: Probability density functions (PDFs) of the nearest-neighbour angular distance (NNAD) $RV M$ between some fixed point and N other points distributed randomly over the celestial sphere. As the sphere gets more densely packed, the probability of finding a small M increases. For each N , we provide the mean point number density n .

exceeds the corresponding proposed outer truncation distance. In such cases, the posterior probability density is 0 too, as it is proportional to the prior probability density. Consequently, the Metropolis acceptance probability vanishes and the proposal is rejected. We do not enter forbidden regions of parameter space. The condition of detailed balance is still respected: probability densities for transitioning *towards* the forbidden region are 0, just as probability densities for *being in* the forbidden region.

Appendix C.4: Likelihood

We assume the likelihood to be Gaussian. To avoid dimensionality errors, we multiply the likelihood by a constant before we take the logarithm:

$$\ln \left(\mathcal{L} \cdot (\sigma \sqrt{2\pi})^{N_r} \right) = -\frac{N_r}{2\sigma^2 N_p} \sum_{i=1}^{N_p} (I_{v,o}[i] - I_{v,m}[i])^2. \quad (\text{C.14})$$

Here, σ is the image noise, $N_r \in \mathbb{R}_{\geq 0}$ is the number of resolution elements in the image, $N_p \in \mathbb{N}$ is the number of pixels in the image, and $I_{v,o}[i]$ and $I_{v,m}[i]$ are the i -th pixel values of the observed and modelled image, respectively. For simplicity, one may multiply the likelihood by a constant factor (or, equivalently, add a constant term to the log-likelihood): the acceptance ratio will remain the same, and the MH MCMC runs correctly.

Appendix C.5: Results for Alcyoneus

We apply the Bayesian model to the 90'' LoTSS DR2 image of Alcyoneus, shown in the top panel of **Figure 9**. Thus, the hyperparameters are $z = 0.24674$, $\nu_c = 144$ MHz (so that $\nu = 180$ MHz), $\theta_{\text{FWHM}} = 90''$, $N = 750$ and $\sigma = \sqrt{2} \cdot 1.16$ Jy deg $^{-2}$. We set the image noise to $\sqrt{2}$ times the *true* image noise to account for model incompleteness. This factor follows by assuming that the inability of the model to produce the true lobe morphology yields (Gaussian) errors comparable to the image noise. To speed up inference, we downsample the image of 2,048 by 2,048 pixels by a factor 16 along each dimension. We run our MH MCMC for 10,000 steps, and discard the first 1,500 steps due to burn-in.

Table C.1 lists the obtained maximum a posteriori probability (MAP) estimates and posterior mean and standard deviation (SD) of the parameters.

Table C.1: Maximum a posteriori probability (MAP) estimates and posterior mean and standard deviation (SD) of the parameters from the Bayesian, doubly truncated, conical radio galaxy lobe model of **Section 3.9**.

parameter	MAP estimate	posterior mean and SD
φ_1	307°	307 ± 1°
φ_2	140°	139 ± 2°
$ \theta_1 - 90^\circ $	54°	51 ± 2°
$ \theta_2 - 90^\circ $	25°	18 ± 7°
γ_1	9°	10 ± 1°
γ_2	24°	26 ± 2°
$d_{i,1}$	2.7 Mpc	2.6 ± 0.2 Mpc
$d_{o,1}$	4.3 Mpc	4.0 ± 0.2 Mpc
$d_{i,2}$	1.6 Mpc	1.5 ± 0.1 Mpc
$d_{o,2}$	2.0 Mpc	2.0 ± 0.1 Mpc
$j_{v,1}(\nu)$	17 Jy deg $^{-2}$ Mpc $^{-1}$	17 ± 2 Jy deg $^{-2}$ Mpc $^{-1}$
$j_{v,2}(\nu)$	22 Jy deg $^{-2}$ Mpc $^{-1}$	18 ± 3 Jy deg $^{-2}$ Mpc $^{-1}$

The proper volumes V_1 and V_2 are derived quantities:

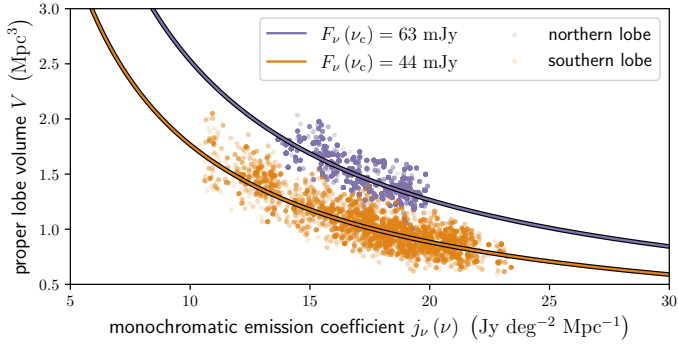
$$V = \frac{\pi}{3} \tan^2 \gamma (d_o^3 - d_i^3), \quad (\text{C.15})$$

just like the flux densities $F_{v,1}(\nu_c)$ and $F_{v,2}(\nu_c)$ at central frequency ν_c :

$$F_v(\nu_c) = \frac{j_v(\nu) V}{(1+z)^3 r_\phi^2(z)}. \quad (\text{C.16})$$

Together, V and $F_v(\nu_c)$ imply a lobe pressure P and a magnetic field strength B , which are additional derived quantities that we calculate through `pysynch`.

Table C.2 lists the obtained MAP estimates and posterior mean and SD of the derived quantities.



For Alcyoneus, this simpler model does not provide a good fit to the data.

Fig. C.2: Our Bayesian model yields strongly correlated estimates for $j_\nu(\nu)$ and V that reproduce the observed lobe flux densities. We show MECs $j_\nu(\nu)$ at $\nu = 180$ MHz and proper volumes V of Metropolis–Hastings Markov chain Monte Carlo samples for the northern lobe (purple dots) and southern lobe (orange dots). The curves represent all combinations $(j_\nu(\nu), V)$ that correspond to a particular flux density at the LoTSS central wavelength $\nu_c = 144$ MHz. We show the *observed* northern lobe flux density (purple curve) and the *observed* southern lobe flux density (orange curve).

Table C.2: Maximum a posteriori probability (MAP) estimates and posterior mean and standard deviation (SD) of derived quantities from the Bayesian, doubly truncated, conical radio galaxy lobe model of **Section 3.9**.

derived quantity	MAP estimate	posterior mean and SD
$\Delta\varphi$	167°	$168 \pm 2^\circ$
V_1	1.5 Mpc^3	$1.5 \pm 0.2 \text{ Mpc}^3$
V_2	0.8 Mpc^3	$1.0 \pm 0.2 \text{ Mpc}^3$
$F_{\nu,1}(\nu_c)$	63 mJy	$63 \pm 4 \text{ mJy}$
$F_{\nu,2}(\nu_c)$	44 mJy	$45 \pm 5 \text{ mJy}$
$P_{\min,1}$	$4.7 \cdot 10^{-16} \text{ Pa}$	$4.8 \pm 0.3 \cdot 10^{-16} \text{ Pa}$
$P_{\min,2}$	$5.4 \cdot 10^{-16} \text{ Pa}$	$5.0 \pm 0.6 \cdot 10^{-16} \text{ Pa}$
$P_{\text{eq},1}$	$4.8 \cdot 10^{-16} \text{ Pa}$	$4.9 \pm 0.3 \cdot 10^{-16} \text{ Pa}$
$P_{\text{eq},2}$	$5.4 \cdot 10^{-16} \text{ Pa}$	$5.0 \pm 0.6 \cdot 10^{-16} \text{ Pa}$
$B_{\min,1}$	45 pT	$45 \pm 1 \text{ pT}$
$B_{\min,2}$	48 pT	$46 \pm 3 \text{ pT}$
$B_{\text{eq},1}$	42 pT	$43 \pm 1 \text{ pT}$
$B_{\text{eq},2}$	45 pT	$43 \pm 3 \text{ pT}$
$E_{\min,1}$	$6.3 \cdot 10^{52} \text{ J}$	$6.2 \pm 0.4 \cdot 10^{52} \text{ J}$
$E_{\min,2}$	$3.7 \cdot 10^{52} \text{ J}$	$4.4 \pm 0.6 \cdot 10^{52} \text{ J}$
$E_{\text{eq},1}$	$6.4 \cdot 10^{52} \text{ J}$	$6.3 \pm 0.4 \cdot 10^{52} \text{ J}$
$E_{\text{eq},2}$	$3.8 \cdot 10^{52} \text{ J}$	$4.4 \pm 0.6 \cdot 10^{52} \text{ J}$

The uncertainties of the parameters and derived quantities reported in **Tables C.1** and **C.2** are not necessarily independent. To demonstrate this, we present MECs and volumes from the MH MCMC samples in **Figure C.2**. MECs and volumes do not vary independently, because their product is proportional to flux density (see **Equation C.16**); only realistic flux densities correspond to high-likelihood model images.

Finally, we explore a simpler variation of the model, in which we force the lobes to be coaxial. In such a case, the true proper length l and projected proper length l_p are additional derived quantities:

$$l = \frac{d_{o,1} + d_{o,2}}{\cos \gamma}; \quad l_p = l \sin \theta. \quad (\text{C.17})$$