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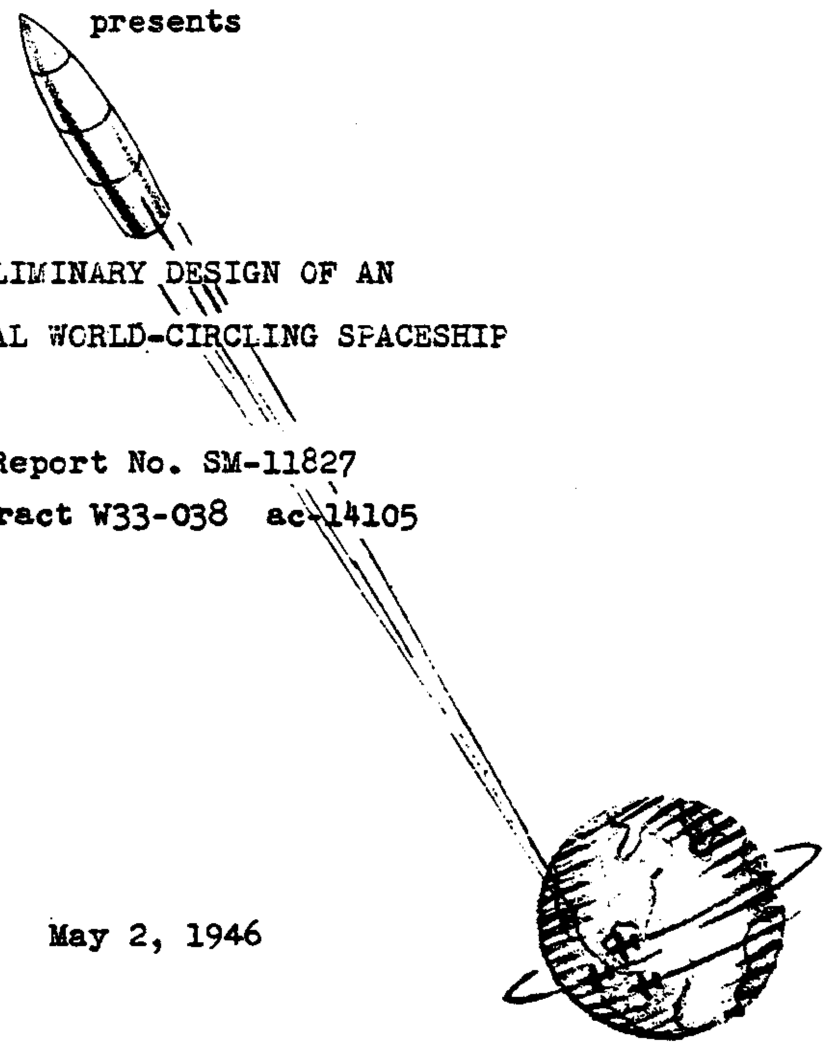

PRELIMINARY DESIGN OF AN EXPERIMENTAL WORLD-CIRCLING SPACESHIP

Report No. SM-11827
Contract W33-038 ac-14105

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SUMMARY

This report presents an engineering analysis of the possibilities of designing a man-made satellite. The questions of power plants, structural weights, multiple stages, optimum design values, trajectories, stability, and landing are considered in detail. The results are used to furnish designs for two proposed vehicles. The first is a four stage rocket using alcohol and liquid oxygen as propellants. The second is a two stage rocket using liquid hydrogen and liquid oxygen as propellants. The latter rocket offers better specific consumption rates, but this is found to be partially offset by the greater structural weight necessitated by the use of hydrogen. It is concluded that modern technology has advanced to a point where it now appears feasible to undertake the design of a satellite vehicle.

ABSTRACT

In this report, we have undertaken a conservative and realistic engineering appraisal of the possibilities of building a spaceship which will circle the earth as a satellite. The work has been based on our present state of technological advancement and has not included such possible future developments as atomic energy.

If a vehicle can be accelerated to a speed of about 17,000 m.p.h. and aimed properly, it will revolve on a great circle path above the earth's atmosphere as a new satellite. The centrifugal force will just balance the pull of gravity. Such a vehicle will make a complete circuit of the earth in approximately 1-1/2 hours. Of all the possible orbits, most of them will not pass over the same ground stations on successive circuits because the earth will turn about 1/16 of a turn under the orbit during each circuit. The equator is the only such repeating path and consequently is recommended for early attempts at establishing satellites so that a single set of telemetering stations may be used.

Such a vehicle will undoubtedly prove to be of great military value. However, the present study was centered around a vehicle to be used in obtaining much desired scientific information on cosmic rays, gravitation, geophysics, terrestrial magnetism, astronomy, meteorology, and properties of the upper atmosphere. For this purpose, a payload of 500 lbs. and 20 cu ft. was selected as a reasonable estimate of the requirements for scientific apparatus capable of obtaining results sufficiently far-reaching to make the undertaking worthwhile. It was found necessary to establish the orbit at an altitude of about 300 miles to insure sufficiently

low drag so that the vehicle could travel for 10 days or more, without power, before losing satellite speed.

The only type of power plant capable of accelerating a vehicle to a speed of 17,000 m.p.h. on the outer limits of the atmosphere is the rocket. The two most important performance characteristics of a rocket vehicle are the exhaust velocity of the rocket and the ratio of the weight of propellants to the gross weight. Very careful studies were made to establish engineering estimates of the values that can be obtained for these two characteristics.

The study of rocket performance indicated that while liquid hydrogen ranks highest among fuels having large exhaust velocities, its low density, low temperature and wide explosive range cause great trouble in engineering design. On the other hand, alcohol though having a lower exhaust velocity, has the benefit of extensive development in the German V-2. Consequently it was decided to conduct parallel preliminary design studies of vehicles using liquid hydrogen-liquid oxygen and alcohol-liquid oxygen as propellants.

It has been frequently assumed in the past that structural weight ratios become increasingly favorable as rockets increase in size, and fixed weight items such as radio equipment become insignificant weight items. However, the study of weight ratios indicated that for large sizes the weight of tanks and similar items actually become less favorable. Consequently, there is an optimum middle range of sizes. Improvements in weight ratios over that of the German V-2 are possible only by the slow process of technological development, not by the brute force methods of increase in size. This study showed that an alcohol-oxygen vehicle could be built whose entire structural weight (including motors, controls, etc.) was about 16% of the gross weight. On the other hand, the difficulties with liquid hydrogen, such as increased tank size, necessitated an entire structural weight of about 25% of the gross weight. These studies also indicated that a maximum acceleration of about 6.5 times that of gravity gave the best overall performance for the vehicles considered. If the acceleration is greater, the increased structural design loads increase the structural weight. If the acceleration is less, rocket thrust is inefficiently used to support the weight of the vehicle without producing the desired acceleration.

Using the above results, it was found that neither hydrogen-oxygen nor alcohol-oxygen is capable of accelerating a single unassisted vehicle to orbital speeds. By the use of a multi-stage rocket, these velocities can be attained by vehicles feasible within the limits of our present knowledge. To illustrate the concept of a multi-stage rocket, first consider a vehicle composed of two parts. The primary vehicle, complete with its rocket motor, tanks, propellants and controls is carried along as the "payload" of a similar vehicle of much greater size. The rocket of the large vehicle is used to accelerate the combination to as great a speed as possible, after which, the large vehicle is discarded and the small vehicle accelerates under its own power, adding its velocity increase to that of the large vehicle. By this means we have obtained an effective decrease in the amount of structural weight that must be accelerated to high speeds. This same idea can be used in designing vehicles with a greater number of stages. A careful analysis of the advantages of staging showed that for a given set of performance requirements, an optimum number of stages exists. If the stages are too few in number, the required velocities can be attained only by the undesirable process of exchanging payload for fuel. If they are too many, the multiplication of tanks, motors, etc. eliminates any possible gain in the effective weight ratio. For the alcohol-oxygen rocket it was found that four stages were best. For the hydrogen-oxygen rocket, preliminary analysis indicated that the best choice for the number of stages was two, but refinements showed the optimum number of stages was three. Unfortunately, insufficient time was available to change the design, so the work on the hydrogen-oxygen was completed using two stages. The characteristics of the vehicles studies are tabulated below. Sketches of the vehicles are shown on the drawings preceding page 203.

Vehicle Powered by Alcohol-Oxygen Rockets

Stage	1	2	3	4
Gross Wt. (lbs.)	233,669	53,689	11,829	2,868
Weight less fuel (lbs.)	93,669	21,489	4,729	1,148
Payload (lbs.)	53,689	11,829	2,868	500
Max. Diameter (in.)	157	138	105	90

Vehicle Powered by Hydrogen-Oxygen Rockets

Stage	1	2
Gross Wt. (lbs.)	291,564	15,364
Weight less fuel (lbs.)	84,564	4,464
Payload (lbs.)	15,364	500
Max. Diameter (in.)	248	167

(had three stages been used for the hydrogen-oxygen rockets, the overall gross weight of this vehicle could have been reduced to about 84,000 lbs. indicating this combination should be given serious consideration in any future study).

In arriving at the above design figures, a detailed study was made of the effects of exhaust velocity, structural weight, gravity, drag, acceleration, flight path inclination, and relative size of stages on the performance of the vehicles so that an optimum design could be achieved or reasonable compromises made.

It was found that the vehicle could best be guided during its accelerated flight by mounting control surfaces in the rocket jets and rotating the entire vehicle so that lateral components of the jet thrust could be used to produce the desired control forces. It is planned to fire the rocket vertically upward for several miles and then gradually curve the flight path over in the direction in which it is desired that the vehicle shall travel. In order to establish the vehicle on an orbit at an altitude of about 300 miles without using excessive amounts of control it was found desirable to allow the vehicle to coast without thrust on an extended elliptic arc just preceding the firing of the rocket of the last stage. As the vehicle approaches the summit of this arc, which is at the final altitude, the rocket of the last stage is fired and the vehicle is accelerated so that it becomes a freely revolving satellite.

It was shown that excessive amounts of rocket propellants are required to make corrections if the orbit is incorrectly established in direction or in velocity. Therefore, considerable attention was devoted to the stability and control problem during the acceleration to orbital speeds. It was concluded that the orbit could be established with sufficient precision so that the vehicle would not inadvertently re-enter the atmosphere because of an eccentric orbit.

Once the vehicle has been established on its orbit, the questions arise as to what are the possibilities of damage by meteorites, what temperatures will it experience, and can its orientation in space be controlled? Although the probability of being hit by very small meteorites is great, it was found that by using reasonable thickness plating, adequate protection could be obtained against all meteorites up to a size where the frequency of occurrence was very small. The temperatures of the satellite vehicle will range from about 40°F when it is on the side of the earth facing the sun to about -20°F when it is in the earth's shadow. Either small flywheels or small jets of compressed gas appear to offer feasible methods of controlling the vehicle's orientation after the cessation of rocket thrust.

An investigation was made of the possibility of safely landing the vehicle without allowing it to enter the atmosphere at such great speeds that it would be destroyed by the heat of air resistance. It was found that by the use of wings on the small final vehicle, the rate of descent could be controlled so that the heat would be dissipated by radiation at temperatures the structure could safely withstand. These same wings could be used to land the vehicle on the surface of the earth.

An interesting outcome of the study is that the maximum acceleration and temperatures can be kept within limits which can be safely withstood by a human being. Since the vehicle is not likely to be damaged by meteorites and can be safely brought back to earth, there is good reason to hope that future satellite vehicles will be built to carry human beings.

It has been estimated that to design, construct and launch a satellite vehicle will cost about $150,000,000. Such an undertaking could be accomplished in approximately 5 years time. The launching would probably be made from one of the Pacific islands near the equator. A series of telemetering stations would be established around the equator to obtain the data from the scientific apparatus contained in the vehicle. The first vehicles will probably be allowed to burn up on plunging back into the atmosphere. Later vehicles will be designed so that they can be brought back to earth. Such vehicles can be used either as long range missiles or for carrying human beings.

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PRELIMINARY DESIGN OF AN EXPERIMENTAL WORLD-CIRCLING SPACESHIP

1. INTRODUCTION

Technology and experience have now reached the point where it is possible to design and construct craft which can penetrate the atmosphere and achieve sufficient velocity to become satellites of the earth. This statement is documented in this report, which is a design study for a satellite vehicle judiciously based on German experience with V-2, and which relies for its success only on sound engineering development which can logically be expected as a consequence of intensive application to this effort. The craft which would result from such an undertaking would almost certainly do the job of becoming a satellite, but it would clearly be bulky, expensive, and inefficient in terms of the spaceship we shall be able to design after twenty years of intensive work in this field. In making the decision as to whether or not to undertake construction of such a craft now, it is not inappropriate to view our present situation as similar to that in airplanes prior to the flight of the Wright brothers. We can see no more clearly all the utility and implications of spaceships than the Wright brothers could see fleets of B-29's bombing Japan and air transports circling the globe.

Though the crystal ball is cloudy, two things seem clear:

1. A satellite vehicle with appropriate instrumentation can be expected to be one of the most potent scientific tools of the Twentieth Century.

2. The achievement of a satellite craft by the United States would inflame the imagination of mankind, and would probably produce repercussions in the world comparable to the explosion of the atomic bomb.

Chapter 2 of this report attempts to indicate briefly some of the concrete results to be derived from a spaceship which circles the world on a stable orbit.

As the first major activity under contract W33-038AC-14105, we have been asked by the Air Forces to explore the possibilities of making a satellite vehicle, and to present a program which would aid in the development of such a vehicle. Our approach to this task is along two related lines:

1. To undertake a design study which will evaluate the possibility of making a satellite vehicle using known methods of engineering and propulsion.
2. To explore the fields of science in an attempt to discover and to stimulate research and development along lines which will ultimately be of benefit in the design of such a satellite vehicle and which will improve its efficiency or decrease its complexity and cost.

This report concerns itself solely with the first line of approach. It is a practical study based on techniques that we now know. The implications of atomic energy are not considered here. This and other possibilities in the fields of science may be the subject of future reports, which will cover the second line of approach.

In the preliminary design study analytical methods have been developed which may be used as a basis for future studies in this new field of astronautical engineering. Among these are the following:

1. Analysis of single and multi-stage rocket performance and methods for selecting the optimum number of stages for any given application.
2. Dimensional analysis of varying size and gross weight of rockets, deriving laws which are useful in design scaling. These laws are also of assistance in appraisal of the effect of shape and proportions on the design of multi-stage rockets.
3. The effect of acceleration and inclination of the trajectory on structural weight and performance of a satellite rocket.
4. Methods of determining the optimum trajectory for satellite rockets.
5. Variation of rocket performance with altitude and its effect on the proportioning of stages.
6. Preliminary study of effect of atmospheric drag on the rocket and how it affects the choice of stages, acceleration, and trajectory.
7. Analysis of dynamic stability and control throughout the entire trajectory.

8. Method of safely landing a satellite vehicle.

It cannot be emphasized too strongly that the primary contributions of this report are in methods, and not in the specific figures in this design study. One point in particular should be high lighted: - the design gross weight, which is of the greatest importance in estimating cost or in comparing any two proposals in this field is the least definitely ascertained single feature in the whole process. This fact is fundamental in the design of a satellite or spaceship, since the slightest variation in some of the minor details of construction or in propulsive efficiency of the fuel may result in a large change in gross weight. The figures in this report represent a reasonable compromise between the extremes which are possible with the data now in hand. The most important thing is that a satellite vehicle can be made at all in the present state of the art. Even our more conservative engineers agree that it is definitely possible to undertake design and construction now of a vehicle which would become a satellite of the earth.

Another important result of this design study is the conclusion on liquid hydrogen and oxygen as fuel versus liquid oxygen and alcohol (the Germans' fuel). The relative merits of these fuels have occasioned spirited controversy ever since liquid fuel rockets have been under development. In the past, the fact which has clinched the arguments has been the difficulty of handling, storing, and using liquid hydrogen. The present design study has approached this subject from another viewpoint. On the assumption that all these nasty problems can be solved, a design analysis has been made for the structure and performance of rockets using both types of fuels. Because of the low density of liquid hydrogen, the greater tankage weight and volume tends to offset the increase in specific impulse. Early in the design study it was necessary to make a choice of the number of stages for both proposed vehicles. Based on the design information available, a decision was made to use four stages for the alcohol-oxygen rocket and two stages for the hydrogen-oxygen rocket. Of these two designs, the alcohol-oxygen rocket proved to be somewhat smaller in weight and size. However, the problem was later re-examined, when more reliable data were available. It was found that, while the choice of four stages for alcohol-oxygen had been wise, the hydrogen-oxygen rocket could have been substantially improved by using three stages. The improvement was sufficient to indicate that the three stage hydrogen-oxygen rocket would have been definitely superior to the four stage alcohol-oxygen rocket. Unfortunately, the work had progressed so far that it was impossible to alter the number of stages for the hydrogen-oxygen rocket.

One of the most important conclusions of this design study is that in order to achieve the required performance it is necessary to have multi-stage rockets for either type of fuel. The general characteristics of both types are shown in the following table:

4 Stage Alcohol-Oxygen Rocket

Payload 500#

Stage	1	2	3	4
Gross weight (lbs.)	233,669	53,689	11,829	2868
Fuel weight (lbs.)	140,000	32,200	7,100	1720

2 Stage Hydrogen-Oxygen Rocket

Payload 500#

	Stage 1	2
Gross weight (lbs.)	291,564	15,364
Total Fuel Weight (lbs.)	217,900	10,000

The design represents a series of compromises. The payload is chosen to be as small as is consistent with carrying enough experimental equipment to achieve significant results. This is done for the purpose of keeping the gross weight within reasonable limits, since the gross weight increases roughly in proportion to the payload above a certain minimum value. The design altitude was originally chosen as 100 miles, since previous calculations indicated that the atmospheric drag there was not great enough to disturb the orbit of the satellite for a few revolutions, and since for communications purposes it was desirable to keep the satellite below the ionosphere. The more refined drag studies made in the present design study show that these early estimates were in serious error, and indicate that the satellite will have to be established at altitudes of 300 to 400 miles to insure the completion of multiple revolutions around the earth.

It is interesting that the design analysis shows that the optimum accelerations are well within the limits which the human body can stand. Further, it appears possible to achieve a safe landing with the type of vehicle which is required. Future developments may bring an increase in payload and decrease in gross weight, sufficient to produce a large manned spaceship able to accomplish important things in a scientific and military way.

We turn now from the design study phase to the basic research approach of the scientists. Our consultants have all made suggestions which have been taken into consideration in the preparation of this report. In the future it is our expectation that the services of these scientists will be of the greatest benefit in planning and initiating broad research programs to explore new fundamental approaches to the problem of space travel.

The real white hope for the future of spaceships is, of course, atomic energy. If this intense source of energy can be harnessed for rocket propulsion, then spaceships of moderate size and high performance may become a reality, and conceivably could even serve efficiently as intercontinental transports in the remote future. We are fortunate in having the consulting services of Drs. Alvarez, McMillan, and Ridenour, well known in scientific circles. Alvarez and McMillan were two of the key men at the Los Alamos Laboratory of the Manhattan Project. With the benefit of their advice, we hope to achieve a degree of competence in the fields of application of nuclear energy to propulsion.

Alvarez and Ridenour, who are also radar experts, have made basic analyses of the radio and radar problems associated with a satellite. These are of service in planning the new equipment which seems to be necessary to make the satellite a useful tool.

Kistiakowsky, a specialist in physical chemistry, has made valuable suggestions for the development of new rocket propellants.

Schiff has contributed to our knowledge of the optimum trajectories to be used in launching the vehicle.

More important than the ideas and suggestions received to date is the fact that these consultants, who are among the leaders in U.S. science, have begun to think and work on these problems. It is our earnest hope that under the terms of this new study and research contract with the Army Air Forces we may be able to enlist the active co-operation of an important fraction of the scientific resources of the country to solve problems in the wholly new fields which man's imagination has opened. Of these, space travel is one of the most important and challenging.

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2. THE SIGNIFICANCE OF A SATELLITE VEHICLE

Attempting in early 1946 to estimate the values to be derived from a development program aimed at the establishment of a satellite circling the earth above the atmosphere is as difficult as it would have been, some years before the Wright brothers flew at Kitty Hawk, to visualize the current uses of aviation in war and in peace. Some of the fields in which important results are to be expected are obvious; others, which may include some of the most important, will certainly be overlooked because of the novelty of the undertaking. The following considerations assume the future development of a satellite with large payload. Only a portion of these may be accomplished by the satellite described in the design study of this report.

The Military Importance of a Satellite - The military importance of establishing vehicles in satellite orbits arises largely from the circumstance that defenses against airborne attack are rapidly improving. Modern radar will detect aircraft at distances up to a few hundred miles, and can give continuous, precise data on their position. Anti-aircraft artillery and anti-aircraft guided missiles are able to engage such vehicles at considerable range, and the proximity fuze increases several fold the effectiveness of anti-aircraft fire. Under these circumstances, a considerable premium is put on high missile velocity, to increase the difficulty of interception.

This being so, we can assume that an air offensive of the future will be carried out largely or altogether by high-speed pilotless missiles. The minimum-energy trajectory for such a space-missile without aerodynamic lift at long range is very flat, intersecting the earth at a shallow angle. This means that small errors in the trajectory of such a missile will produce large range errors in the point of impact. It has been suggested that the accuracy can be increased by firing such a missile along the same general course as that being followed by a satellite, and at such a time that the two are close to one another at the center of the trajectory of the missile. Under these circumstances, precise observations of the position of the missile can be made from the satellite, and a final control impulse applied to bring the missile down on its intended target. This scheme, while it involves considerable complexity in instrumentation, seems entirely feasible. Alternatively, the satellite itself can be considered as the missile. After observations of its trajectory, a control impulse can be applied in such direction and amount, and at such a time, that the satellite is brought down on its target.

There is little difference in design and performance between an intercontinental rocket missile and a satellite. Thus a rocket missile with a free space-trajectory of 6,000 miles requires a minimum energy of launching which corresponds to an initial velocity of 4.4 miles per second, while a satellite requires 5.1. Consequently the development of a satellite will be directly applicable to the development of an intercontinental rocket missile.

It should also be remarked that the satellite offers an observation aircraft which cannot be brought down by an enemy who has not mastered similar techniques. In fact, a simple computation from the radar equation shows that such a satellite is virtually undetectable from the ground by means of present-day radar. Perhaps the two most important classes of observation which can be made from such a satellite are the spotting of the points of impact of bombs launched by us, and the observation of weather conditions over enemy territory. As remarked below, short-range weather forecasting anywhere in the vicinity of the orbit of the satellite is extremely simple.

Certainly the full military usefulness of this technique cannot be evaluated today. There are doubtless many important possibilities which will be revealed only as work on the project proceeds.

The Satellite as an Aid to Research - The usefulness of a satellite in scientific research is very great. Typical of the outstanding problems which it can help to attack are the following:

One of the fastest-moving fields of investigation in modern nuclear physics is the study of cosmic rays. Even at the highest altitudes which have been reached with unmanned sounding balloons, a considerable depth of atmosphere has been traversed by the cosmic rays before their observation. On board such a satellite, the primary cosmic rays could be studied without the complications which arise within the atmosphere. From this study may come more important clues to unleashing the energy of the atomic nucleus.

Studies of gravitation with precision hitherto impossible may be made. This is possible because for the first time in history, a satellite would provide an acceleration-free laboratory where the ever present pull of the earth's gravitational field is cancelled by the centrifugal force of the rotating satellite. Such studies might lead to an understanding of the cause of gravitation - which is now the greatest riddle of physics.

The variations in the earth's gravitational field over the face of the earth could be measured from a satellite. This would supply one very fundamental set of data needed by the geologists and geophysicists to understand the causes of mountain-building, etc.

Similarly, the variations in the earth's magnetic field could be measured with a completeness and rapidity hitherto impossible.

The satellite laboratory could undertake comprehensive research at the low pressures of space. The value of this in comparison with pressures now attainable in the laboratory might be great.

For the astronomer, a satellite would provide great assistance. Dr. Shapley, director of the Harvard Observatory has expressed the view that measurements of the ultra-violet spectrum of the sun and stars would contribute greatly to an understanding of the source of the sun's surface energy, and perhaps would help explain sunspots. He also looks forward to the satellite observatory to provide an explanation for the "light of the night sky."

Astronomical observations made on the surface of the earth are seriously hampered by difficulties of "seeing," which arise because of variations in the refractive index of the column of air through which any terrestrial telescope must view the heavens. These difficulties are greatest in connection with the observation of any celestial body whose image is an actual disk, within which features of structure can be recognized: the moon, the sun, the planets, and certain nebulae. A telescope even of modest size could, at a point outside the earth's atmosphere, make observations on such bodies which would be superior to those now made with the largest terrestrial telescopes. Because there would be no scattering of light by an atmosphere, continuous observation of the solar corona and the solar prominences should also be possible. Astronomical images could, of course, be sent back to the earth from an unmanned satellite by television means.

From a satellite at an altitude of hundreds of miles, circling the earth in a period of about one and one half hours, observations of the cloud patterns on the earth, and of their changes with time, could be made with great ease and convenience. This information should be of extreme value in connection with short-range weather forecasting, and tabulation of such data over a period of time might prove extremely valuable to long-range weather forecasting. A satellite on a North-South orbit could observe the whole surface of the world once a day, and entirely in the daylight.

The properties of the ionosphere could be studied in a new way from such a satellite. Present ionospheric measurements are all made by studying the reflection of radio waves from the ionized upper atmosphere. A satellite would permit these measurements to be extended by studying the transmission properties of the ionosphere at various frequencies, angles of incidence, and times. Reflection measurements could also be made from the top of the ionosphere. Since we now know that disruption of the ionosphere accompanying auroral displays is caused by the impact

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of a cloud of matter from space, the satellite could determine the nature, and maybe the source of that cloud.

Biologists and medical scientists would want to study life in the acceleration-free environment of the satellite. This is an important pre-requisite to space travel by man, and it may also lead to important new observations in lower forms of life.

The Satellite as a Communications Relay Station - Long-range radio communication, except at extremely low frequencies (of the order of a few kc/sec), is based entirely on the reflection of radio waves from the ionosphere. Since the properties of the earth's ionized layer vary profoundly with the time of day, the season, sunspot activity, and other factors, it is difficult to maintain reliable long-range communication by means of radio. A satellite offers the possibility of establishing a relay station above the earth, through which long-range communications can be maintained independent of any except geometrical factors.

The enormous bandwidths attainable at microwave frequencies enable a very large number of independent channels to be handled with simple equipment, and the only difficulty which the scheme appears to offer is that a low-altitude (300 mile) satellite would remain in the view of a single ground station only for about 2100 miles of its orbit.

For communications purposes it would be desirable to operate the satellites at an altitude greater than 300 miles. If they could be at such an altitude (approximately 25,000 miles) that their rotational period was the same as that of the earth, not only would the "shadow" effect of the earth be greatly reduced, but also a given relay station could be associated with a given communication terminus on the earth, so that the communication system problem might be very greatly simplified.

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An idea of the potential commercial importance of this development may be gained from the fact that the ionosphere is now used as the equivalent of about $10,000,000,000. in long-lines, and is jammed to the limit with transmissions.

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The Satellite as a Forerunner of Interplanetary Travel - The most fascinating aspect of successfully launching a satellite would be the pulse quickening stimulation it would give to considerations of interplanetary travel. Whose imagination is not fired by the possibility of voyaging out beyond the limits of our earth, traveling to the Moon, to Venus and Mars? Such thoughts when put on paper now seem like idle fancy. But, a man-made satellite, circling our globe beyond the limits of the atmosphere is the first step. The other necessary steps would surely follow in rapid succession. Who would be so bold as to say that this might not come within our time?

Chapter 3

3. GENERAL CHARACTERISTICS OF A SATELLITE VEHICLE

Within the limits of our everyday experience, the trajectories of freely moving objects are nearly parabolic. The departures from truly parabolic trajectories are caused largely by air resistance. However, there is an additional factor whose influence is small at low speeds but rapidly becomes larger as the speed increases. This factor is the curvature of the earth. Because of it, even a vehicle traveling parallel to the earth is subjected to a centrifugal force and at high speeds this force can become of equal importance to the force of gravity. Since gravitational force is inward and the centrifugal force is outward, there is a speed at which the two would just balance and the vehicle would revolve about the earth like a new satellite. The speed to accomplish this is easily calculated. If, for the moment, we disregard aerodynamic forces, then a satellite near the surface of the earth would be balanced between a gravitational attraction of mg and a centrifugal force of $\frac{mv^2}{R}$, where m and v are the mass and velocity of the satellite and g and R are the acceleration of gravity and the radius of the earth. Placing $\frac{mv^2}{R} = mg$ and using the equatorial values of $R = 3{,}963$ miles and $g = 32.086$ ft. per sec.^{2}, we readily find that $v = 25{,}810$ ft. per sec. or 17,600 miles per hour. If this motion were to take place in the plane of the equator we would have to add or subtract the velocity of rotation of the earth, depending on whether the vehicle were rotating with or against the earth. These new values are 24,285 ft./sec. and 27,335 ft./sec. These values are only approximately correct because the effect of the earth's rotation on the gravitational attraction of stationary objects has been neglected.

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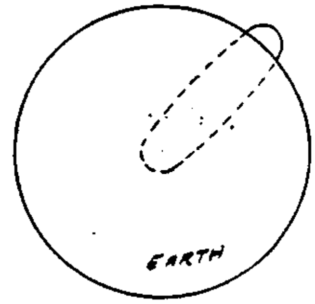
A more detailed calculation is given in Appendix C. Traveling at these speeds, the times to make a complete circuit of the earth would be 1 hour and 30 min. and 1 hour and 20 min. respectively. It is of course impractical to attempt to move at such great speeds within the atmosphere of the earth. However, at a height of 300 miles above the surface of the earth, the air is so thin that such speeds are practical. If we repeat our calculations for this altitude, taking into account that the attraction of gravity falls off as the square of the distance from the center of the earth, we find that the new velocities are 23,655 ft. per sec. and 26,705 ft. per sec. and the new times for complete circuits of the earth are 1 hour and 32 min. and 1 hour and 22 min. Interestingly enough, the energy required to establish an orbit at an altitude of 200 miles is not very much larger than that required at the surface of the earth because, although the potential energy is considerably greater, it is partly compensated by the lower kinetic energy of the higher orbit.

It is interesting to note that in our equation $\frac{mv^2}{R} = mg$ the mass occurs on both sides and cancels out. Consequently, the speeds for orbital motion do not depend on the mass of the object nor on the material from which it is made.

As mentioned above, we normally expect the trajectories of freely moving objects to be parabolic. However, if we take strict account of the curvature of the earth, our mathematics tells us that <u>all</u> such trajectories are arcs of



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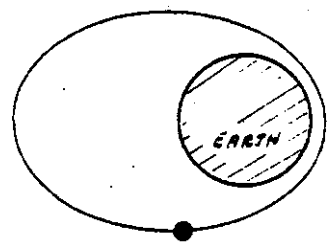
Keplerian* ellipses. If the velocity is small, the trajectory is only a small portion of the outer end of the ellipse, as shown in the preceding figure. This tip portion of an elongated ellipse is very nearly but not quite parabolic. As the speed increases, the portion of the ellipse lying outside the earth likewise increases and the first trajectory lying entirely outside the earth is the circular one whose speed was computed above. As the speed increases still further, the orbits will become ellipses extending far out into space as shown in the figure at the right. Our own moon is, of course, traveling in an orbit that is very nearly circular.



So far, only the effect of velocity on the orbit has been mentioned. However, there is another factor of importance in determining the characteristics of an orbit, namely the initial direction with which the body was launched. This, in turn, will determine whether the orbit is a long flattened ellipse or a nearly circular one. Both kinds can correspond to the same velocity of launching, differing only with the direction of launching.

Suppose now that our satellite, mentioned above, is launched directly upward with the same velocity, instead of on a circular orbit parallel to the surface of the earth. The simple equation of its motion shows that it will travel out into space a distance equal to the diameter of the earth before returning to the earth. If the velocity is increased, the vehicle

*After the noted astronomer, Johannes Kepler (d. 1630)

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will, of course, travel even farther out. When the initial velocity has been increased to a value equal to the $\sqrt{2}$ (or 1.41) times the orbital speed of 25,810 ft. per sec. or 36,500 ft. per sec., it will travel out beyond the influence of our planet and never return. This speed is appropriately called the escape velocity.

Returning now to a more detailed examination of the characteristics of a vehicle rotating in a circular orbit a few hundred miles above the surface of the earth, we note that the balance between gravitational and centrifugal forces exists not only for the vehicle itself, but also for all objects within the vehicle. Consequently there will be no "up" or "down". Everything will float weightless inside the vehicle.

When we consider the possible orbits in which the vehicle could travel, as seen from the earth, we realize that they must all be great circle paths, i.e. in planes passing through the center of the earth. Of all such paths, only the one lying in the plane of the equator will repeat itself on each revolution because for all the others when the vehicle has completed a circuit in approximately 1-1/2 hours, the earth has turned under it 1/16 of a revolution and the vehicle is over a new spot on the earth's surface. Consequently, the first attempts at establishing satellites will be around the equator so that they may be repeatedly observed from fixed ground stations.

So far, we have purposely avoided considering the means of supplying the enormous energies necessary to obtain the speeds calculated above. This is such an important problem that it will be given special consideration in the next chapter.

Chapter 4

4. POWER PLANT SUITABLE FOR SATELLITE VEHICLES

In order to be able to establish a vehicle in a satellite orbit, a power plant must be capable, not only of lifting its own weight and that of its fuel and the associated structure and payload, but also to accelerate these components sufficiently to attain the enormous velocities calculated in the preceding chapter. Clearly this will require a power plant capable of producing thrusts many times its own weight. At the present time, the only quasi-conventional power plants that meet this requirement are the rocket, the turbo-jet and the ram-jet.

The turbo-jet and the ram-jet both depend upon atmospheric air for their combustion. Their maximum thrusts fall off rapidly with altitude so that their useful range is well below 100,000 ft. When speeds of the order of 24,500 ft. per sec. (Approximately a Mach number of 25) are considered, the compression and friction of the air give calculated temperatures of the order of 49,000°F.* Even at 100,000 ft. the density of the air is sufficiently great to burn up the vehicle in short order. Consequently it would appear that the turbo-jet or the ram-jet could be used only in the very initial stages of launching man-created satellites.

It is conceivable that these power plants may be found to serve a useful purpose as initial launching engines. However, for the present investigation, this scheme has been left out of consideration in order to avoid the complication.

*Long before such temperatures are reached, the conventional methods of calculation become invalid. However, the conclusion that the temperatures are prohibitively high is still valid.

Chapter 4

The rocket motor, carrying its own propellants, can traverse the atmosphere at limited speeds and after entering the rarefied ionosphere be free to accelerate to the speeds required for orbital motion. The V-2 has demonstrated the practicality of such a scheme. The greatest question to be answered is whether within the stern accounting of engineering reality, successors to the V-2 can be built capable of accelerating to speeds of the order of 25,000 ft. per sec.

Before attempting to answer this question, it will be of interest to examine rocket power plants in some detail. At present these power plants can be divided into two general classes. The first is the familiar solid propellant type of rocket used extensively in Fourth of July celebrations. When used to obtain high performance, the propellant containers must withstand such great pressures that their weight becomes prohibitive where weight is an important consideration. This has led to the development of the liquid propellent rocket in which the propellants are forced into the combustion chamber under gas pressure (frequently compressed nitrogen) or by means of pumps as in the V-2. For installations where large thrusts are required this latter system has proved to be of lighter weight.

It is helpful to have an understanding of the parameters which are used for evaluating the performance of a rocket motor and which, since they are unique to the field of jet propulsion, may be unfamiliar to the reader. From Newton's familar second and third laws, it may readily be shown that the thrust T is equal to the product of the exhaust velocity, c and the mass rate of propellant consumption, $\frac{dm}{dt}$, thus $T = c\,\frac{dm}{dt}$. The quantity $\frac{dm}{dt}$ may be made as large as we please since it is only a matter of arranging adequate means for delivering and burning the desired amounts of propellants. However, this is not the case with the exhaust velocity which is more strictly a characteristic of the propellants used. The exhaust velocity is determined to a large extent by the molecular weight, the temperature, and the specific heats of the combustion products. For a given fuel we have little control over these quantities. The pressure in the combustion chamber, the external atmospheric pressure and the overall efficiency of the power plant (which are the factors over which we have greatest control) also affect the exhaust velocity but to such a lesser degree that it is possible to assess the exhaust velocities of an installation largely from a knowledge of the propellants used.

It will be seen later that the exhaust velocity of a rocket installation is of prime importance in determining its suitability for use as a satellite-producing power plant. In addition to the exhaust velocity c, two other parameters are frequently used. The equation for the thrust of a rocket motor shows that a given quantity of propellants, if consumed under comparable conditions, represents an ability to produce a given impulse, either as a large thrust for a short time or a small thrust for a proportionately larger time. Consequently, it is in order to ask for the pounds of thrust obtained per pound of propellant per second. It is seen at once that this parameter, known as the specific impulse I, is given by the formula $I = \frac{T}{g\frac{dm}{dt}} = \frac{c}{g}$, i.e. it is obtained from the exhaust velocity simply by division by the acceleration of gravity (I is the same in both c.g.s. and ft.-lb.-sec.systems since it contains units of force

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in both numerator and denominator). Again, we may ask for the pounds of propellant consumed per pound of thrust per second. This is known as the specific fuel consumption and is merely the reciprocal of the impulse: $s.f.c. = \frac{1}{I} = \frac{g}{c}$. Typical values of these parameters are $c = 6{,}434$ ft. per sec., $I = 200$ sec., and $s.f.c. = .005$ $sec.^{-1}$

In the discussion above, it was mentioned that external atmospheric pressure played a lesser role in determining the exhaust velocity. While this role is small, it is not insignificant and enters into the problem of establishing a man-made satellite in a very helpful fashion. As we go to higher altitudes, the atmosphere exerts less of a back pressure on the exhausting gases, allowing their velocity to increase until at extreme altitudes it has increased by some 20% to 30%. This will be found to be of significant magnitude in our problem of determining if rocket motors are capable of imparting a sufficiently large momentum to the proposed satellite vehicle, a problem to which we now return.

Chapter 5

5. DYNAMICS OF ACHIEVING ORBITAL MOTION

Let us begin by considering the simplest possible case of a rocket motor accelerating our vehicle to high speed. We shall temporarily neglect gravity and air resistance in order to determine what are the fundamental factors occuring in our problem. If m is the mass of the vehicle at any instant, $\frac{dV}{dt}$ the acceleration and T the thrust, then $m\frac{dV}{dt} = T$. In the preceding chapter, we saw that $T = -c\frac{dm}{dt}$.* Placing this in our equation, we have $m\frac{dV}{dt} = -c\frac{dm}{dt}$. This can be integrated to give $\Delta V = c \log \frac{m_o}{m_1}$ where ΔV is the change in velocity of the vehicle that the rocket produces and m_o and m_1 are the masses at the beginning and end of the acceleration, their difference being the fuel used in the process. This formula, although it will be successively modified numerous times, brings into focus the two most fundamental parameters of our problem; namely, the exhaust velocity and the mass ratio. In fact, these two parameters are so vital that the next two chapters will be devoted entirely to a critical engineering analysis of what values we can reasonably expect to achieve.

It is clear that the gain in velocity of the vehicle is directly proportional to the exhaust velocity and any improvements in this factor will be immediately reflected in the performance of the vehicle. The mass ratio, entering the logarithm would appear to be a factor of minor importance. However, this appearance is quite deceptive as we shall presently see.

If we put W equal to the initial gross weight of the vehicle, P equal to the payload and S equal to the entire structural, power plant, tank and control weight (i.e. S includes all items except the fuel and the payload)

*The minus sign is necessary here because $\frac{dm}{dt}$ is the rate of change of mass of the vehicle (which is negative) while in Chapter 4 it was the rate of propellant consumption.

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then our formula becomes $\Delta V = -c \ln\left(\frac{S}{W} + \frac{P}{W}\right)$. As we have seen before, c cannot be made arbitrarily large, but is limited by the state of development of our technology. Likewise, $\frac{S}{W}$, the ratio of the entire structural weight to the gross weight, cannot be chosen arbitrarily small but is limited by technological progress. Consequently, the quantity within the parentheses of the logarithm has a smallest value when the payload is zero (this will make the logarithm, with a negative sign in front, have its greatest value). Actually, in engineering application we usually must view this the other way around; that is, the payload is given and the gross weight must be varied. This has been illustrated on the accompanying graph. Here we have plotted the ratio of velocity increase to exhaust velocity against the ratio of gross weight to payload (i.e. the gross weight for a 1 lb. payload) for various values of the structural weight ratio. The extreme importance of this latter parameter is immediately apparent.

This graph also illustrates another characteristic that will confront us time and again; namely, the extreme variability of the gross weight for a fixed payload when we attempt to obtain high performances. For example, suppose we could obtain an exhaust velocity of 11,000 ft. per sec. and build the complete structure for only 5% of the gross weight. Then to accelerate a payload of 1 lb. to 24,750 ft. per sec. ($\frac{\Delta V}{c} = 2.25$) would require a vehicle having a gross weight of 200 lbs. However, if the desired velocity had been only 2% greater, the smallest vehicle with which

[1]Here and throughout the rest of the report we shall refer to both the fuel and the oxidizer simply as the fuel and designate its weight by F. If we call them propellants and designate their weight by P, we should have a conflict with our designation for payload.

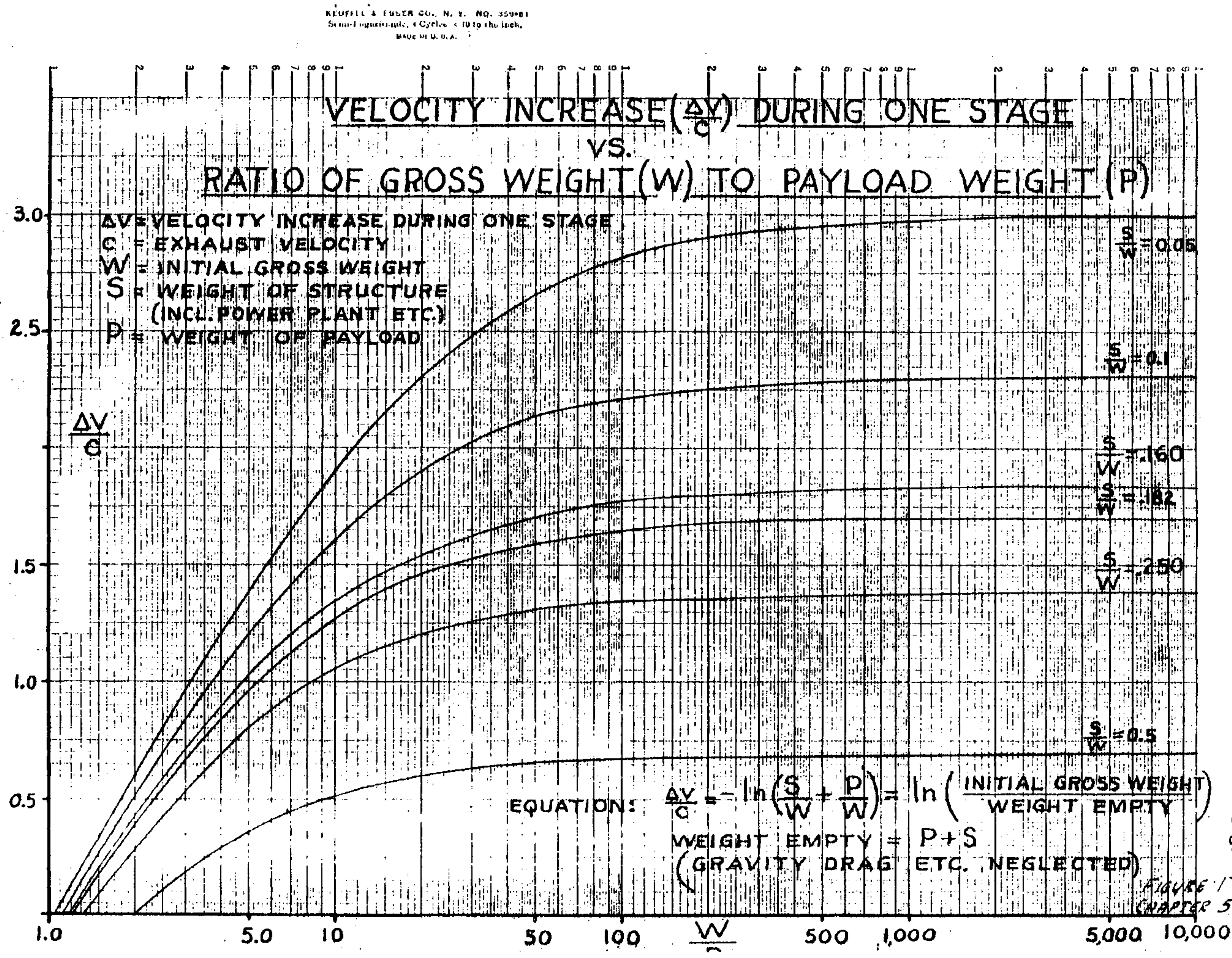

FIGURE 1
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we could accomplish this would have a gross weight of 1000 lbs., a fivefold increase. The reason for such extreme sensitivity is clear; the performance gain was made by adding a bit more fuel at the expense of payload and then enlarging the entire project until the payload returned to its original value. What was a fraction of a percent increase in fuel amounted to 80% of the payload. Consequently the multiplication factor was five. Simple clarity of reason does not alter the fact that the gross weight is a variable of questionable reliability.

We are now in a position to make an elementary examination of the feasibility of using rockets to establish new satellites. To do this we shall anticipate a few of the results of the next two chapters. There we shall find that by using alcohol and liquid oxygen (these were the propellants used in the V-2), we can obtain average exhaust velocities of about 8,500 ft./sec. and a corresponding entire structural weight of about 16% of the gross weight. Both of these figures have had a certain amount of optimism injected in them, to represent what we might reasonably expect to accomplish in the foreseeable future. If we select 500 lbs. as our goal for a payload, then our formula shows that a vehicle of 5000 lbs. initial gross weight could be accelerated to 11,420 ft. per sec. If the size of vehicle is increased to 50,000 lbs. gross weight, the velocity is 15,090 ft. per sec. and a 500,000 lb. vehicle only gives an increase to 15,510 ft. per sec.* All of these velocities are impressively large, but fall considerably short of our round figure of 24,500 ft. per sec. required for orbital velocities.

*Even if the vehicle were made indefinitely large the velocity could not exceed 15,600 ft. per sec.

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The question immediately arises: By using liquid hydrogen, the fuel that tops the list with an exhaust velocity of about 13,500 ft./sec., can we achieve our desired velocity? Unfortunately, liquid hydrogen has a number of characteristics (which will be discussed in detail later) that necessarily cause an increase in structural weight. Our figure of 16% for structural weight is increased to 25% for use with liquid hydrogen. The following table summarizes the velocities calculated for both alcohol and liquid hydrogen:

Gross wt. for 500 lb. payload	Velocity of Vehicle Using Alcohol	Velocity of Vehicle Using Liquid Hydrogen
5,000 lbs.	11,420 ft./sec.	14,180 ft./sec.
50,000 lbs.	15,090 ft./sec.	18,160 ft./sec.
500,000 lbs.	15,510 ft./sec.	18,620 ft./sec.
Indefinitely large	15,600 ft./sec.	18,700 ft./sec.

The liquid hydrogen shows improvement* over the alcohol but is still considerably short of producing the orbital velocity figure of 24,500 ft. per sec.

We are forced to conclude that a realistic appraisal of the problem shows that our technology, even allowing a reasonable note of optimism to creep in, has not sufficiently advanced as yet to permit us to build a single unassisted vehicle capable of acquiring sufficient speed to remain in space as a satellite. This is doubly emphasized when we remember that as yet we have neglected entirely the effects of air resistance and gravity.

Since we cannot attain our goal with an unassisted vehicle, we next examine the problem of giving the vehicle enough initial speed so that it

*This is true only for single assisted vehicles using the simplified analysis presented here. When the multistage vehicles (to be considered presently) are compared using a refined analysis, the conclusion is different.

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can subsequently attain orbital velocities under its own power. Since rocket power plants have been shown capable of supplying more than half of the velocity required, it appears logical to ask if they cannot be used to supply the other half. To answer this question in the affirmative we introduce the concept of a multistage rocket. We shall find this idea fundamental to our later work. To illustrate this concept, let us consider a two-stage rocket. The primary vehicle will be carried along as the "payload" of a larger secondary vehicle. When this larger vehicle has exhausted its fuel, and hence its usefulness, it will be discarded and the smaller vehicle will continue to accelerate under its own power, adding its own velocity increase to that imparted by the larger stage. The particular example selected above is a special case of a much more general idea, namely, that of discarding weight once it has served its purpose and is no longer necessary. A moment's reflection shows that this can be of great aid, because as the fuel is used up, the structural weight and the payload, initially insignificant, become major items and if substantial reductions in the structural weight are possible at this point, the remaining fuel will be capable of supplying correspondingly greater accelerations and velocities.

In place of the method proposed above, it is conceivable that the fuel could be contained in multiple tanks and as each is drained in turn, it and its associated structure would be jettisoned. With this reduced weight, the acceleration would increase considerably and it might be desirable to shut down a portion of the rocket power plant to keep the loads on the remainder of the structure within reasonable limits. The remainder

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of the fuel would be used to produce smaller thrust over a longer period of time. If this is done, it is of course advisable also to jettison the idle power plants.*

It will be readily appreciated that such staging schemes are limited only by the fertility and ingenuity of the designer's imagination. For the sake of definiteness, we have confined our attention in this report to the clearcut scheme originally proposed, but it is not intended to imply that this is a final arrangement.

Let us return to the problem of examining the possibilities of achieving orbital velocities. We found for a single stage, that $V = -c \ln \frac{S + P}{W}$. If we now have a two stage rocket, and we designate the larger vehicle, which is fired first, by the subscript 1 and the smaller by the subscript 2, then

$$V_{total} = -c \ln \frac{S_1 + W_2}{W_1} - c \ln \frac{S_2 + P_2}{W_2} .$$

Here we have used W_2, the gross weight of the smaller vehicle, as the payload of the larger. With this notation W_1 is the gross weight of the entire aggregate.

It is now logical to ask: For a given payload and a fixed value of aggregate weight, what is the correct proportioning of the two stages to give the greatest total velocity? If the large stage can be built so that its entire structural weight is the same percentage of its gross weight as that of the smaller stage, then simple differentiation shows

*Viewed from this standpoint, our original proposal of a series of progressively larger vehicles each carrying the preceding member as payload, consists of building tanks, power plants and structure in associated size units and jettisoning them as units.

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the greatest total velocity is obtained when the ratio of payload to gross weight is the same for each stage, i.e. $\frac{W_2}{W_1} = \frac{P}{W_2}$.

If we apply these results to our alcohol and liquid oxygen powered vehicle, (and assume that the entire structure of the large stage can also be built for 16% of its gross weight) we can achieve the following velocities with the corresponding aggregate combinations:

TWO STAGE ROCKET VEHICLE, USING ALCOHOL AND LIQUID OXYGEN AND CARRYING A PAYLOAD OF 500 LBS.

Gross Weight of 1st Stage	Gross Weight of 2nd Stage	Total Velocity
50,000 lbs.	5,000 lbs.	22,849 ft./sec.
5,000,000 lbs.	50,000 lbs.	30,180 ft./sec.
500,000,000 lbs.	500,000 lbs.	31,020 ft./sec.

This table illustrates two salient points:

1st, a two stage rocket vehicle, using feasible values of exhaust velocity and structural weights has been shown to have a reasonable margin over the minimum essential requirement to attain orbital speeds. It only remains to be seen if this margin is sufficient to account for the effects of air resistance, gravity and the like.

2nd, we notice, upon comparing this table with the results of our single stage calculations that for a given total weight, (e.g. 50,000 lbs.) we can attain a greater total velocity from two stages (22,840 ft./sec.) than we can from one stage (15,090 ft./sec.). And this is in spite of the fact that we have the weight of two machines instead of one.

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This second point immediately poses the following questions: Is it always better to use two stages than one? If two stages are superior, would three or more stages give even greater velocities for a fixed aggregate weight? These questions are answered by the accompanying graph,* on which has been plotted the total velocity with the available exhaust velocity taken as a unit, against the gross weight of the aggregate for a one pound payload for 1, 2, 3, 4, and 5 stages. This has been computed on the assumption that each vehicle could be built for an entire structural weight of 16% of its gross weight. In each case, the stages have the optimum proportions mentioned above.

We see immediately that two stages are not always superior to one. For small aggregate weights, a single stage is better, but at higher weights the two-stage curve crosses over and gives higher velocities. For a better understanding of the reasons behind this it is helpful to refer back to our remarks on the great variability of the gross weight of a single stage. There we saw that in our attempt to get higher and higher performances from a fixed exhaust velocity, we were exchanging payload for fuel and then swelling the size of the entire vehicle to return the payload to its specified value. As the payload became a diminutive portion of the vehicle, its exchange for fuel could affect the performance but little, while the multiplication in size became astronomical. It is at this point of diminishing returns that it is

*For additional graphs of the same type but with $\frac{S}{W}$ = .1, .143, .182, and .25, see Chapter 8.

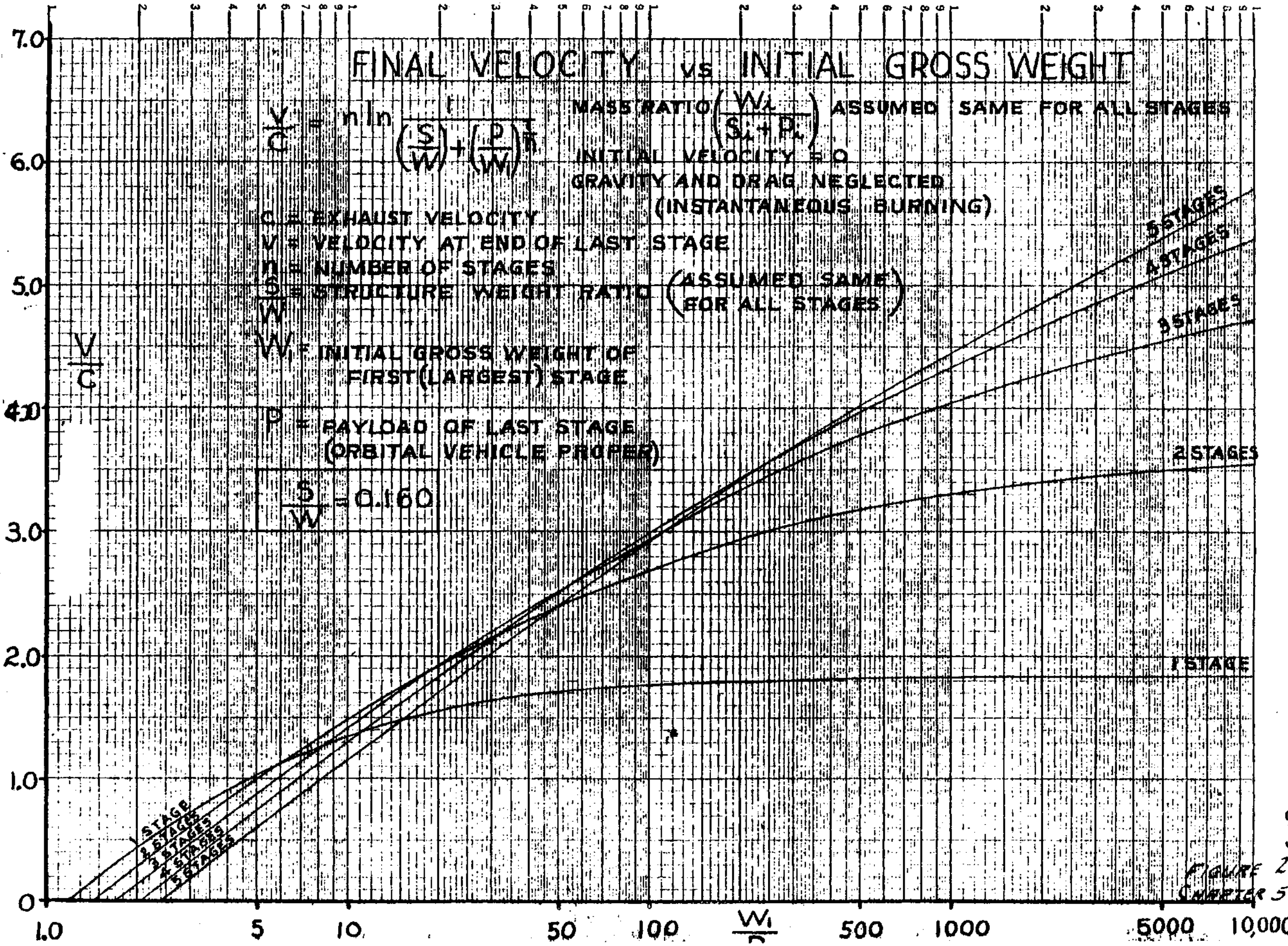

FIGURE 2
CHAPTER 5

better to use two stages. This same line of reasoning answers our question about larger numbers of stages, because as the two-stage vehicle reaches its point of diminishing returns, it is advantageous to use 3 stages and so on for 4, 5, 6 and higher numbers of stages. It is interesting to note that this simplified analysis would indicate that the Germans could have accomplished the mission of the V-2's with an approximate 25% decrease in total weight if they had used two stages instead of one. Undoubtedly, with all factors taken into account, including the urgency of the situation, they were well justified in using a single stage missile.

Thus far, by neglecting the "practical" details of gravity, air resistance, variation of exhaust velocity with altitude, inclination of flight path, control, maneuvering and the like, we have indicated the possibility that our technology has advanced sufficiently for us to launch a new satellite into space. Now we must determine how great will be the influence of these "practical" details.

First, let us consider the effect of gravity. So far, it has made no difference whether we used our fuel to produce a large thrust for a short time or a small thrust for a longer time. All that mattered was the velocity of the exhaust products and not the consumption rate. However, when the vehicle is accelerating vertically upward, this is no longer the case. If the thrust is insufficiently large to exceed the weight of the vehicle, the rocket will ineffectually expel its fuel, accomplishing little more than a display of fireworks. It can easily be shown that for vertical acceleration, larger velocities will be attained as greater thrusts are used for shorter times. As the thrust becomes infinite, the velocity will approach that calculated by our simplified analysis. This concept of an infinite thrust, frequently encountered in more abstract treatises on rocket vehicles, would indicate that the effect of gravity could be made negligibly small. However, closer examination shows this is not the case. As we increase the thrust, the weight of the rocket combustion chamber, pumps, piping, controls and associated structure goes up. Furthermore, the remaining structure such as tanks and supports is subjected to increasing loads as the thrust increases, with a consequent increase in weight of these items. Since we have seen that the performance is critically sensitive to the structural weight ratio S/W, the increase of this parameter will rapidly nullify the benefits of increased acceleration; in fact, we would anticipate that an optimum acceleration exists, representing the best compromise between the advantages of high thrust and the accompanying disadvantages of high structural weight. Unfortunately we have not as yet laid the foundation of structural analysis necessary to pursue this investigation further at this point.

If we attempt to examine the other "practical" factors in detail we shall find that corresponding foundation data are lacking for them too. Consequently, it is advisable to turn our attention now to a detailed examination of the capabilities of the rocket power plant and an analysis of the feasible weights of structures. Later we shall resume our investigation of the "practical" details. In the analytical work that preceded the writing of this report, performance studies, structural analysis, and the assessment of rocket power plant capabilities all proceeded hand in hand. Consequently, in the next two chapters, which deal in turn with rockets and structural weights, we shall find frequent references to the results of our more detailed performance analysis which will be presented later. Unfortunately there appears to be no way of avoiding this lack of straightforwardness in the presentation of a subject whose parts are so closely interrelated.

As an aid to the reader, a few words of coordination may prove helpful. It was decided to investigate two vehicles. One employed alcohol and liquid oxygen rockets as representative of an established technique founded on the Germans' experience with the V-2. The second employed liquid hydrogen and liquid oxygen rockets as representative of the top class of high velocity propellants. It was found best to use a four stage vehicle when using alcohol and oxygen and a two stage vehicle when using hydrogen and oxygen.

Chapter 6

6. ROCKET POWER PLANTS AND FUELS

The importance of selecting propellants which give high exhaust velocities is obvious from Chapter 5. High exhaust velocity cannot be the sole criterion, however. One or both propellants of every system proposed to date possesses physical properties which are so extreme as to present major engineering or operational problems, in some cases, to a degree almost precluding use of the propellant. Consequently, along with a consideration of specific impulse must go a careful weighing of the other advantages and disadvantages of a particular system. The disadvantages of some properties such as inflammability, corrosivity, toxicity, sensitivity to detonation, availability and handling and storing qualities are obvious. Others, such as high vapor pressure, low density, low boiling point, high average molecular weight of the products of combustion are not so obvious and require a few words of explanation.

Two types of liquid propellant systems are used: bipropellant and monopropellant. In the bipropellant system a fuel and an oxidizer, both of which may be a mixture of two or more compounds, are mixed and burned in the combustion chamber. In the case of the monopropellant system a liquid or a mixture, which is stable at ordinary temperatures, is injected into the combustion chamber where, after ignition it decomposes at the temperatures and pressures prevailing. The bipropellant system is more complicated than the monopropellant since it presents problems of designing injectors to give good mixing, of feeding the propellants at a constant mixture ratio and of providing tanks, tubing and pumps for two propellants. The monopropellants have, in general, lower specific impulses and are inherently unstable, decomposing explosively under high temperatures or shock.

Motors operating continuously for periods longer than about 30 seconds must be provided with cooling. One method, known as regenerative cooling, brings one or both propellants to the combustion chamber through ducts in the motor walls. This system is limited by the ability of the propellants to absorb the necessary heat without boiling or decomposing. Another method called film cooling injects a liquid, preferably one of the propellants, through numerous small orifices so as to provide a cool film between the hot gases and the motor walls. This system was used on the German V-2 motor in addition to regenerative cooling with alcohol. Temperature of the gases may also be reduced by using an excess of fuel or oxidizer, by addition of water to the fuel, or by injecting water directly into the chamber. If carried to extreme the latter methods are costly in specific impulse.

Unless gas pressurization is used pumps are required to supply the propellants to the combustion chamber at high pressures and mass flow rates. To keep the weight of the pumping system low it is desirable to use high speed centrifugal pumps and as few pump stages as possible. Weight saving along these lines is limited by cavitation. Since cavitation appears on the blade at the point where the pressure drops to the vapor pressure of the fluid, a propellant with high vapor pressure leads to lower rotative speeds and more stages and so to excessive feed system weights. Low density of the propellant also increases pump weights. This is due to the fact that lower density reduces the pressure rise per stage of a centrifugal, pump so that more stages are required.

The specific impulse of a propellant system at optimum expansion ratio can be calculated from the formula

$$(1) \qquad I = 6.94\sqrt{\frac{T_c}{M}}\left\{\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}}$$

if T_c, M and γ (respectively the temperature, average molecular weight, and ratio of the specific heats of the gases in the chamber) are known for the pressure ratio p_e/p_c. Now p_e/p_c, with minor reservations, can be chosen without regard to the propellant system and, although γ varies some for the different systems, its effect is comparatively small. Hence the ratio T_c/M accounts for the major part of the variation in specific impulse exhibited in table (1). Stoichiometric mixture ratios give a maximum for T_c but not necessarily for T_c/M. For example, stoichiometrid mixture ratio for liquid hydrogen and liquid oxygen occurs at approximately 89% by weight of oxygen, but figure (1) shows that the maximum of T_c/M as reflected in I lies at about 76% which corresponds to nearly five moles of hydrogen to one of oxygen instead of the stoichiometric ratio of 2 to 1. The reason, of course, is that the low molecular weight of the excess hydrogen in the gases reduces M and more than offsets the decrease in temperature.

Chapter 6

TABLE 1. - Summary of Rocket Propellants*

Bipropellant Systems

	System (wt. percent)	Spec. Grav.	P_c atmos.	T_c $^oR.$	T_e $^oR.$	M	I sec.
(1)	23.9% liquid hydrogen, 76.1 liquid oxygen	.248	23.0	4,960	2,650	8.36	362
(2)	45.8% hydrazine, 54.2% liquid fluorine	1.061	20.4	6,850			292
(3)	60.1% hydrazine, 39.9% liquid oxygen	1.061	20.4	5,550	3,090		264.0
(4)	32.6% methyl amine, 67.4% liquid oxygen	.985	20.4	6,100	3,560		251.5
(5)	31.9% liquid ammonia, 8.1% liquid acetyline, 60% liquid oxygen	--	20.4	5,880			257
(6)	25.4% liquid acetylene, 74.6% liquid nitrogen tetraoxide		23.0	6,230	3,960		256
(7)	41.5% liquid ammonia, 61.6% liquid oxygen		20.4				249
(8)	58.5% hydrazine, 58.6% hydrogen peroxide	1.237	20.4	4,890	2,900		249
(9)	9.8% liquid acetylene, 21.1% liquid ammonia, 69.1% liquid nitrogen tetraoxide		23	5,530	3,600		244
(10)	40% ethyl alcohol, 60% liquid oxygen	.966	20.4	5,720			243
(11)	71.5% liquid oxygen, 28.5% gasoline	.978	20.4	5,930	3,460	22.66	242.0
(12)	24.0% liquid acetylene, 31.4% liquid ammonia, 44.6% liquid oxygen		21.4	4,140	2,070		240
(13)	19.4% liquid propane, 80.6% liquid nitrogen tetraoxide		23	5,580	3,600		238

*Table I (slightly revised) from "Fuel Systems for Jet Propulsion" (prepared for Commander in Chief, U. S. Fleet) by Alexis W. Lemmon. Jr.

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TABLE 1. - Summary of Rocket Propellants (Cont.)

Bipropellant Systems

	System (wt. percent)	Spec. Grav.	P_c atmos.	T_c °R.	T_e °R.	M	I sec.
(14)	46.6% liquid ethylene, 53.4% liquid oxygen	.774	20.4	4,040		15.00	236
(15)	40% nitromethane, 60% hydrogen peroxide		20.4	5,350		24.3	227
(16)	70% nitromethane, 21% hydrogen peroxide, 4% water, 5% methyl alcohol		20.4	4,950		21.1	226
(17)	92.9% nitromethane, 7.1% liquid oxygen	1.139	20.4	5,160	2,910		225.5
(18)	21.44% methyl alcohol, 78.56% hydrogen peroxide	1.239	20.4	4,590	2,960		225
(19)	22.2% gasoline, 54.5% liquid oxygen, 23.3% liquid nitrogen	.931	20.4	5,290	3,020	23.92	221.5
(20)	57.1% methyl alcohol 42.9% liquid oxygen	.911	20.4	4,120	2,350		221
(21)	25% aniline, 75% red fuming nitric acid	1.390	20.4	5,525		25.41	220.5
(22)	17.9% mono-ethyl aniline, 82.1% mixed acid	1.396	23.0	5,060	3,400		210.0
(23)	33.6% liquid diborane, 66.4% water	.706	20.4				200
(24)	24.4% ethylene diamine, 55.4% hydrogen peroxide, 20.2% water	1.174	20.4	3,140	1,780		196.3
(25)	48.4% liquid ethane, 51.6% liquid oxygen	.760	20.4	1,910		12.40	180

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TABLE 1. - Summary of Rocket Propellants (Cont.)

Monopropellant Systems

	System (wt. percent)	Spec. Grav.	P_c atmos.	T_c °R.	T_e °R.	M	I sec.
(1)	Nitromethane (100%)	1.139	20.4	4,590		20.3	222
(2)	80% methyl nitrate, 20% methyl alcohol		20.4	4,370		20.0	221
(3)	70% nitroglycerine, 30% nitrobenzene		20.4	4,950		22.9	217
(4)	Nitromethane (100%)	1.139	20.4	4,430	2,400	20.34	216.5
(5)	Diethylene-glycol dinitrate (100%)	1.483	20.4	4,590		21.8	215
(6)	Diethylene-glycol dinitrate (100%)	1.483	20.4	4,540	2,520		213.1
(7)	89.6% nitromethane, 10.4% nitrobenzene	1.181	20.4	4,450	2,400		212
(8)	90% nitromethane, 10% nitrobenzene		20.4	3,980		19.4	211
(9)	83% nitromethane, 17% nitroethane	1.123	20.4	3,940	2,050		206
(10)	90% diethylene-glycol dinitrate, 10% nitrobenzene		20.4	3,960		20.6	204
(11)	Ethyl nitrate (100%)		20.4	3,530		18.2	203
(12)	61.9% nitromethane, 38.1% nitroethane	1.105	20.4	3,310	1,650		195.9
(13)	Hydrogen peroxide (100%)	1.463	20.4	2,258	1,173	22.68	146.
(14)	87% hydrogen peroxide, 13% water	1.381	20.4	1,668	840		126

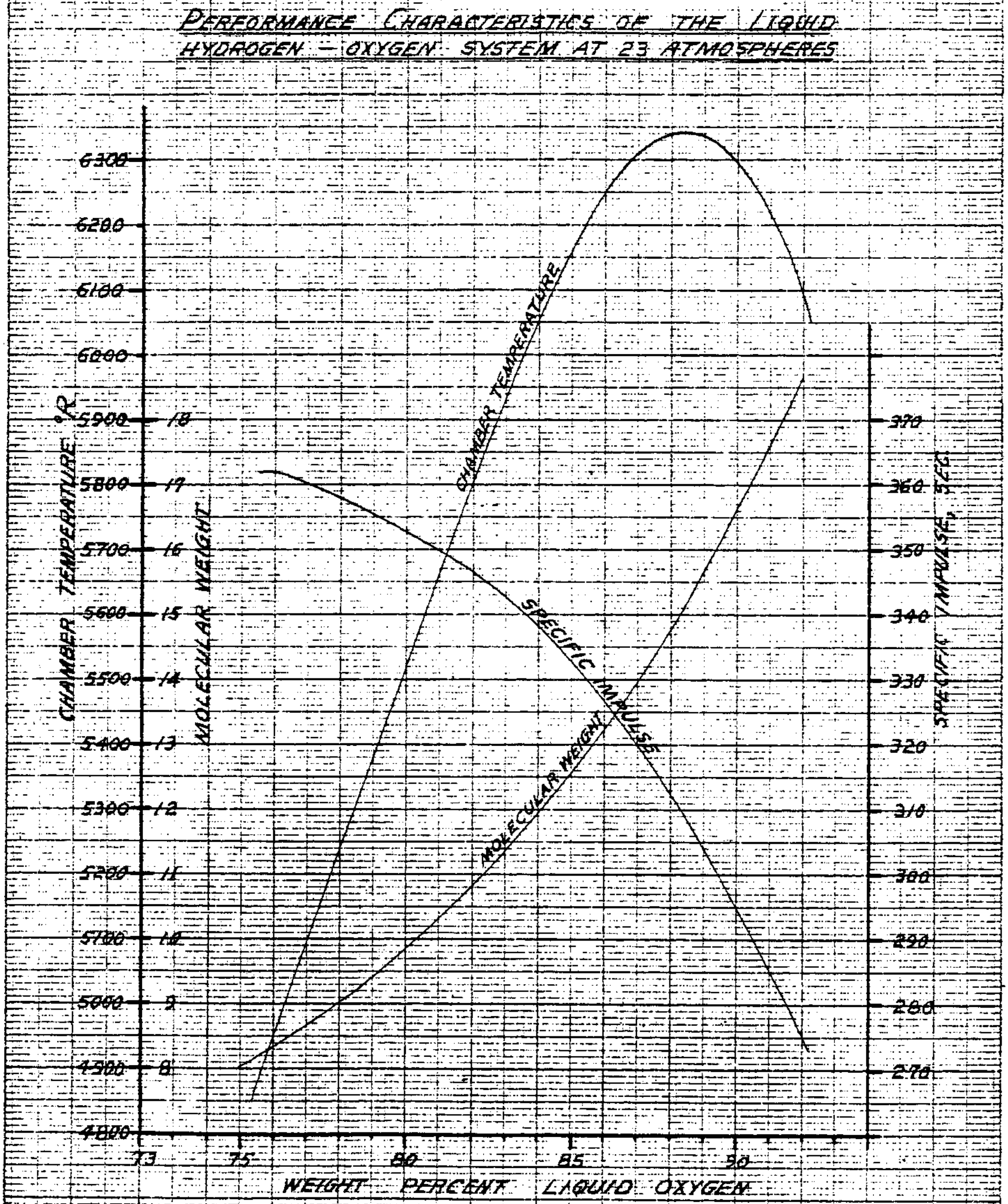

FIGURE 1
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The quantities T_c, M and γ of formula (1) are calculable for a given chamber pressure. Their values along with specific impulse and density are given in table (1) for a number of propellant systems. This list does not contain all possible systems but is representative of rockets obtaining their energy from combustion. At first glance it might seem that, in view of the variety of fuels available for consideration, the performance might well rise beyond the limits indicated by the table as unnoticed fuels with higher heats of combustion are brought to attention. That the problem is not quite so simple is shown by comparison of liquid oxygen-alcohol with liquid oxygen-gasoline. The heat of combustion of gasoline is appreciably higher (60% higher if n-octane is used for gasoline) than ethyl alcohol. Yet the specific impulses of the two systems are approximately equal. The underlying reason is the appearance of dissociation at about 4500°R which absorbs large amounts of energy. Both systems are composed of the elements carbon, hydrogen and oxygen. In addition to the equations of oxidization of alcohol and gasoline to carbon dioxide and water, the reversible reactions of dissociation

$$2\,H_2O \rightleftarrows H_2 + 2OH,$$

$$H_2O \rightleftarrows H_2 + O\ ,$$

$$2\,H_2O \rightleftarrows 2H_2 + O_2\ ,$$

$$H_2 \rightleftarrows 2H\ ,$$

$$CO_2 + H_2 \rightleftarrows CO + H_2O,$$

enter into the equilibrium of the products of combustion for the two cases. The dissociation processes are accompanied by the absorption of large amounts of heat so that the greater heat of combustion of gasoline is absorbed chiefly by increased dissociation. From this example of a general behaviour it is evident that the common oxidizers and fuels composed principally of carbon, hydrogen, oxygen, and nitrogen must be fairly represented by the examples of table (1) since the same dissociations must appear to limit the chamber temperatures. The use of liquid ozone, for instance, which has a negative heat of formation, instead of liquid oxygen may increase specific impulses but not significantly.

Since common fuels and oxidizers promise nothing phenomenal it is natural to examine the uncommon reactants. If we turn to the halogens, fluorine is the logical choice because of its low molecular weight. Furthermore, hydrogen fluoride dissociates less easily than water. High molecular weights, however, limit the choice of fuels to the non-carbonaceous, since the best carbon compound which could appear in the products of combustions is carbon tetrafluoride which has a molecular weight of 88, twice that of its oxygen-formed counterpart, carbon dioxide. Another disadvantage of fluorine is that it is one of the most reactive substances known and therefore extremely difficult to handle and store. Also hydrogen fluoride is sufficiently toxic to have had consideration as a weapon of chemical warfare.

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Metals have also been considered as fuels because of their high heats of combustion. However when the molecular weights of their oxides are weighted against their heats of combustion it is not clear that this approach leads to higher specific impulses. Dr. A. J. Stosick* calls attention to the fact that the heat of formation of the gaseous form of the metal oxides is considerably less than that of the solid form. The latter is the one usually quoted in the present connection.

Table (1) shows that the specific impulse of liquid hydrogen and liquid oxygen exceeds by an appreciable margin that of any other system listed. If we consider that liquid oxygen is pure oxidizer, that T_c cannot be increased appreciably and that an excess of hydrogen is the most effective practical means of obtaining low average molecular weight, it seems probable that the oxygen - hydrogen system will maintain its theoretical supremacy in specific impulse for some time to come. The system has, however, a large number of disadvantages which must be overcome before use in a rocket motor. To begin with the density of the system is far below that of any other system. Low density increases the size and therefore the weight and drag of the vehicle. The boiling point of hydrogen is of course very low, -259.18°C. which means that the vapor pressure will be high. The combination of high vapor pressure and low density makes a light weight simple pumping system almost impossible.

*Most of the above argument against the likelihood of remarkable propellant systems was gained through a verbal discussion with Dr. Stosick of GALCIT Jet Propulsion Laboratory. Any inaccuracies in fact or theory must be charged to misunderstanding or misquotation on the part of the writer.

The low boiling point, small temperature range (6.4°C) of the liquid phase and low heat of vaporization of liquid hydrogen make almost imperative use of thermos piping and pumps. High diffusivity of hydrogen makes sealing leads nearly impossible which, combined with the fact that hydrogen and oxygen are violently explosive in mixture ratios anywhere from 2% to 98%, makes accidents inevitably frequent. Cooling a hydrogen rocket is especially difficult. Neither liquid oxygen nor liquid hydrogen are usable for regenerative cooling because of their low boiling points. Liquid oxygen could not be used for film cooling since the more nearly stoichiometric mixture formed along the wall from the excess hydrogen would give intense heat. Nor could liquid hydrogen be used since it diffuses too rapidly to form an insulating film. Experimental investigation is difficult and hazardous because the excess hydrogen in the rocket exhaust forms a huge ball of flame on coming into contact with the atmosphere.

The difficulties emmerated tend to reduce the effective engineering use of the hydrogen-oxygen motor. In fact, the German V-2 engineers, from whom some of the information on hydrogen and oxygen was obtained, state that comparative designs, made for a rocket using hydrogen-oxygen and for the final rocket using alcohol-oxygen showed that the alcohol-oxygen rocket was superior in overall performance when all factors were taken into account.

While the difficulties of using liquid hydrogen as a fuel are discouraging, no one of them can be said to be impossible of satisfactory solution. It is conceivable that our technology may advance to a point where pumps can be replaced by a lighter pressure feed system such as a gas generation system. The need for cooling may be reduced by more heat resistant materials and the inclusion of a third fluid for the specific purpose of film cooling, and so on down the list. Of all the disadvantages of hydrogen, only the effect of low density on the size and weight of the vehicle is an irremovable difficulty. Consequently in a study of the feasibility of a satellite vehicle, the liquid hydrogen-liquid oxygen motor must be included as an evaluation of the worth and necessity of a high performance motor.

By the same token it is desirable to include in the present study some motor with less spectacular performance, but which has had sufficient development to insure that this somewhat lower performance can actually be attained in practice. Only four systems, liquid oxygen-alcohol, acid-aniline, hydrogen peroxide-alcohol (with hydrazine hydrate) and liquid oxygen-gasoline, have passed out of the theoretical-experimental stage and become production or semi-production motors. One of these, oxygen and gasoline, is dubiously placed in this class since satisfactory cooling has not yet been achieved. The most successful motor, to date, particularly from the important standpoint of specific impulse, is the V-2 motor which used liquid oxygen and alcohol. The theoretical value of the specific impulse is seen from table (1) to be appreciably higher than any of the other four except oxygen and gasoline, the least successful of all. Consequently, if the choice of motor is restricted to those now available, both theoretical and past performance force the selection of the liquid oxygen-alcohol motor.

The theoretical value of the specific impulse of a rocket motor, as would be expected, is never reached in practice. It is generally agreed that 90% of the maximum theoretical impulse is obtainable. This figure

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is supported by experience with acid-aniline motor which has been subject to extensive investigation and development. By improving the cooling so as to allow use of the most favorable mixture ratio and by improving the injector system so as to obtain better mixing, the specific impulse of the acid-aniline motor has been brought up to 90% of its theoretical maximum. It is reasonable, therefore, to suppose both oxygen-alcohol and oxygen-hydrogen will ultimately be brought to the same degree of perfection which is equivalent to assuming specific impulses of about 220 and 326 sec.* respectively. In the case of oxygen-alcohol, 220sec. is not overly optimistic since the V-2 motor is estimated to have had a specific impulse of about 215 sec.

Available time permitted studies of satellite vehicles employing both an oxygen-alcohol motor with the aforementioned expected impulse of 220 sec. and an oxygen-hydrogen motor with a specific impulse of 326 sec. Had time been available a study based on the liquid oxygen-hydrazine would have been interesting as a happy mean between the high performance and excessive disadvantages of the oxygen-hydrogen motor and the lower performance but proven feasibility of the oxygen-alcohol motor. According to table (1) the specific impulse of the system with 60% hydrazine is 264 sec. and the propellant density is 1.061. Both figures are higher than those for oxygen and alcohol. The boiling point of hydrazine is 113.5°C which is also higher than alcohol. Another advantage lies in the fact that the motor can be cooled by using an excess of fuel without seriously

*These values are those obtained at sea level with a combustion chamber pressure of 20 atmospheres which is representative of current rocket designs.

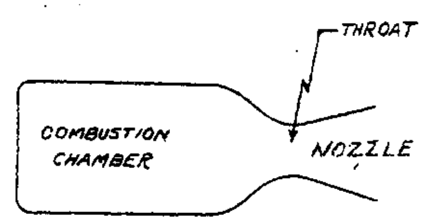
lowering the specific impulse; the loss in chamber temperature being partially offset by the appearance of free hydrogen in the gases. No obvious disadvantages enter except unavailability of hydrazine in large quantities and some toxicity. The former is said to be due to lack of commercial demand and to be easily overcome.

A few items in connection with the propellant feed system, the chamber pressure and the configuration of the combustion chamber and nozzle remain for discussion. Time has not permitted a study of the feed system but, on the basis of present knowledge and experience, turbine driven centrifugal pumps should be the most economical in weight for each stage of the satellite vehicle. Recently the notion of using a gas generator for providing the pressure has been considered and some work has been done on developing the method. Such a scheme looks promising from the weight standpoint. At present the turbine driven pump is the more advanced, although the margin is rather small since only a few pump-feed systems have been designed.

Gases for the turbine could be generated by burning the propellants in a firepot separate from the combustion chamber. However, gases generated from most systems must be cooled by some means such as introducing water as a third component. A notable exception is the monopropellant hydrogen peroxide which under the action of suitable catalysts decomposes to steam and oxygen. This system was used to generate steam for the V-2 turbines.

An optimum chamber pressure exists for any given installation. This optimum is fixed by two factors, the favorable increase in specific impulse and the unfavorable increase in weight of chamber and pumping system as the chamber pressure increases. Since, for most installations, the optimum works out to be about 300 psia, this value was assumed for the satellite vehicles considered. A motor designed to operate at a given chamber pressure may also be run at a lower pressure, as long as the lower limit at which the propellants burn stably is not passed. Consequently throttling to a lower thrust is possible, a maneuver which will be seen in a later chapter to offer an advantage in reduced structural weight. Fig. (2) shows the behaviour of T, p_c and I with throttling for an acid-aniline motor. Since no suitable data were quickly available on the throttled characteristics of either alcohol-oxygen or hydrogen-oxygen rockets, we shall use for later investigations of this problem variations similar to those shown for acid-aniline.

An important parameter governing the configuration of the nozzle is the ratio of the exit area to the throat area, called the expansion ratio. For each value of p_e/p_c an expansion ratio exists for which the specific impulse is a maximum. In the case of the satellite vehicle for which the motor must operate at a constantly increasing altitude and therefore constantly decreasing pressure ratio, a compromise between the optimum expansion



ratios at highest and lowest altitudes of operation of the motor must be made. Figures (3) and (4) show the variation in specific impulse with altitude, for alcohol-oxygen and hydrogen-oxygen rockets respectively. For these figures, the chamber pressures are assumed to be 20 atmospheres and the specific impulses at sea level for optimum expansion ratio are

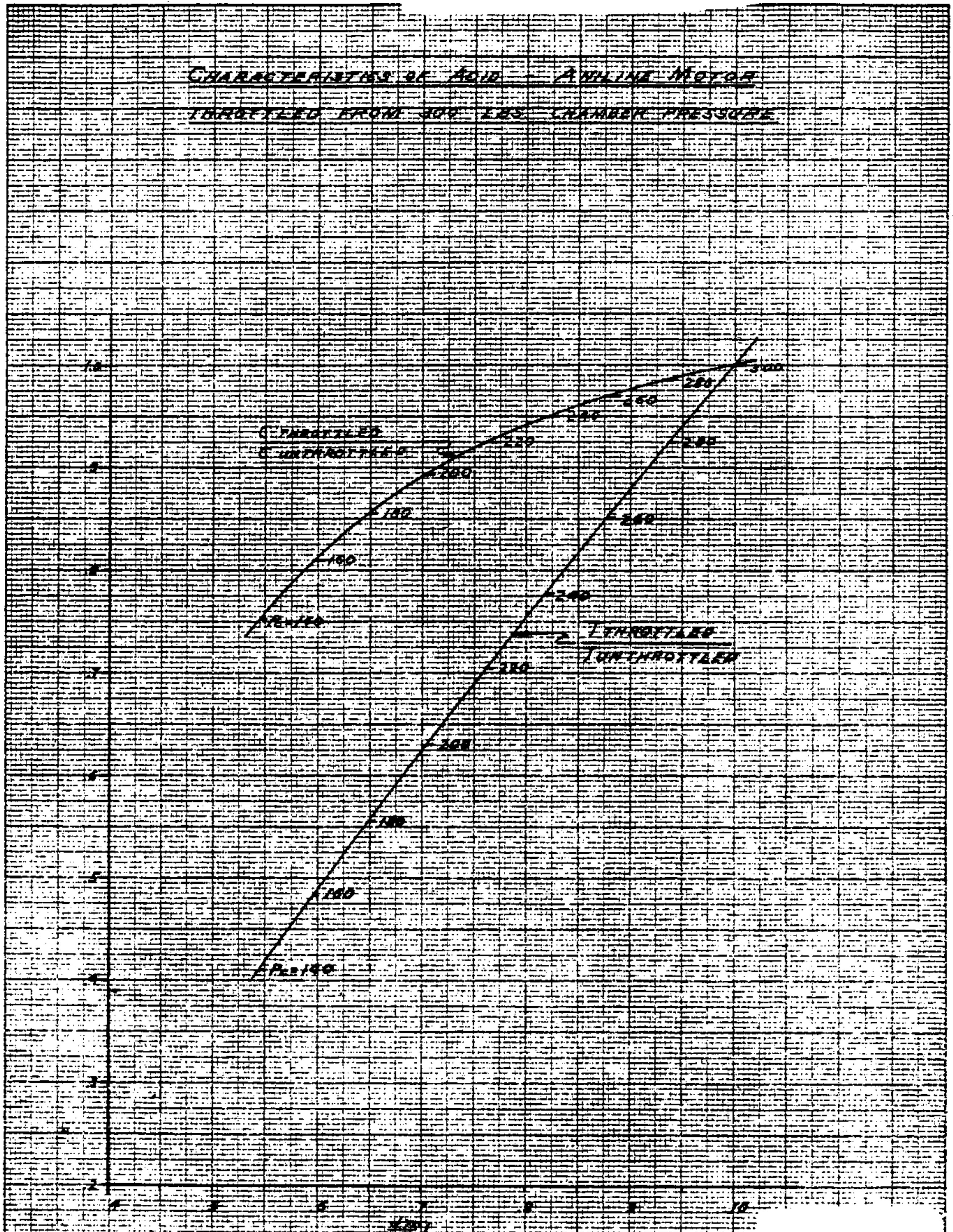


FIGURE 2
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LIQUID OXYGEN — ALCOHOL

VARIATION OF SPECIFIC IMPULSE WITH ALTITUDE FOR SEVERAL EXPANSION RATIOS

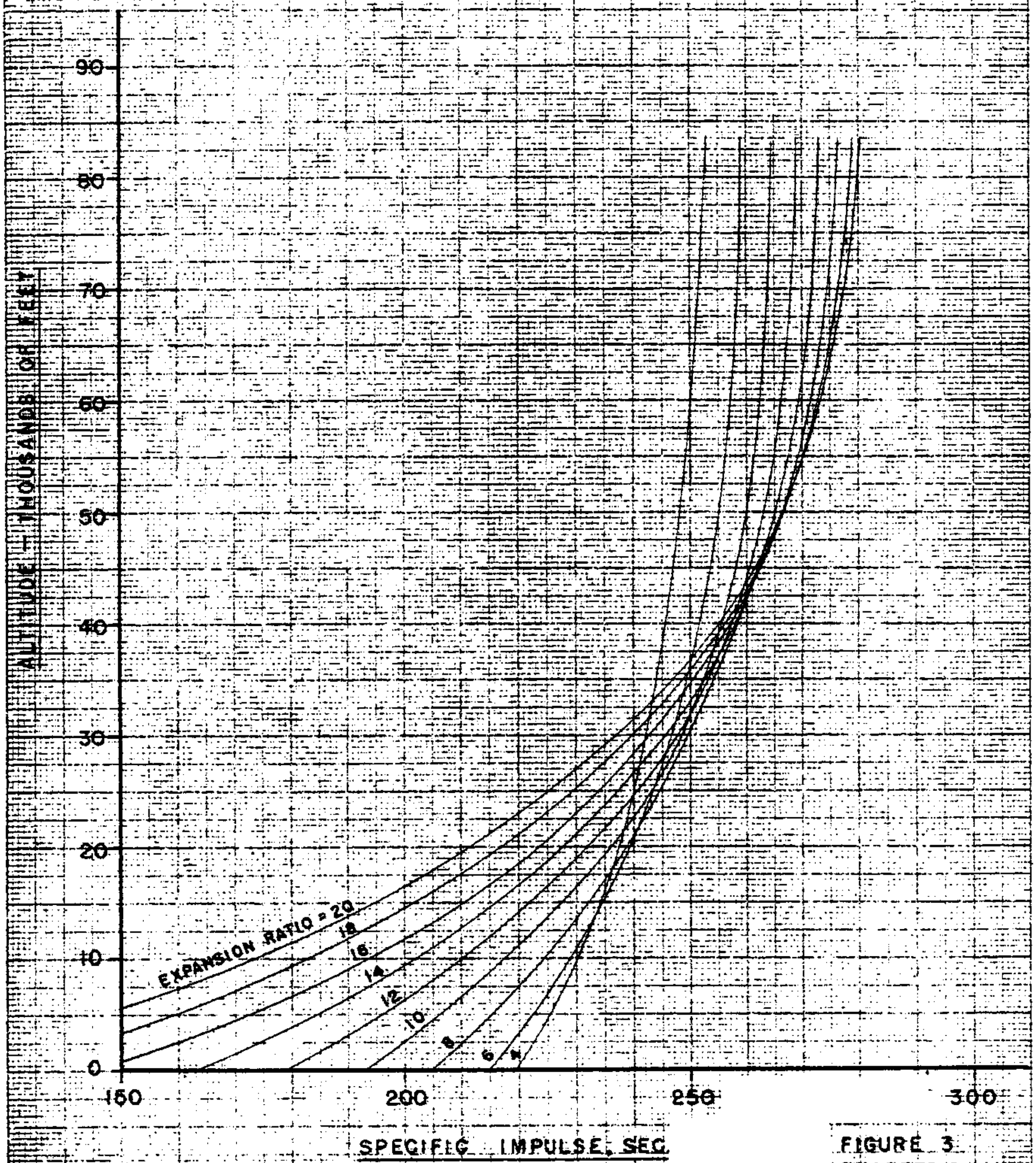

FIGURE 3
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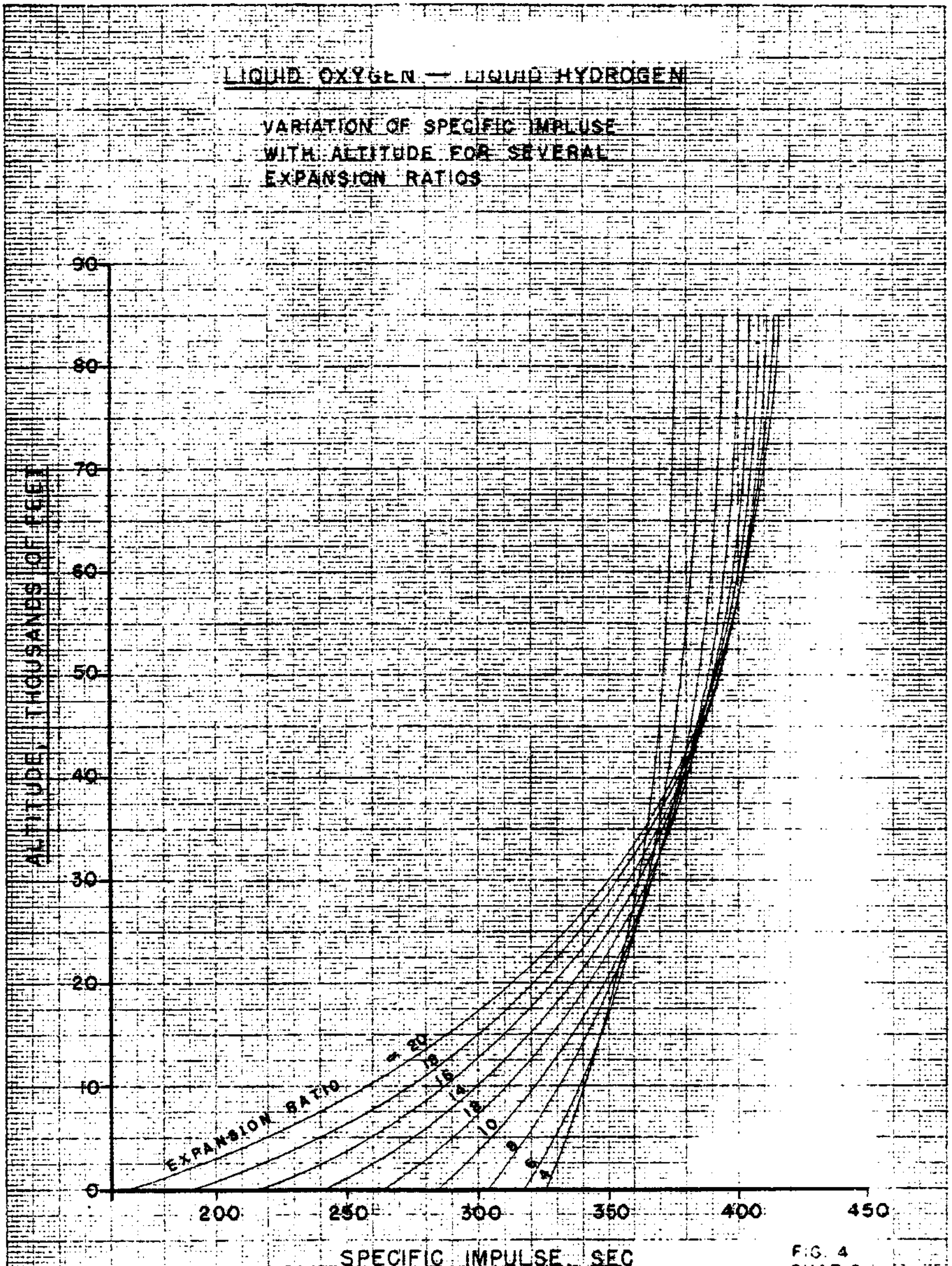
LIQUID OXYGEN — LIQUID HYDROGEN
VARIATION OF SPECIFIC IMPLUSE WITH ALTITUDE FOR SEVERAL EXPANSION RATIOS
ALTITUDE, THOUSANDS OF FEET
0
10
20
30
40
50
60
70
80
90
SPECIFIC IMPULSE, SEC
200
250
300
350
400
450
EXPANSION RATIO = 20
18
16
14
12
10
8
6
4

FIG. 4
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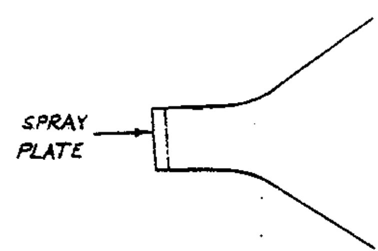
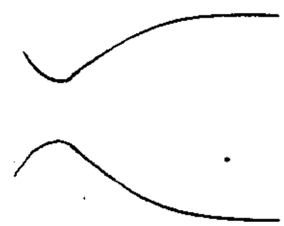
taken as 220 and 326 seconds, in agreement with our earlier values. It is clear from these curves that for the first stage of the satellite vehicle a smaller expansion ratio is required than for later stages. In the case of the alcohol-oxygen powered vehicle an expansion ratio of 6 was used for calculating the trajectory of the first stage; in the case of hydrogen-oxygen, 8. For the later stages in both cases the expansion ratios were arbitrarily limited to 20 for the preliminary calculations. As the design progressed, it became apparent that somewhat larger expansion ratios were both desirable and possible for these later stages. However the work necessary to change the calculations at this time was felt to be unwarranted.

The chief consideration governing the shape of the combustion chamber is the necessity for allowing sufficient time for mixing and burning of the propellants while still in the chamber. As improvements in mixing are made, the dimensions of the chamber tend to decrease since the time for combustion becomesless. This principle was carried to a high degree of development by the Germans, who by the end of the war, succeeded in reducing the chamber dimensions to such an extent that they were able to use the so-called throatless combustion chamber shown in the sketch. For the V-2, this combustion chamber was less than half as heavy as the one used on production models.



-pter 6

For most rocket motors, where the expansion ratio is not excessive, it is sufficient to use a straight conical expansion for the nozzle. However, when expansion ratios of the order of 20 are reached, a conical diffusor must be made undesirably long in order to avoid losses in nozzle efficiency caused by the large radial components of jet momentum. In order to avoid this it is advantageous to use a nozzle shaped as shown at the right. This type nozzle was used on the proposed design.



7. CONSIDERATION OF STRUCTURAL WEIGHT

On The Influence of Size on Structural Weight of Rockets. If two geometrically similar structures of different size are compared for strength the smaller is usually the relatively stronger. The laws governing this relationship are expressed by the doctrine of mechanical similitude, which considers the dimensional correlations imposed by the invariance of certain of the physical properties involved.

Assume for instance, that the geometrical similarity extends to all structural details, especially the degree of subdivision of structural members. If the loads are primarily derived from volume forces such as weights and inertia and if the two structures will be subjected to identical accelerations, assuming also that the structures are made of identical materials, then the following relationships obtain between dimensions M mass, L length, T time

M/L^3 = const for invariant material density

L/T^2 = const for invariant acceleration

The product of both implies

M/L^2T^2 = const, invariance of force per unit volume

Since stress is force per unit area it does not remain invariant but increases as M/LT^2 = const x L. Therefore where strength is governed by stress, as for instance in members carrying tensile or bending stresses the ratio of stress to given strength increases in linear proportion with size. To assure equal strength, the larger member will have to be made huskier, hence heavier. If wall thicknesses are increased, the structural weight per volume of vehicle would tend to go up approximately in direct proportion; actually this starts a vicious circle inasmuch as the increase in gross weight will encroach on performance.

Where structural members are endangered by limits of structural stability as in column compression, there the critical carrying capacity is also increased at the rate of the square only instead of the cube of size so that the disadvantage in strength as well increases linearly with size exactly as in the case of tensile stress members. However, where strengthening can be done by increasing column diameters, this would suffice at the rate of the five-fourths power of size instead of wall thickness increase at the rate of the square of size.

Loads originating from aerodynamic action which are suffered by surface impingement, increase only proportional to the square of linear size. They can therefore, as far as velocities are invariant as integrals of acceleration ("Invariance of time scale") be suffered without additional burden. However, they will not evoke equal transverse accelerations, hence less path curvature, in inverse proportion to linear size. Hence it follows that such inertia loads as are derived from lift (and neither from thrust nor gravity) can be carried without beefing up the structural members concerned beyond proportionality with size.

Where the load components due to gravity are negligible compared to the axial inertia loads there it becomes preferable to abandon the invariance of acceleration, retaining the invariance of corresponding velocities by adopting a time scale proportional to size; T = L. Now all inertia forces will vary just like area forces; the stresses and the strength of all structural members will be independent of size. Thrust will also have to increase with area, not with volume, hence the throat loading of a jet nozzle will be invariant (whereas it had to increase linearly with size under the assumption of equal acceleration). The thrust process will now take longer in proportion to the linear size and the range traveled under power will similarly be larger, but this may not be detrimental. It will reduce maneuverability because the same velocities will be attained at lesser air densities.

This analysis has a bearing on the choice of the best acceleration peak value for a given vehicle size as this choice must be governed by a compromise between those factors which derive an advantage from quick acceleration and those which favor keeping it slow. The structural weight of members carrying the inertia load belong into the latter group. In a vehicle of the V2 (A4) type they - tanks and fuselage - are estimated to make up about 5% of the gross weight. This weight component will have to be expected to go up in linear proportion to axial acceleration. The weight of the thermodynamic and mechanical machinery of the power plant which make up about 10% of the missile's gross weight, should essentially be linearly proportional to thrust, thus similarly to axial acceleration. Since tank loads diminish as fuel is burned at a constant rate, their strength is dictated essentially by the initial acceleration of their own stage or by the early surge of it occasioned by the gain of motor thrust efficiency with altitude. However, they must also be capable of withstanding their full fuel load at the peak accelerations of all preceding stages. All subsequent stages will therefore require relatively stronger structures than the first one, unless the motors are throttled during all of the powered flight stages except the last one. This means that a margin has to be applied in any attempt to extrapolate a multi-stage aggregate from a single stage prototype.

In terms of the whole vehicle it may be desirable to strike a compromise to balance the advantages and disadvantages. A slightly heavier structure is balanced by the relatively lighter power plant when the thrust or acceleration is decreased approximately at some ratio like $L^{-2/3}$. The following table gives a rough estimate of the weight change in % of the ptototype gross weight, assuming the "tanks and tank-like structures" made up 5% and the "power plant" 10% of the prototype gross weight.

Case	linear scale factor of geometrically similar enlargement:	1.4	2	2.8	4	
1	Retaining the prototype acceleration schedule, structural weight increases to: while power plant weight remains unchanged	2	5	9	15	%
2	Reducing the thrust loading inversely with enlargement, namely to: saves on power plant weight by: and incurs no increase of structural weight	70 -3	50 -5	35 $-6\frac{1}{2}$	25 $-7\frac{1}{2}$	% %
3	To offset structural weight increase by saving in power plant weight would require reduction of thrust loading to	80	64	50	40	%

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However, any drastic reduction of thrust loading can only be considered where the prototype acceleration is many times gravity in the first place. It would disastrously encroach on performance when weight alone exacts a large fraction of the thrust loading. Obviously, a reduction of the apparent acceleration of the V2 to 50% of its initial value of 2 g would leave it burning itself out sitting on the ground. Actually, considerations of the influence of acceleration changes cannot be separated from performance calculations; they will be treated in considerable detail in the next chapter.

On the other hand, any complete vehicle of a typical design will be composed of various components which may be divided into several groups whose weights vary essentially with some more or less established exponents n of size, or of other characteristic parameters and these components will make up certain fractions $\alpha, \beta,$ of the gross weight.

Assume for instance, that tank weight is proportional to the nth power of the fuel weight. (It was shown above that under certain assumptions $n = 4/3$; under others part of the tank and fuselage structures may have $n = 5/4$ or some value between 4/3 and 1; an average may well be less than 1.30). Assume that all other components weigh proportionally to gross weight. Let αW denote the fuel weight. Denote the quantities in a known prototype breakdown by index $_o$ so that the prototype fuel weight is $\alpha_o \beta_o$ and the prototype tank weight $(1-\alpha_o-\beta_o)\ W_o$ where $\beta_o W_o$ is the weight of everything that is neither fuel nor tank and assumed to weigh proportional to gross weight. Then in any article

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geometrically enlarged (except for beefing up where necessary) the fuel must be a lesser fraction of the gross weight, namely αW and the tank weight $(1-\alpha-\beta)W$. These quantities will then compare in the proportion $(\alpha W/\alpha_0 W_0)^n$

$$\frac{(1-\alpha-\beta)W}{(1-\alpha_0-\beta_0)W_0} = \left(\frac{\alpha W}{\alpha_0 W_0}\right)^n$$

hence

$$\frac{1-\alpha-\beta}{1-\alpha_0-\beta_0} \cdot \left(\frac{\alpha_0}{\alpha}\right) = \left(\frac{W}{W_0}\right)^{n-1}$$

The following table of values of gross over payload for exponents from 1.3 down to 1.1 will give an idea of the order of magnitude of the reduction of fuel capacity necessary and also of the sensitivity of the result to the choice of the assumption of n.

Values of gross weight/payload.

n =	1.10		1.15		1.20		1.25		1.30	
β =	.20	.25	.20	.25	.20	.25	.20	.25	.20	.25
α_0 = .70	1	1	1	1	1	1	1	1	1	1
α = .65	130	2310	26	179	12	50	7.3	23	5.4	14
.60	5580	322000	331	4950	81	613	35	176	21	80
.55			2860	65600	415	4350	130	850	61	292
.50					1830	23500	435	3340	168	920

Actually there will also be some parts of the missile which will not require enlargement or even duplication on mother stages, for instance the "brains". These could well be taken out of the structural weights class and lumped with the ultimate payload. It is estimated that about 1½% of the V2 may be in this category, which would bring a worthwhile improvement of the mother stages' mass ratio and can be pitted against the weight increases entailed by enlargement previously discussed. However, the advantage thus afforded eventually fades into insignificance when enlargement is carried to extremes. The fact therefore remains that unlimited geometrical enlargement of a rocket will eventually bring a penalty in weight. This is contrary to the contention advanced by some that structural efficiency will indefinitely increase with size.

The very fact that some parts of the prototype need not be enlarged as the prototype is enlarged works a hardship when an attempt is made to reduce the size. Indeed some parts cannot be reduced proportionally or not at all. They may have attained practical or otherwise determined minimum sizes. This is a very real problem in the manufacture of miniature models. For this reason it appears that the real weight per unit volume increases towards both the small and the large end of the scale. There is an optimum somewhere in the realm of "moderate" sizes. This optimum is presumably rather flat, its exact position will sensitively depend on minor variations of the components.

Thus far only geometrically similar "blow-up" with size has been considered. The disadvantages attending this method of enlargement arise from the fact that pressures due to volume forces go up with size. This applies to all hydrostatic pressures in tanks and the critically thin supports of mass loads. It would equally apply to power plant parts built to withstand pressures, notably the burning chamber if the latter were to handle a thrust proportional to the volume through a throat area proportional only to the cross sectional area of the vehicle. The latter is not feasible thermodynamically, there being no reason why higher pressures and temperatures should become easier to handle as the article is enlarged in size. Within a limited degree it may be possible to increase the nozzle diameters more than in geometrical proportion to the rest of the vehicle, but when it surpasses the caliber then geometrical similarity of the configuration is violated.

Both the hydrostatic pressure increase and the nozzle thrust loading increase are avoided if the vehicle were to be enlarged in cross section area only and not at all in height. It would then become fatter at the rate of the square root of the gross weight increase, but all weight proportions would remain essentially the same. It is as though a plurality of the prototype vehicles were arrayed, L^2 in parallel only and not also L in series. Actually this method of fattening cannot be carried to several mother stages as the grandmother would look like a mushroom. Aerodynamic drag considerations might weigh heavily against such malformation. The idea is nevertheless fruitful in that it points the way to a compromise: As the vehicle is enlarged, it may to advantage be fattened a little, thus reducing the hydrostatic and nozzle penalties without growing out of bounds in girth. If for instance the heights (lengths) are increased by the one-fifth power and the diameter by the two-fifth, instead of each by the 1/3, the hydrostatic penalties should be reduced to 2/3, yet the fineness ratio would drop only to 1/2 for every 32 fold increase in weight. However, any such violation of geometric similarity conjures up new problems of structural subdivision, large bulkheads, anti-sloshing devices and other structural exigencies whose weight penalty has to be carefully watched lest it encroach on the gain to be derived from the whole scheme.

The question may be posed: How will a change of fuel density ρ affect the tank weight? If the increased bulk is to be accommodated by geometrically similar enlarged tanks, then the linear tank dimensions increase as $\rho^{-1/3}$; the hydrostatic pressure (at any given acceleration) will actually decrease namely as $\rho^{-1/3} \cdot \rho = \rho^{2/3}$. Since for a given material strength the wall thickness has to vary with the product of pressure by radius, and the latter varies as $\rho^{-1/3}$, the wall thickness will also decrease as $\rho^{2/3} \cdot \rho^{-1/3} = \rho^{1/3}$. The tank weight is proportional to the surface and the wall thickness viz to $\rho^{-2/3} \cdot \rho^{-1/3}$. Hence it increases with linear dimension or volume $^{1/3}$. This is a strike against liquid hydrogen fuel which weighs only 7% of hydrazine or about 8½% of alcohol per unit volume. This disadvantage is aggravated by the flimsiness of the thinner walls of larger vessels.

If again the tanks are to be enlarged in width only and not in height, then the tank radius varies indirectly and the required wall thickness directly with the square root of the fuel density, so that the tank weight would remain unchanged. The progressive fattening of successive stages would rapidly grow prohibitive. On the other hand fewer stages are required to accomplish the same performance with the higher exhaust velocity of the lighter fuel and vice versa so that the overall picture may not be radically affected.

It is noteworthy that the optimum proportion of a cylindrical tank from a viewpoint of minimum wall weight to volumetric content is more squat for hydrostatic pressure than for uniform (gas) pressure. As is well known, the flat headed cylinder of least surface per volume is as high as its diameter, $(h = 2r)$.

In order to make the tank heads stand up under any uniform internal pressure, they should be bulged. Hence they would have a surface $K_s r^2 \pi$ each and a calotte volume of $K_v r^3 \pi$. For equal wall thickness the bulge radius would be twice the cylinder radius and the coefficients $K_s = 1.072$ and $K_v = .274$. The lightest shape (neglecting seams) would be attained with a cylinder height of $h = (2K_s - 3K_v)r$, here $= 1.32r$ and the total height including caps $H \doteq 1.86r$, somewhat shallower than the flat headed cylinder.

On the other hand, if the tank is to stand hydrostatic pressure which increases linearly with height and if the walls could be suitably tapered, the weight of the top would be negligible but the bottom would have to be bulged slightly more than to the double cylinder radius if it was to be made of the same thickness as the lowest part of the cylinder wall. If we also neglect this bulge for the sake of a first rough approximation, then the weight of the tank will be indicated by

$$W = (h + r) r^2 \pi \ wnfh/s$$

where w is the specific weight of the tank material, s its allowable stress;

n the load factor and f the specific weight of the fuel. Defining $H = V/r^2\pi$ by the volume V and the radius r,

$$W = (V/r^2\pi + r)\ Vwnf/s$$

$$dW/dr = (-2V/r^3\pi + 1)\ Vwnf/\pi s$$

Equating this to zero yields

$$V = \tfrac{1}{2} r^3\pi = r^2 \pi h$$

$$h = \tfrac{1}{2} r$$

which is four times as squat as the square cylinder of uniform wall thickness.

If the bottom is bulged to a radius equal to the cylinder diameter and made thicker in the sump according to the hydrostatic pressure increase, then the lightest proportion turns out to be $h = .325\,r$ and $H = .593\,r$ which is rather squat. However, the influence of a variation from the optimum is not large and other considerations such as manufacture, bracing and lid, safeguards against sloshing, etc., militate against extremely shallow vessels. The lid cannot be made weightless, the walls cannot be ideally tapered, seams and anti-sloshing means have to be provided. Hence practical vessels will probably be of proportions H/r ranging somewhere around $1 \pm \frac{1}{2}$.

The preceding discussion of scale effects is useful for giving an overall view of the rocket design problem. However, in order to make weight estimates for preliminary designs a greater amount of detail is necessary. An attempt is made in the following pages to consider various parts of the vehicle weight separately, applying a separate scale factor to each part. Up to the present time, the best (in fact, the only) long range rocket is the V2. For this reason the V2 is used as a basis or standard for calculation. Some features of the present multi-stage designs do not appear in V2, and separate weight allowances must be made in such cases. Since no past experience or present design practice exists for staged rockets, various reasonable appearing assumptions must be made.

It is to be expected that the art of estimating weights for long range, low acceleration rockets will progress rapidly as designs reach the layout stage on the drawing board.

Chapter 7

Weight Estimation for Rocket Design Study. As a starting point for this study, use is made of a breakdown of weights of V-2 as given by Gilliland*. Percentages corresponding to those weights are listed in column (1) of Table I below. Since they correspond to a mass ratio $\left(\frac{\text{Gross Wt.}}{\text{Gross Wt.-Fuel}}\right)$ of only 3.25, a new set of percentages has been assigned in column (2) to raise the mass ratio to 4, a ratio which was achieved for V-2 at the end of the war.

Based on a gross weight of 27,305# as given by Gilliland, the percentages of column (2) yield weights for the various items as given in column (3). The "Radio and Instruments" and "Warhead" of the above reference are lumped under "Payload" below.

Table I

V-2 Major Weight Breakdown

		(1) Mass Ratio 3.25	(2) Mass Ratio 4	(3)	
1.	Tanks and Piping	5.4%	4.5%	1230	lbs.
2.	Pumping Unit	5.7	4.7	1280	lbs.
3.	Nozzle and Combustion Chamber	3.8	2.5	680	lbs.
4.	Controls and Surfaces	4.9	4.1	1120	lbs.
5.	Fuels	69.1	75.0	20,480	lbs.
6.	"Payload"	11.1	9.2	2510	lbs.
	Total	100.0	100.0	27,305	lbs.

Although the figures of Table I above are admittedly inaccurate, they represent the best information available to this writer at the present time.

*Gilliland, E. R., Rocket-Powered Missiles, Jet Propelled Missiles Panel, May 1945, page 45.

Items of Table I are further broken down in Table II, below, still following **Gilliland's outline**, but using revised figures.

Table II

V-2 "Detail" Breakdown

Item	Subdivision	Weight lbs.	Sub-totals lbs.
1. Tanks & Piping			1230
	a. Integral Tanks	1160	
	b. Distributing Pipes & valves	70	
2. Pumping Unit			1280
	a. Power unit & tanks	730	
	b. Mounting & end frame	210	
	c. Shell structure	340	
3. Nozzle & Combustion Chamber		680	680
4. Controls & Surfaces			1120
	a. Fins	630	
	b. Internal controls	390	
	c. External controls	100	
5. Fuels			20,480
	a. Oxygen	11,500	
	b. Alcohol	8,980	
6. "Payload"			2510
	a. Radio compartment & frames	270	
	b. Radio Equipment	130	
	c. Instruments, wiring	260	
	d. Compressed air bottles	70	
	e. Warhead	1780	

Chapter 7

It is convenient, in the analysis which follows, to regroup the items and subdivisions of Table II, so that quantities which vary alike with scale of the vehicle can be lumped. Such a regrouping is given in Table III, with sub-totals that apply to V-2.

Table III

Weights Regrouped for Analysis

Group	Contains subdivisions (From Table II)	Weight
T. Tanks & Structures	1a, 2b, 2c, 6a	1980
M. Miscellaneous Structure	not in V-2	- -
N. Nozzle, chamber, pumps	1b, 2a, 3	1480
H. Provisions for H_2	not in V-2	- -
C. Controls	4a, 4b, 4c	1120
B. "Brains"	not in V-2	- -
F. Fuels	5a, 5b	20,480
P. Payload	6b, 6c, 6d, 6e	2240
Total		27,300

Groups as listed in Table III are separately discussed and analyzed below. A list of notation is given here to facilitate such discussion.

a = design acceleration for structure (no. of "g's")

f_t = applied tensile stress, #/in^2

F_t = allowable tensile stress, #/in^2

k_n = constant, may or may not be dimensionless

L = scale dimension of length

ℓ = length of fuel tank, inches

(o) = subscript, referring to V-2 as a basis for calculations

ρ = fuel density, #/in^3

p = fuel pressure #/in^3

r = radius of fuel tank, inches

T, M, N, etc. = group weights as captioned in Table III

t = wall thickness of tank or shell, inches

W = Gross weight of a stage, considering the sum of all succeeding (smaller) stages as payload

Stage no.: Taken in order of firing, i.e. #1 is the largest, #4 the smallest

τ = duration of burning for a stage, seconds

Group T. Tanks and Structure

Since a high percentage of the gross weight (60 - 70% is in fuel, it is to be expected that fuel tank weights will have a major bearing on the overall structural weight. For this reason, the structural items listed by Gilliland for the V-2 projectile are all lumped and assumed to vary as the fuel tank weight.

For our purposes, the fuel tanks are assumed to be integral with the structure, rather than separate, although this point has by no means been finally settled.

Consider the fuel tanks in a manner similar to the discussion (p. 58 to 69 of this report) on the influence of size on the structural weight of rockets.

$$p = k_1 \rho a l$$

$$f_t = F_t = \frac{pr}{t} = \text{constant for a given wall material}$$

$$\therefore t = k_2\, pr = k_3 \rho a \ell r$$

(a) Side walls of tank

$$\text{Area} = 2\pi \ell r$$

$\therefore T_a$ = wt of side walls

$= t \times \text{area} = 2 \;\; k_3 \, \rho a \, l^2 r^2$

(b) Tank bottom

$\text{Area} = \pi r^2$

$\therefore T_b = \pi k_3 \, \rho a \, \ell r^3$

Total $\quad T = T_a + T_b$

$= \pi k_3 \, \rho a \, (2\ell^2 r^2 + \ell r^3)$

F (= fuel weight) $= \rho \pi r^2 l$

$$\therefore \frac{T}{F} = k_3 \, a(2\ell + r) \quad = k_4 a(\ell + \frac{r}{2})$$

From V-2:

$T_o = 1980, \quad F_o = 20{,}480$

$\ell = 243, \quad r = 31.5 \qquad a = 2.0$

$\therefore k_4 = .000187$

$$\therefore \frac{T}{F} = .000187 \quad a(\ell + \frac{r}{2}) \qquad (1)$$

Using approximate dimensions for design studies of (a) a four stage alcohol, oxygen rocket and (b) a two stage hydrogen, oxygen vehicle, group T weights have been calculated and are given in Table IV below. In applying a value for a, it should be noted that the design acceleration for the first stage is at the start of that stage (minimum acceleration, full tanks) whereas for succeeding stages the design acceleration occurs at the end of the first stage burning (maximum acceleration, full tanks).

For stage 4 of the alcohol, oxygen system, it is considered that the extrapolation from V-2 is too great for a simple scale effect formula. Therefore an independent estimate based on a reasonable minimum shell thickness is shown for this case in Table IV.

Table IV

Group T (tank and structure weights (pounds)

Stage	4	3	2	1
Alcohol & Oxygen	120	590	4350	14,300
Hydrogen & Oxygen	- -	- -	2250	36,000

Group M. Miscellaneous Structure

This weight group is provided to allow for structural items that do not appear in V-2. There are two major components considered. First, to allow for coupling of stages, an amount of 3% W is allowed for each stage. Second, to allow for minimum gauges and general miscellaneous, weight is assumed which is 4% W for W = 1000 pounds and zero for W = 27,000 pounds and above.

Group M weights as described above are shown in figure 1, an arbitrary curve.

Group N. Nozzle, Chamber, Pumps

Weights placed within this group are those which depend upon the rate of fuel flow for their size. It has been found in past designs that the complete power plant varies nearly directly with the mass flow rate of fuel.

For V-2, 20,480 pounds of fuel are burned in 60 seconds and N is 1480 pounds.

$$\therefore N = 1480 = \frac{60}{20,480} \cdot \left(\frac{F}{\tau}\right)$$

$$= 4.33 \frac{F}{\tau} \qquad (2)$$

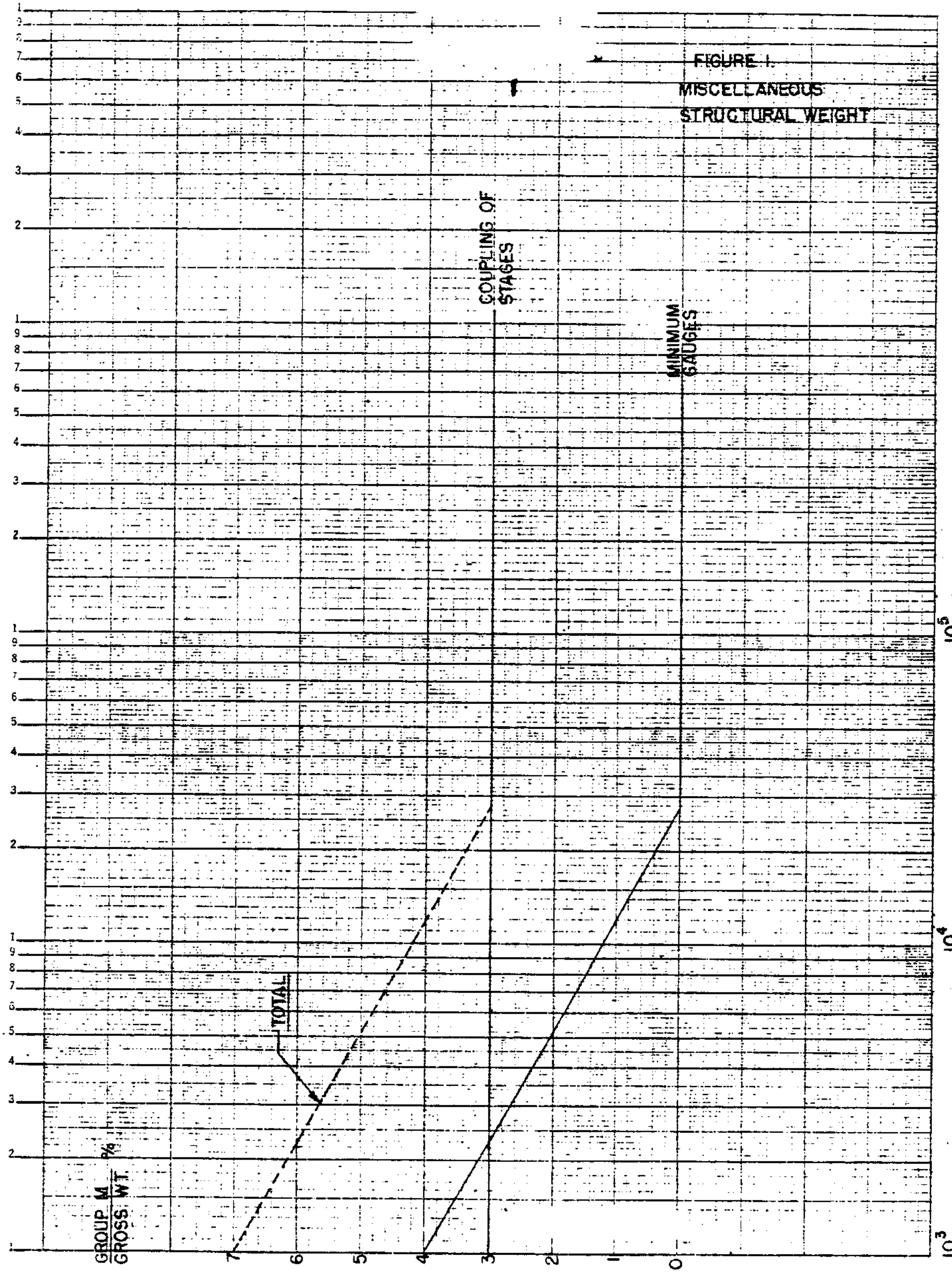
FIGURE I.
MISCELLANEOUS
STRUCTURAL WEIGHT
COUPLING OF STAGES
MINIMUM GAUGES
TOTAL
GROUP M / GROSS WT. %
7
6
5
4
3
2
1
0
10^3
10^4
10^5

Group C. Controls

Percentage of gross weight taken up by electrical, mechanical and structural controls and surfaces has been found to vary roughly with the square root of a linear scale dimension, based upon past experience with aircraft design.

Thus $\frac{C}{W} = k_5 L^{\frac{1}{2}}$

But L varies as $W^{1/3}$

$$C = k_6 W^{7/6}$$

To avoid handling large numbers, write this in the form

$$C = C_o \left(\frac{W}{W_o}\right)^{7/6} = 1120 \left(\frac{W}{27,300}\right)^{7/6} \qquad (3)$$

Group H. Provisions for H_2

For the hydrogen burning rocket only, it is believed that special provisions will be necessary to (a) maintain the liquid state inside the hydrogen tank (b) prevent escape of the liquid and (c) prevent explosions due to the wide explosive mixture range. Although no logical basis now exists for calculating the weight of such provisions, a reasonable amount may be 1% of the gross weight of each stage.

Group B. "Brains"

By "Brains" are meant the central guiding units which furnish commands to the control system in order to guide the vehicle on its trajectory. A weight of 200 pounds is arbitrarily allowed for such equipment. This item is applied only to the last stage of each rocket system, since a single set of "Brains", with proper control system connections, should serve all stages.

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Group F. Fuels

The total fuel weight for each stage is determined from a percentage of gross weight, which in turn is derived from trajectory calculations for the particular fuels and number of stages employed. Trajectory calculations are set forth elsewhere in this report. It will suffice to say here that for alcohol-oxygen 4-stage systems the fuel weight is taken as 60% of the gross weight for each stage whereas for hydrogen-oxygen 2-stage systems the fuel is 71% of the gross weight.

Group P. Payload

For the final stage, the payload has been set arbitrarily at 500 pounds. For other stages, the payload of each stage is the gross weight of the succeeding stages. Since the gross weight of stage 2 (say) includes the weights of stages 3 and 4 it can be said that the payload of stage 1 is simply the gross weight of stage 2, and so on for the successive stages.

Gross weight is the sum of groups T, M, N, C, H, B, F and P. Using the formulas and assumptions described above it is possible to tabulate the weights for a 4-stage alcohol-oxygen rocket and a 2-stage hydrogen-oxygen rocket. These are given in tables V and VI, respectively. Since the solution for gross weight, fuel and structure in terms of each other is a trial anderror process, these figures are not completely accurate or consistent, however, they are close enough for preliminary design purposes.

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Table V Weight Summary, Weight in lbs.

4-stage Alcohol-Oxygen Rocket

Group	Stage	4	3	2	1
T.	Tanks & Structure	120	590	4350	14,300
M.	Miscell. Structure	160	473	1610	7,000
N.	Nozzle, chamber, pumps	91	376	1550	7,780
C.	Controls & Surfaces	77	422	2150	11,100
H.	Provisions for H_2	None			
B.	Brains	200	- -	- -	- -
F.	Fuels	1720	7100	32,200	140,000
P.	Payload	500	2868	11,829	53,689
	Gross	2868	11,829	53,689	233,669
	S = T + M + N + C + H	448	1861	9660	39,980
	S/W	.156	.157	.18	.168

Table VI Weight Summary, Weight in lbs.

2-stage Hydrogen-Oxygen Rocket

Group	Stage	2	1
T.	Tanks & Structure	2,250	36,000
M.	Miscell. Structure	555	8,740
N.	Nozzle, chamber, pumps	250	5,620
C.	Controls & Surfaces	559	16,090
H.	Provisions for H_2	150	2,750
B.	Brains	200	- -
F.	Fuels	10,900	207,000
P.	Payload	500	15,364
	Gross	15,364	291,564
	S = T + M + N + C + H	3764	69,200
	S/W	.246	.238

Chapter 8

8. INVESTIGATION OF DESIGN PROPORTIONS

This chapter has a double purpose. 1). The first is to continue and develop the study of the dynamics of orbital vehicles which was initiated in Chapter 5. Use will be made of the results gained in Chapters 6 and 7 concerning power plants and structural weights. 2). The second purpose is to apply the general theory thus developed to the design of two actual vehicles. In this chapter we shall be concerned only with the basic features such as number of stages, weight of stages and maximum thrusts to be used. In the following chapter, these values will be combined with the results of trajectory calculations to give a final integrated design.

General Dynamics of Orbital Vehicle. Single Stage Vehicle.- We shall improve on the analysis of Chapter 5 by taking into consideration the practical details which were left out in that chapter; namely gravity, inclination, dependency of structural weight on load factor, drag and throttling.

Gravity - First let us consider a vertical trajectory. It will be necessary to add a term, -g (the acceleration of gravity) to the right right hand side of the equation of motion, presented in Chapter 5. We obtain

$$(1) \quad \frac{dV}{dt} = -\frac{c}{m}\frac{dm}{dt} - g$$

which integrates to

$$(2) \quad V_F = c \ln \frac{m_i}{m_f} - gt_B + V_o$$

Where t_B is the burning time and subscripts "F" and "o" denote "final" and "initial" respectively.

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For the purposes of our later work we shall eliminate t_B in terms of the maximum load factor n defined by

(3) $ng = \left(\frac{dV}{dt}\right)_{max} + g$

The missile reaches its maximal acceleration (i.e. thrust - mass ratio) at the end of burning time (we assume constant thrust). Hence it follows that

$$ng = \frac{c}{S+P} \times \frac{F}{t_B} \text{ or}$$

(4) $gt_B = \frac{Fc}{(S+P)n}$.

Thus (3) may be written as

(5) $\frac{\Delta V}{c} = \ln \frac{W}{S+P} - \frac{F}{n(S+P)}$

(Note that $\frac{W}{S+P} = \frac{m_i}{m_f}$).

The following numerical example shows the importance of n. Let $\frac{F}{W} = .6$ and $n = 6.5$ (we shall later see that these are reasonable figures). Then $\ln \frac{W}{S+P} = \ln \frac{1}{1-.6} = .916$ and $\frac{F}{n(S+P)} = \frac{.6}{6.5\,(1-.6)} = .231 = 25\%$ of .916. Thus if the exhaust velocity c is 8,500 ft/sec, the velocity increase during one stage is 7,800 ft/sec neglecting gravity, but if gravity is considered, the velocity increase is only 75% of this or 5850 ft/sec.

<u>Inclination</u> - In most practical cases the trajectory will have variable inclination. In this case the formula for acceleration along the path of flight is

(6) $\frac{dV}{dt} = -\frac{c}{m}\frac{dm}{dt} - g \sin \theta$

There θ is the angle the trajectory makes with the horizontal. This equation is not readily integrable unless θ is considered constant.

We shall make this assumption, because in spite of its inaccuracy, it will furnish us valuable information on how θ affects our choice of n. θ will be referred to as the average inclination. Instead of (5) we then obtain

$$(7) \quad \frac{\Delta V}{c} - \ln \frac{W}{S+P} - \frac{F \sin \theta}{n(S+P)}$$

Clearly, the more horizontal the flight path, the less is the loss in performance caused by finite acceleration.

Dependency of Structural Weights on Load Factors - The example above shows that a low acceleration like 6.5g has a detrimental effect on performance. On the other hand, it was shown in Chapter 7, that a higher load factor necessitates a heavier structure and the resulting lower value of the mass-ratio parameter gives a lower value of $\ln \frac{mi}{mf}$. To study how these factors balance each other, let us consider a missile whose total initial weight W is fixed and whose weight empty may be expressed in the form

(8) $S+P = Q+R.n$ where Q is that portion of the weight which is unaffected by the maximum acceleration and $R \cdot n$ is the weight of the remaining structure which is assumed to be directly proportional to n. Actually, Q represents essentially the weight of the payload and the controls, whereas $R \cdot n$ is the weight of fuel tanks, power plants and accessories, etc. Then (7) reads

$$(9) \quad \frac{\Delta V}{c} = \ln \frac{W}{Q + Rn} - \sin \theta \times \frac{W - (Q + Rn)}{n(Q + Rn)}$$

In figure 1 we see how $\frac{\Delta V}{c}$ is affected by n in a vertical trajectory where we have used the values $Q/W = .275$ and $R/W = .019$. These figures correspond to a payload which is 20% of gross weight, a structure independent of acceleration of 7.5% (.20 + .075 = .275) and a remaining structure which weighs 1.9% of gross weight for each gross acceleration.

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It can be seen from the figure that the advantages of high acceleration to reduce the loss from the effect of gravity are counteracted by the poorer structural weight in such a manner that an optimum acceleration exists. For this particular example, the optimum acceleration is 7.g. This could also have been obtained by the aid of calculus. Namely, if $\frac{\partial \Delta V}{\partial n}$ is equated to zero, one obtains the following cubic equation for n

$$(10)\quad \left(\frac{R}{W}\right)^2 n^3 + \frac{R}{W}\left(\frac{Q}{W} + a\frac{R}{W}\right)n^2 + 2a\frac{R}{W}\left(\frac{Q}{W} - 1\right)n + a\frac{Q}{W}\left(\frac{Q}{W} - 1\right) = 0, \text{ where } a = \sin\theta.$$

In Figure 2 the solution of this equation has been plotted against R/W with parameters of $\frac{Q}{W}$ and θ. This chart is very useful for a rapid determination of the approximate value of the optimal acceleration. Two corrections will have to be added for a more refined analysis: 1). A correction based on a more exact weight formula. 2). A correction for drag.

From the discussion in Chapter 7 it is evident that the expression of the weight empty as a linear function of n is over-simplified. However, if we use a more accurate formula for the weight variation, we are forced to abandon general analytical methods and shall have to reduce ourselves to a numerical study of a concrete example. In choosing this example we anticipate some results to be established later. The oxygen-alcohol missile to be proposed in this report will be a four-stage missile. Its first (largest) stage will have a gross weight W_1 of 233,669 lbs., its payload (W_2) will be 11,829. We select this stage as one example. The two weights mentioned will be kept

FINAL VELOCITY vs LOAD FACTOR

EQUATION: $$\frac{V}{C} = -\ln\left\{\frac{P+S}{W} - \frac{1-\frac{P+S}{W}}{n\left(\frac{P+S}{W}\right)}\right\}$$

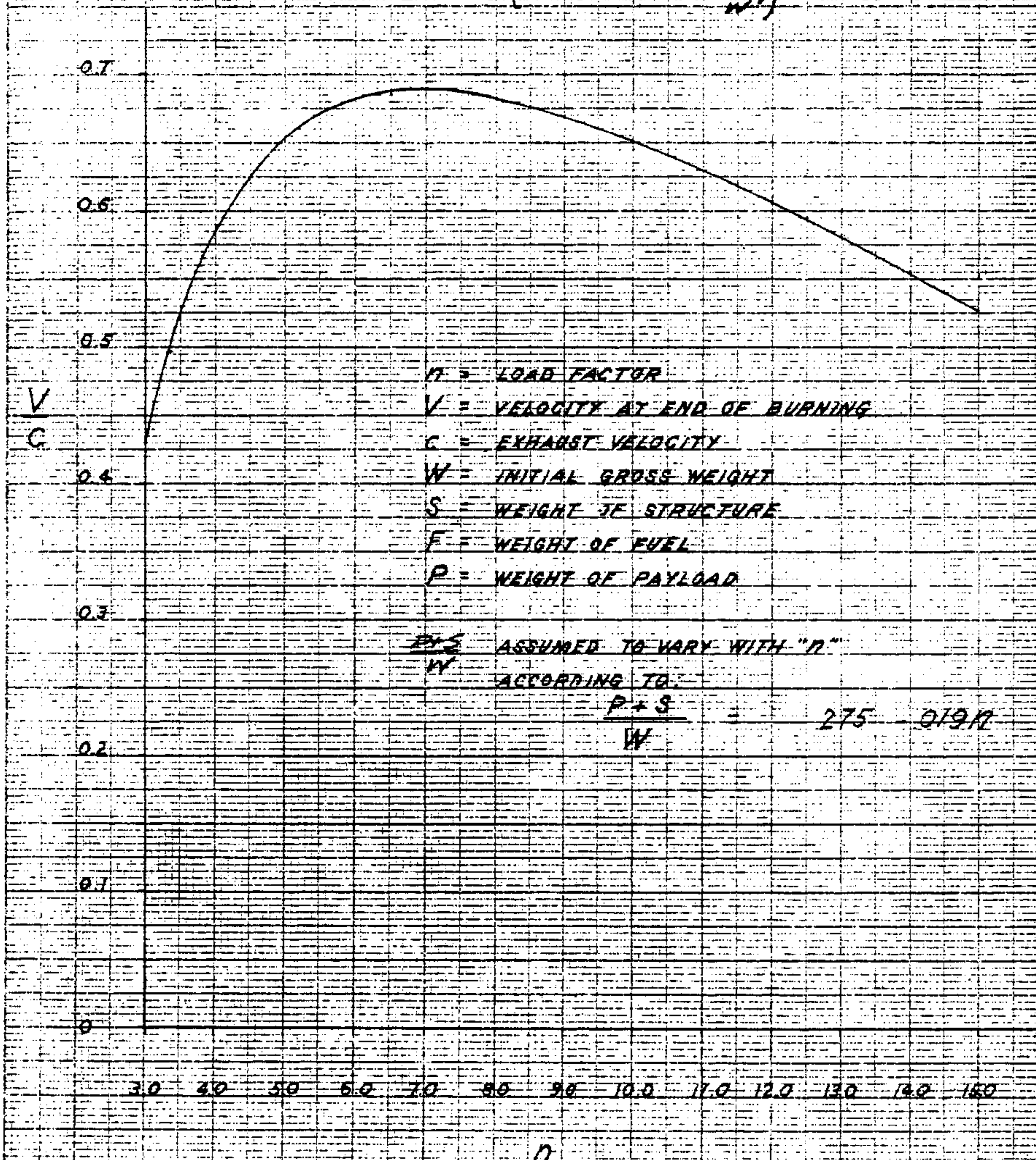


FIGURE 1
CHAPTER 8

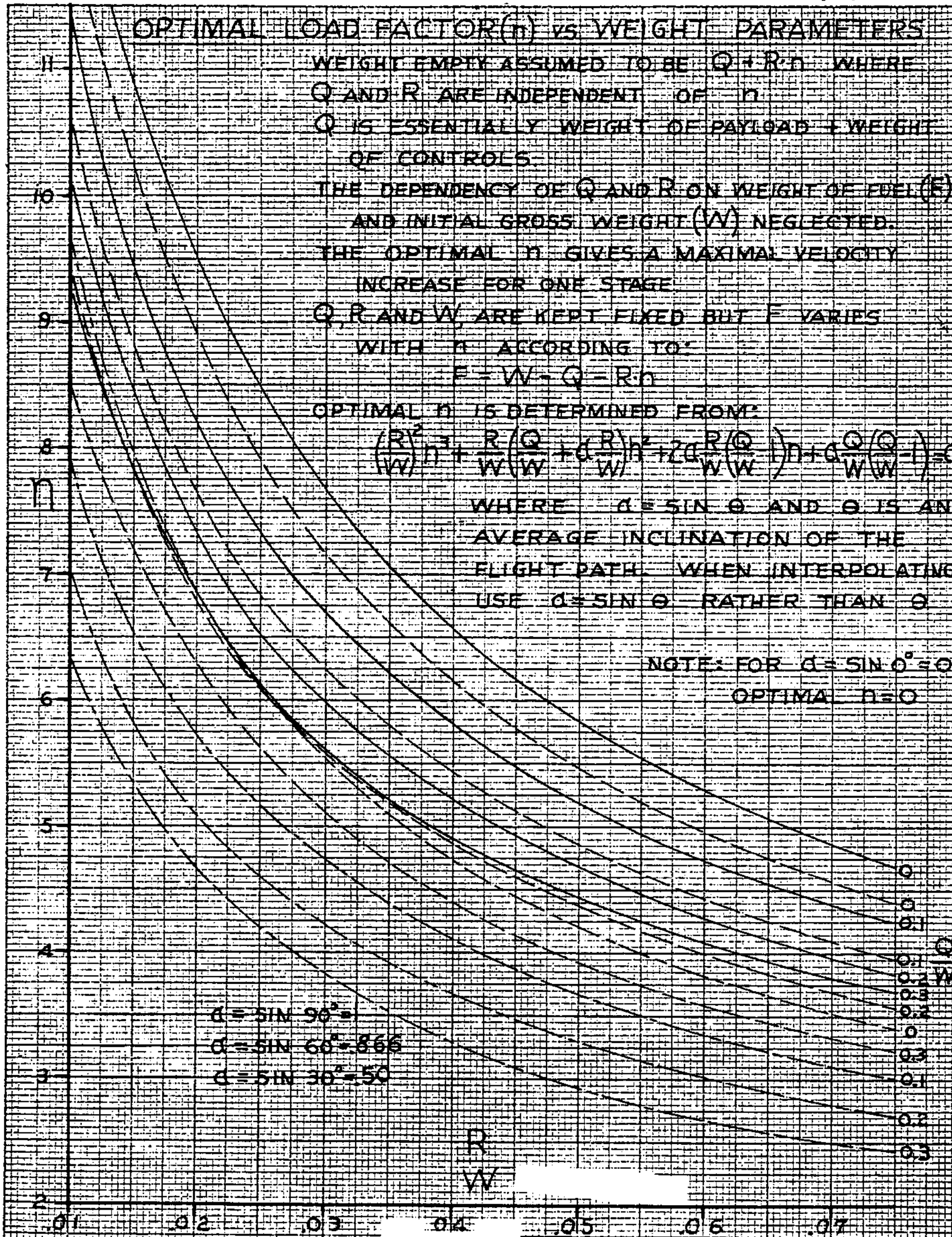


FIGURE 2 CHAPTER 8

constant, but the load factor will be varied from its design value 6.5.

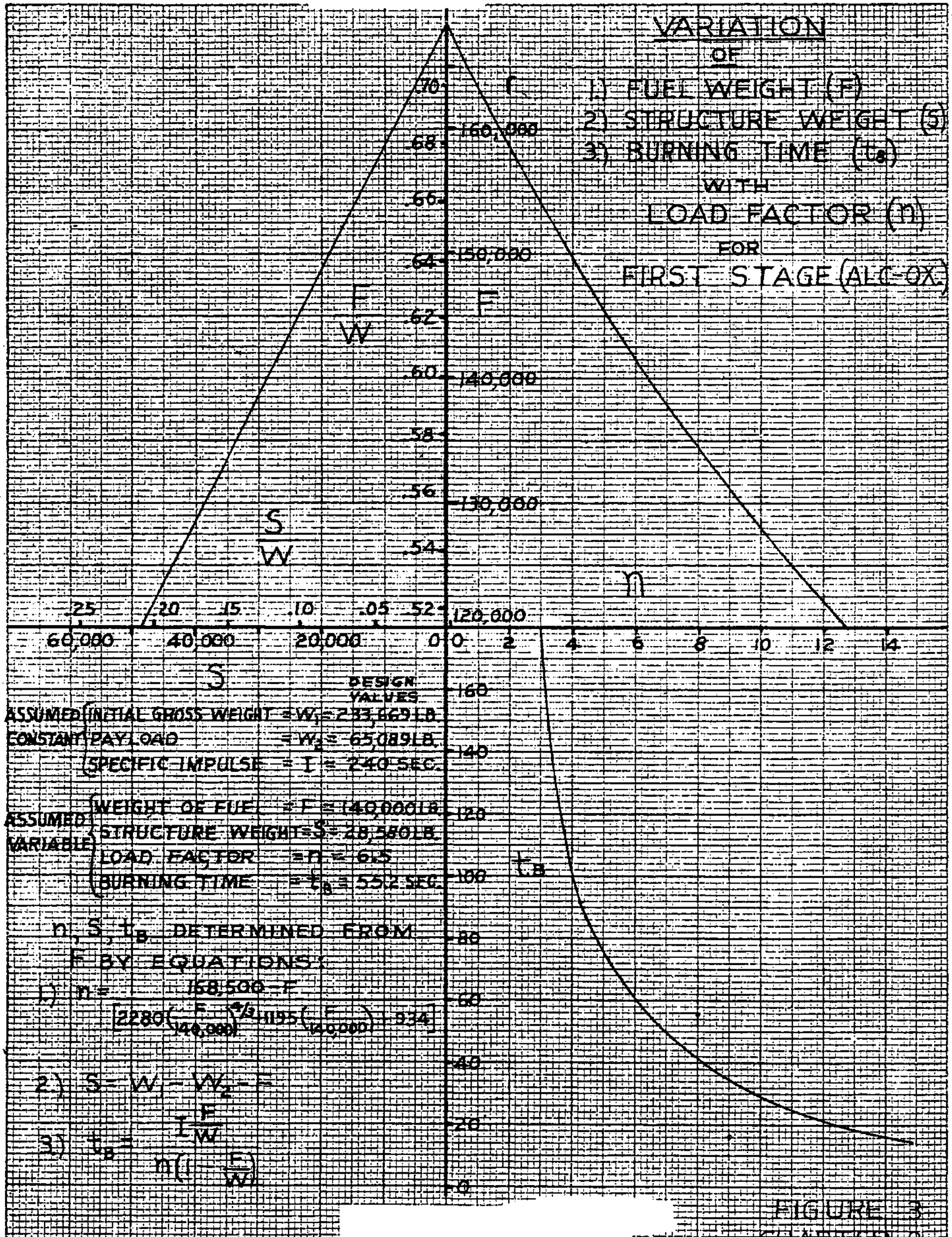
The analysis given in Chapter 7 shows that the weight of the structure S_1 will vary with the load factor n according to the equation

$$(11)\quad S_1 = 14,300\left(\frac{F_1}{140,000}\right)^{4/3}\frac{n}{6.5} + 7,780 \times \frac{55.2}{t_B}\left(\frac{F_1}{140,000}\right) + 11,400\left(\frac{W_1}{233,669}\right)^{7/6} + \frac{3.0}{100}\,(233,669)\,\frac{n}{6.5}$$

The terms on the right hand side represent in order the weight of tanks, power plant, controls and various miscellaneous weights. The design values of these weights are 14,800 lbs., 7,780 lbs, 11,400 lbs and 3.0% of W_1 respectively. 55.2 and 6.5 are the design values of t_B and n. If we select a value of I of 240* sec and put W_1 and W_2 equal to their design values, then every term on the right hand side is a function of n only. The other variables may be eliminated with the aid of the equations $W_1 = S_1+F_1+W_2$ and $t_B = \dfrac{I \times F_1}{n(W_1-F_1)}$. The result of this rather cumbersome numerical calculation is given in Figure 3. Once F_1, S_1 and t_B are known as functions of n, the final velocity V_F may be computed from (2) (with c = 32.2 x 240 = 7,750 fps). The result has been plotted as curve 1 in Figure 4 (the other graphs in Figure 4 will be discussed shortly). The optimal value of n is seen to be 7.5; V_F does not fall more than one percent below its maximum if n is kept between 6.5 and 8.75.

*This represents an average value during first stage.

VARIATION OF 1) FUEL WEIGHT (F) 2) STRUCTURE WEIGHT (S) 3) BURNING TIME (t_B) WITH LOAD FACTOR (n) FOR FIRST STAGE (ALC-OX)



	DESIGN VALUES
ASSUMED CONSTANT: INITIAL GROSS WEIGHT	$= W_1 =$ 233,669 LB.
PAYLOAD	$= W_2 =$ 65,089 LB.
SPECIFIC IMPULSE	$= I =$ 240 SEC.
ASSUMED VARIABLE: WEIGHT OF FUEL	$= F =$ 140,000 LB.
STRUCTURE WEIGHT	$= S =$ 28,580 LB.
LOAD FACTOR	$= n =$ 6.5
BURNING TIME	$= t_B =$ 55.2 SEC.

n, S, t_B DETERMINED FROM F BY EQUATIONS:

1) $n = \dfrac{168{,}500 - F}{\left[2280\left(\frac{F}{140{,}000}\right)^{4/3} + 1195\left(\frac{F}{140{,}000}\right) + 934\right]}$

2) $S = W_1 - W_2 - F$

3) $t_B = \dfrac{I\frac{F}{W}}{n\left(1 - \frac{F}{W}\right)}$

FIGURE 3
CHAPTER 8

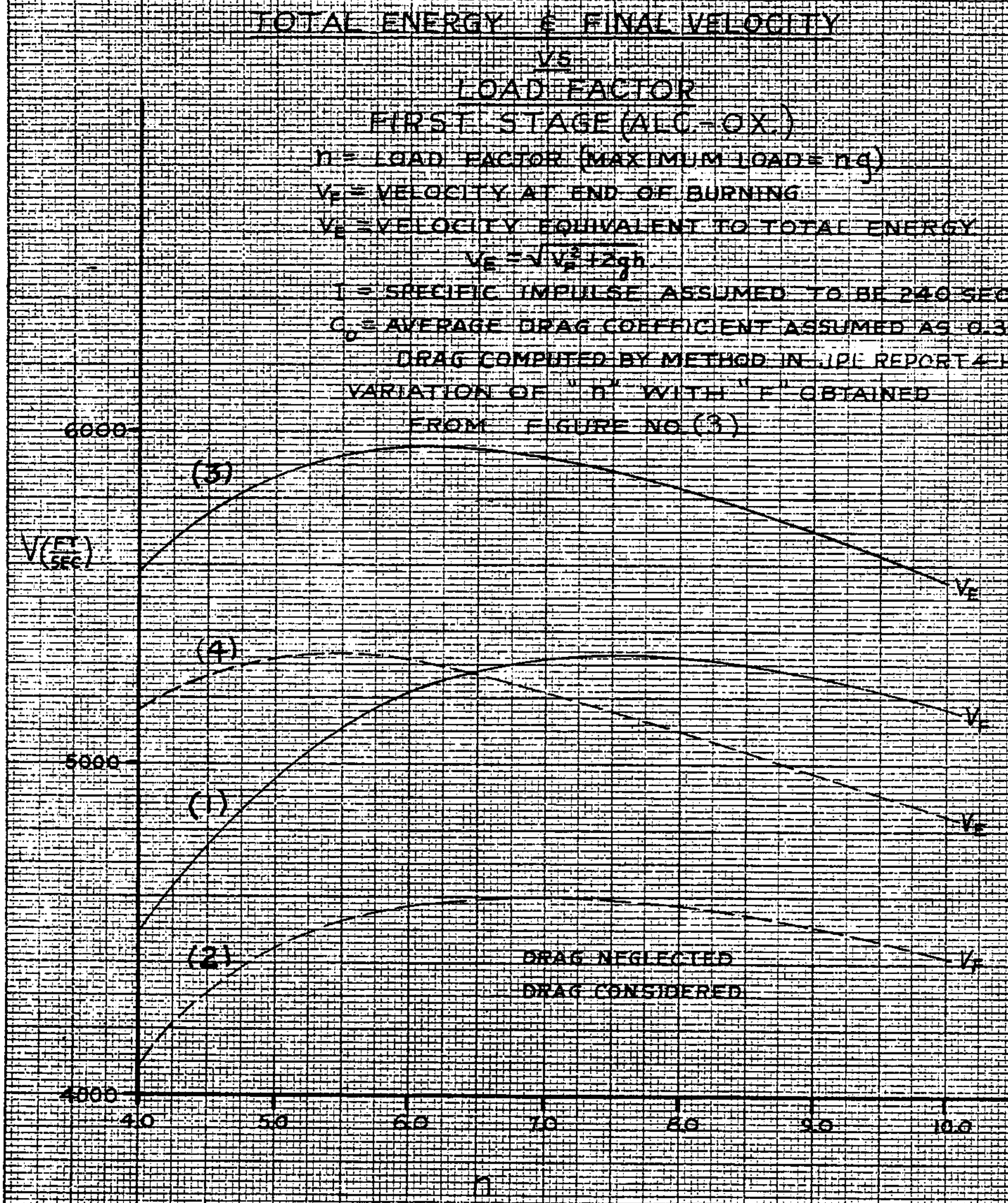


FIGURE 4
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<u>Drag</u> - For a rapid estimate of the effect of drag on performance, the method of successive approximations is recommended. The equation for a vertical trajectory is (D denotes drag):

$$(12) \quad \frac{dV}{dt} = - \frac{c}{m} \frac{dm}{dt} - g - \frac{D}{m} .$$

A zero order solution is obtained by putting $D = 0$. Then (12) reduces to (1) and by integration one obtains the vacuum trajectory, i.e., velocity and altitude as functions of time. Using these functions, $\frac{D}{m}$ may be expressed as a function of time. The right hand side then depends on time only, and by integrating one obtains the first order solution for V as a function of time. The method has been elaborated in JPL-GALCIT Report No. 4-11, "Vertical Flight Performance of Rocket Missiles" by W. Z. Chien. It has been applied to the example discussed above (first stage) and the result is plotted as curve 2 in Figure 4. It can be seen that drag moves the maximum from $n=7.5$ to $n=7.0$.

<u>Total Energy</u> - So far, we have measured performance in terms of the final velocity V_F. However, it would seem that one really should consider the total energy gained which consists of kinetic + potential energy. This total energy may conveniently be represented by an "equivalent velocity" V_E, defined by $V_E = \sqrt{V_F^2 + 2gh}$ where h is the altitude gained. The equivalent velocity has also been plotted with and without drag as curves 3 and 4 in Figure 4. The values of n for maximum V_F are 6.0 (vacuum) and 5.5, as compared with the values 7.5 and 7.0 for maximum V_F.

If our vehicle actually were a single stage rocket V_E would clearly be the significant value. However, if it is one of the initial stages of a multi-stage rocket, the situation is more complicated. The reason is that a rocket motor is characterized by its thrust T which is essentially independent of flying conditions. Its power on the other hand is T x V and hence directly proportional to the speed of the vehicle, and the total energy gained during one stage is equal to the work done $= \int F ds = \int_0^{t_B} F \times V dt$ (neglecting drag and gravity). This shows the importance of the initial velocity, but the initial velocity of the second stage is the final velocity of the first stage. Thus by trying to get as much total energy as possible during the first stage one might lose out on later stages.

Returning to our specific example we conclude that the optimal n lies somewhere between the value for $V_E(7.0)$ and that for $V_F(5.5)$. Considering the flatness of the graphs, the design value n = 6.5 represents a reasonable compromise.

Throttling - On the basis of the above discussion of the influence of maximum acceleration on weight, it seems logical to investigate whether weight can be saved by throttling during the later portions of the burning period where the accelerations have increased considerably. However, a moment's reflection shows that this will not reduce the critical design condition on some of the structural components. Motors and pumps will have to be designed for maximal thrust rather than maximal acceleration and hence nothing is saved in the weight of the power plant. The load on the tanks of the first stage is greatest during the first part of the burning period when the tanks are full. Consequently, the weight of these tanks will be unaffected by throttling.

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Calculations have shown that if the maximum acceleration is reduced from 6.5g to 4g by throttling, the gain in structural weight is sufficiently greater than the loss in efficiency of the rocket, that a gain of about 3% on velocity is obtained. However, it is felt that this gain is not sufficient to warrant the additional complication in the present study.

Chapter 8

Proportioning of Stages

Multi-stage Vehicle - In Chapter 5 the concept of a multi-stage rocket was introduced. A simplified analysis showed that if the structural weight ratio is the same for all stages the performance is a maximum if the payload and the weights of the various stages form a geometric progression:

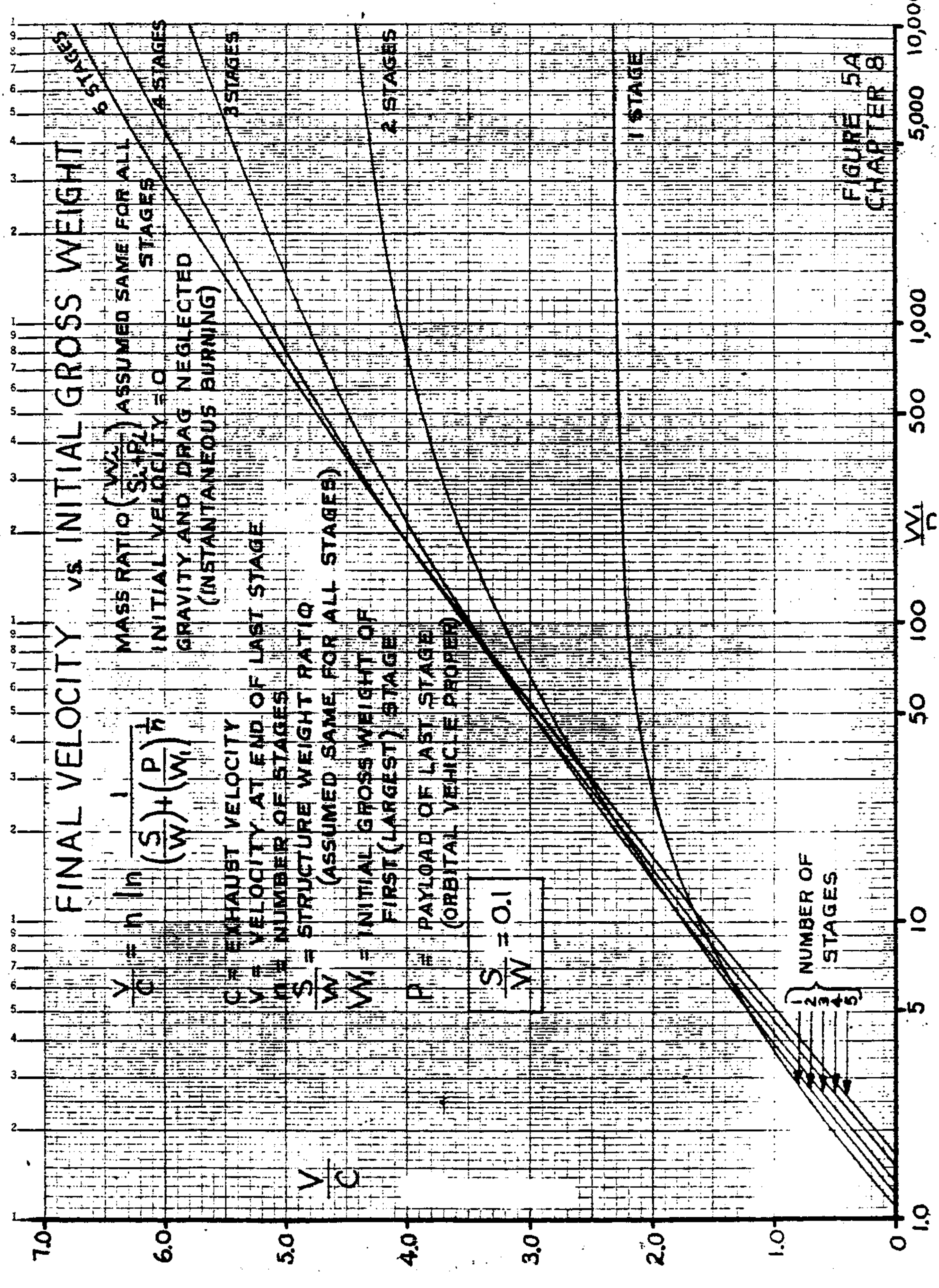
$$(11) \quad \frac{W_n}{P} = \frac{W_{n-1}}{W_n} = \dots \frac{W_1}{W_2}$$

The final velocity at the end of burning of the n^{th} stage is thus

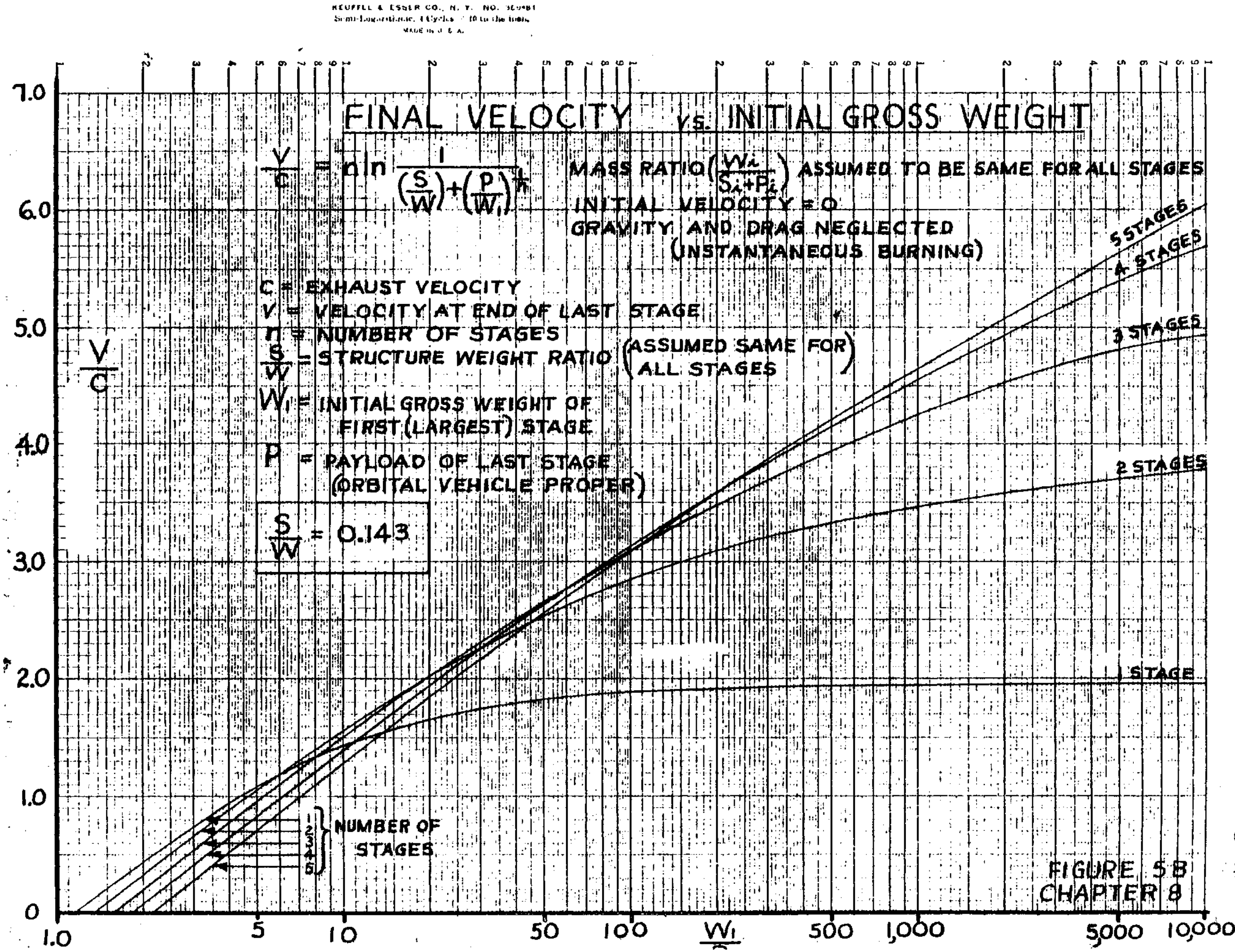
$$(12) \quad \frac{V_F}{c} = -n \ln\left(\frac{S}{W} + \left(\frac{P}{W}\right)^{\frac{1}{n}}\right)$$

This basic formula was plotted in Chapter 5 for n = 1,2,3,4,5 and $\frac{S}{W}$ = .16. On the next pages four additional charts of this type are given for $\frac{S}{W}$ = .1, .143, .182 and .25 (Figures 5A - 5D). The value .182 was achieved by the Germans in their later redesign of the V-2.

We now turn to a detailed consideration of the problem of optimal proportioning of the stages, when more realistic assumptions are introduced. It will turn out that factors like inclination, variable exhaust velocity, etc., will cause deviations of the optimal proportions given by the geometric series. However, these deviations will in general be small, sometimes insignificant. It will also be seen that the maxima



FINAL VELOCITY vs. INITIAL GROSS WEIGHT
MASS RATIO $\left(\frac{W_L}{S_L+P_L}\right)$ ASSUMED SAME FOR ALL STAGES
INITIAL VELOCITY = O
GRAVITY AND DRAG NEGLECTED
(INSTANTANEOUS BURNING)
$\frac{V}{C} = n \ln \frac{1}{\left(\frac{S}{W}\right)+\left(\frac{P}{W_1}\right)^{\frac{1}{n}}}$
C = EXHAUST VELOCITY
V = VELOCITY AT END OF LAST STAGE
n = NUMBER OF STAGES
$\frac{S}{W}$ = STRUCTURE WEIGHT RATIO
(ASSUMED SAME FOR ALL STAGES)
W_1 = INITIAL GROSS WEIGHT OF FIRST (LARGEST) STAGE
P = PAYLOAD OF LAST STAGE
(ORBITAL VEHICLE PROPER)
$\frac{S}{W} = 0.1$
6 STAGES
5 STAGES
4 STAGES
3 STAGES
2 STAGES
1 STAGE
NUMBER OF STAGES
1
2
3
4
5
$\frac{V}{C}$
$\frac{W_1}{P}$
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
1.0
5
10
50
100
500
1,000
5,000
10,000
FIGURE 5A
CHAPTER 8



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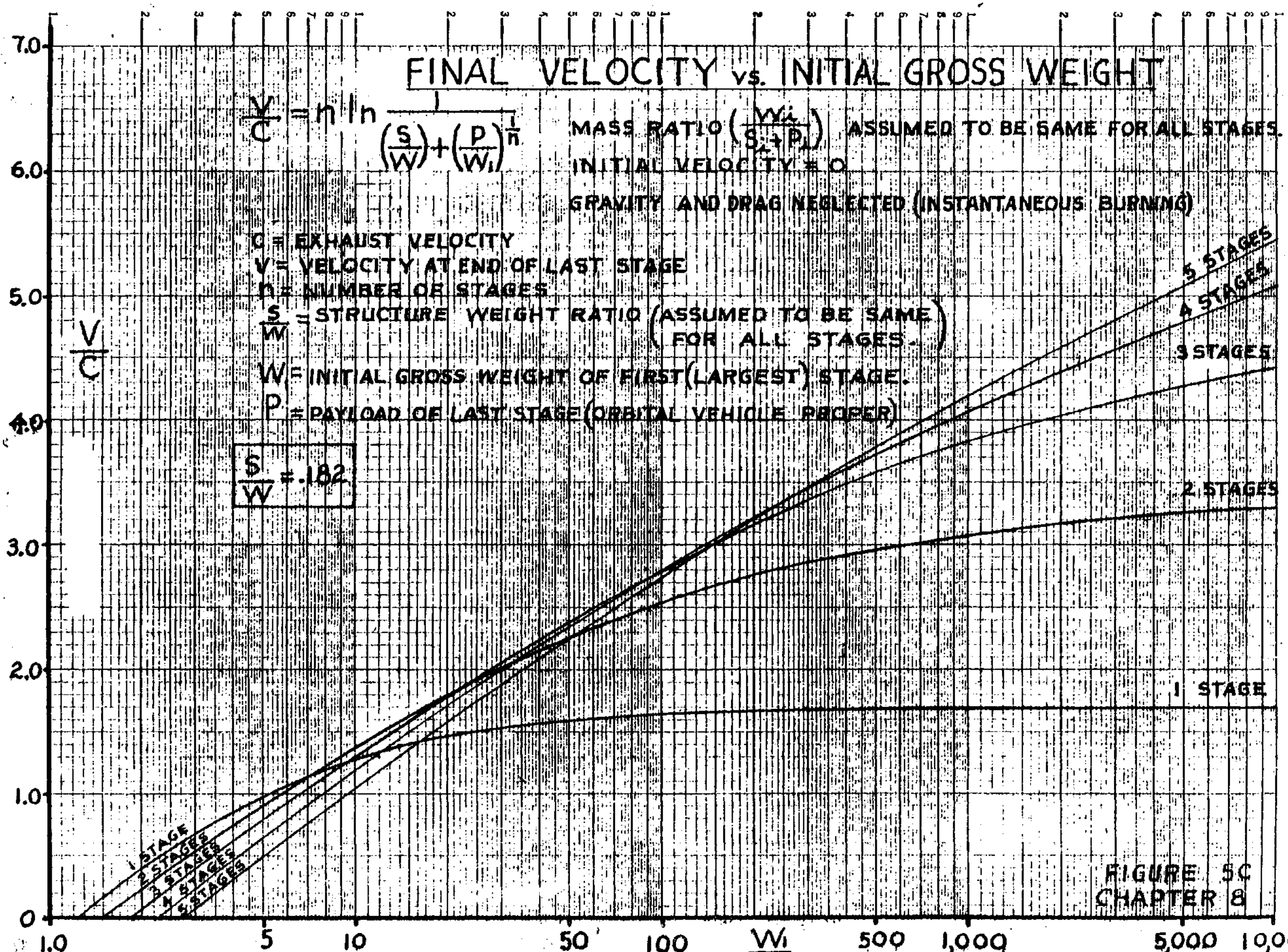
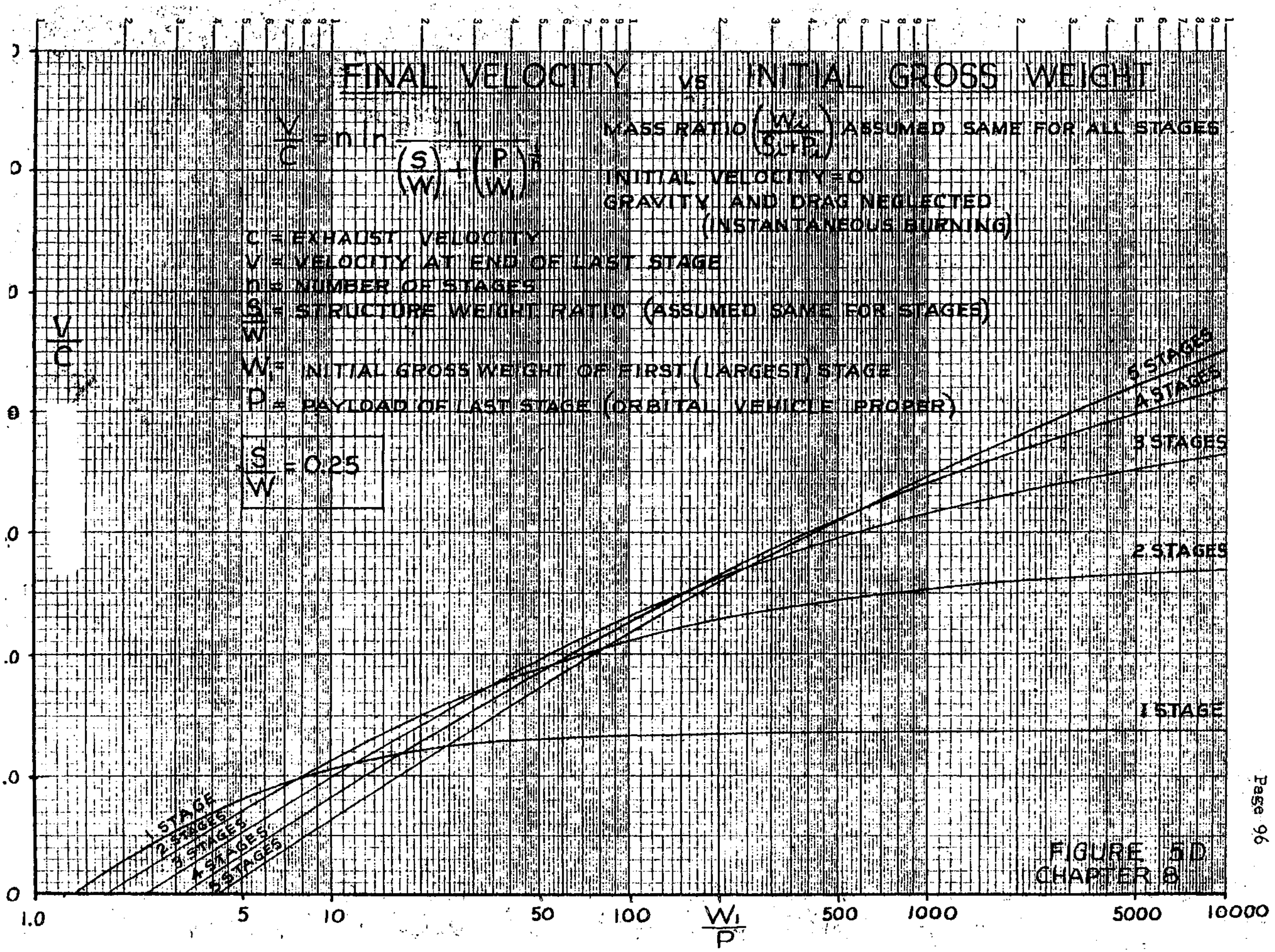
FINAL VELOCITY vs. INITIAL GROSS WEIGHT
$\frac{V}{C} = n \ln \frac{1}{\left(\frac{S}{W}\right) + \left(\frac{P}{W_1}\right)^{\frac{1}{n}}}$
MASS RATIO $\left(\frac{W_i}{S_i + P_i}\right)$ ASSUMED TO BE SAME FOR ALL STAGES
INITIAL VELOCITY = 0
GRAVITY AND DRAG NEGLECTED
(INSTANTANEOUS BURNING)
C = EXHAUST VELOCITY
V = VELOCITY AT END OF LAST STAGE
n = NUMBER OF STAGES
$\frac{S}{W}$ = STRUCTURE WEIGHT RATIO (ASSUMED SAME FOR ALL STAGES)
W_1 = INITIAL GROSS WEIGHT OF FIRST (LARGEST) STAGE
P = PAYLOAD OF LAST STAGE (ORBITAL VEHICLE PROPER)
$\frac{S}{W} = 0.143$
5 STAGES
4 STAGES
3 STAGES
2 STAGES
1 STAGE
1 2 3 4 5 NUMBER OF STAGES
$\frac{V}{C}$
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
1.0
5
10
50
100
W_1
500
1,000
5,000
10,000
FIGURE 5B
CHAPTER 8


FINAL VELOCITY vs. INITIAL GROSS WEIGHT
$\frac{V}{C} = n \ln \frac{1}{\left(\frac{S}{W}\right) + \left(\frac{P}{W_1}\right)^{\frac{1}{n}}}$
MASS RATIO $\left(\frac{W_i}{S_i + P_i}\right)$ ASSUMED TO BE SAME FOR ALL STAGES.
INITIAL VELOCITY = O
GRAVITY AND DRAG NEGLECTED (INSTANTANEOUS BURNING)
C = EXHAUST VELOCITY
V = VELOCITY AT END OF LAST STAGE
n = NUMBER OF STAGES
$\frac{S}{W}$ = STRUCTURE WEIGHT RATIO (ASSUMED TO BE SAME FOR ALL STAGES.)
W = INITIAL GROSS WEIGHT OF FIRST (LARGEST) STAGE.
P = PAYLOAD OF LAST STAGE (ORBITAL VEHICLE PROPER)
$\frac{S}{W} = .182$
$\frac{V}{C}$
$\frac{W_1}{P}$
5 STAGES
4 STAGES
3 STAGES
2 STAGES
1 STAGE
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
1.0
5
10
50
100
500
1,000
5,000
10,000
FIGURE 5C
CHAPTER 8


FINAL VELOCITY vs INITIAL GROSS WEIGHT
$\frac{V}{C} = n \ln \frac{1}{\left(\frac{S}{W}\right) + \left(\frac{P}{W_1}\right)^{\frac{1}{n}}}$
MASS RATIO $\left(\frac{W_1}{S+P}\right)$ ASSUMED SAME FOR ALL STAGES
INITIAL VELOCITY = 0
GRAVITY AND DRAG NEGLECTED
(INSTANTANEOUS BURNING)
C = EXHAUST VELOCITY
V = VELOCITY AT END OF LAST STAGE
n = NUMBER OF STAGES
$\frac{S}{W}$ = STRUCTURE WEIGHT RATIO (ASSUMED SAME FOR STAGES)
W_1 = INITIAL GROSS WEIGHT OF FIRST (LARGEST) STAGE
P = PAYLOAD OF LAST STAGE (ORBITAL VEHICLE PROPER)
$\frac{S}{W} = 0.25$
$\frac{V}{C}$
$\frac{W_1}{P}$
1.0
5
10
50
100
500
1000
5000
10000
1 STAGE
2 STAGES
3 STAGES
4 STAGES
5 STAGES
FIGURE 5D
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in general are fairly flat i.e. deviations from the theoretical maxima will cause but small loss in performance. In order to bring out the trends clearly, the various factors will be considered separately. When concrete numerical computations are made a two-stage rocket will be considered. If values are desired for rockets having a great number of stages, these results may be applied to the stages in pairs. The following formulation of the problem applies to all such two-stage rockets: The gross weight of the first stage (W_1) and the pay load of the second stage (P) will be fixed. The weight of the second stage (W_2) will be varied and the performance plotted against W_2 (or some equivalent variable like $\frac{W_2}{P}$). Quantities like n, c, $\frac{S}{W}$ will also be considered fixed except when otherwise stated. They may or may not be the same for both stages, depending on what factor is being studied.

<u>Gravity. Inclination</u> - If gravity is considered the optimal proportioning of the stages is still a geometrical progression as long as performance is being gauged by the final velocity V_F. (We thereby assume that the maximal load factor n rather than the burning time t_B is kept the same for both stages.) This result may be proved by standard methods of calculus or by a qualitative method to be discussed later in this chapter. A plot for an actual concrete case (Figure 6, curve 1) shows this maximum to be rather flat. If the equivalent velocity V_E is considered instead, a small displacement of the maximum to the right takes place as shown by curve 2.

In the same figure the effect of inclination is shown by curves 3, 4 and 5. The first stage is assumed vertical. If the second stage has an

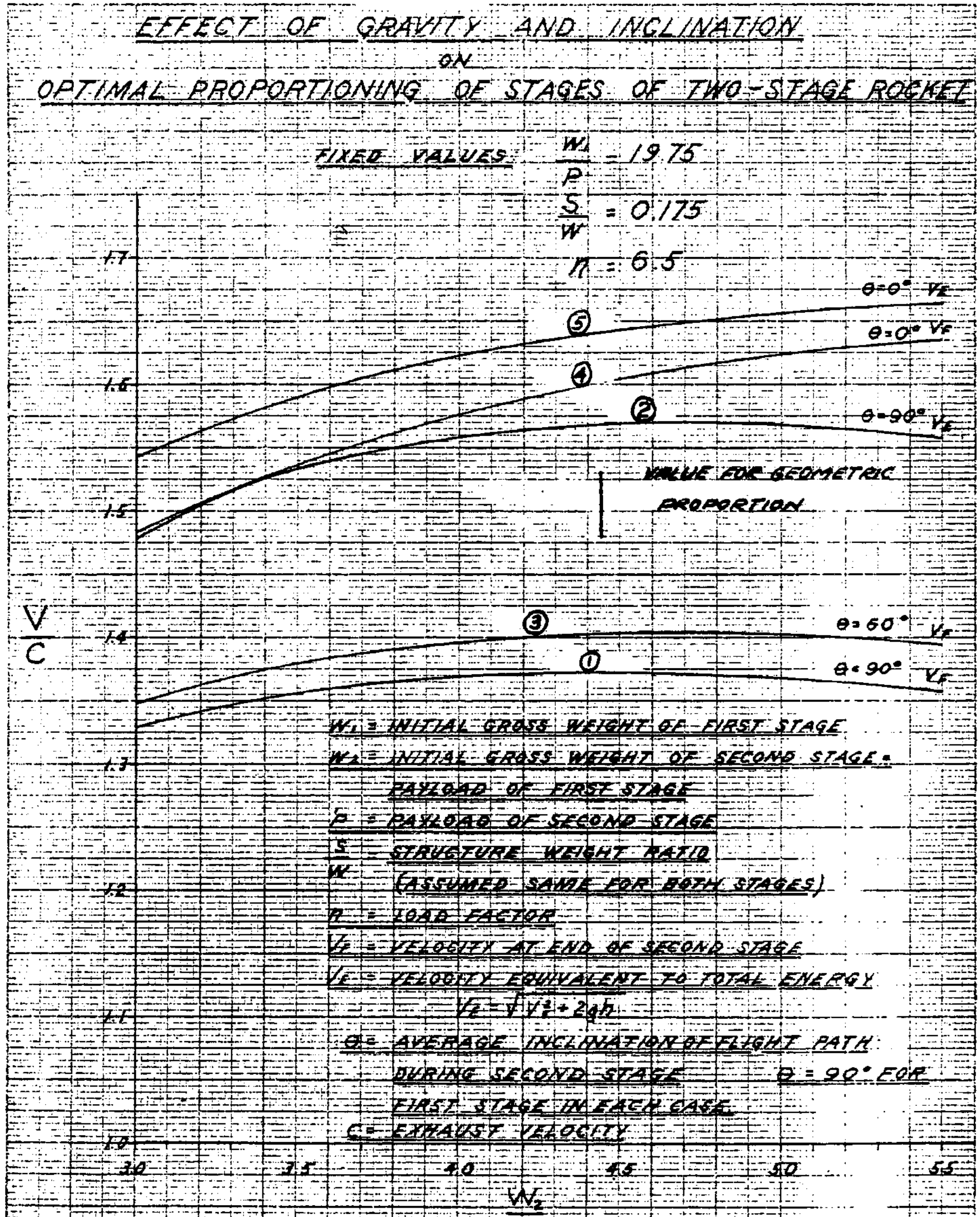

FIGURE 6
CHAPTER 8

average inclination with the horizontal of 60°, a rather insignificant displacement of the maximum to the right takes place (curve 3). In order to bring out the trend more clearly, the exaggerated case of a horizontal second stage was considered (curves 4 and 5). In this case, the displacement to the right of the maximum of both V_F and V_E is considerable.

Exhaust Velocity - Due to altitude effect on power plants the average exhaust velocity of the second stage is usually larger than that of the first stage (a 15% increase is a typical value). Multi-stage rockets have also been proposed where the second stage employs fuel with considerably higher exhaust velocity than that of the first stage.

For a two-stage rocket, the following formula for the optimal proportioning was obtained by calculus:

$$(13)\quad \frac{W_2}{\sqrt{PW_1}} = \frac{1}{2r}\left(\frac{W}{S}\right)\left[-\sqrt{\frac{P}{W_1}}\,(r-1) + \sqrt{\frac{P}{W_1}\,(r-1)^2 + 4\left(\frac{S}{W}\right)^2 r}\,\right] \quad \text{where } r = \frac{C_2}{C_1}$$

For values of r near one, this formula can be linearized to give

$$(14)\quad \frac{W_2}{\sqrt{PW_1}} = 1 - \frac{1}{2}(r-1)\left[1 + \frac{W}{S}\sqrt{\frac{P}{W_1}}\right]$$

Plots of values obtained from these equations are given in Figures 7A and 7B. For the typical values of $\left(\frac{S}{W}\right) = .143$ and $\frac{W_1}{P} = 20$ and the exhaust velocity of the second stage 15% larger than that of the first stage, the optimal value of W_2 is about 16% smaller than the geometric mean of P and W_1.

OPTIMAL PROPORTIONING OF STAGES OF TWO-STAGE ROCKET WHEN EXHAUST VELOCITIES OF THE TWO STAGES ARE DIFFERENT

W_1 = INITIAL GROSS WEIGHT OF FIRST STAGE

W_2 = INITIAL GROSS WEIGHT OF SECOND STAGE = PAYLOAD OF FIRST STAGE

P = PAYLOAD OF SECOND STAGE

$\frac{S}{W}$ = STRUCTURE WEIGHT RATIO (SAME FOR BOTH STAGES)

C_1 = EXHAUST VELOCITY DURING FIRST STAGE

C_2 = EXHAUST VELOCITY DURING SECOND STAGE

$\pi = \frac{C_2}{C_1}$

GRAVITY AND DRAG NEGLECTED

W_1, P AND $\left(\frac{S}{W}\right)$ ASSUMED FIXED

W_2 CHOSEN SO THAT TOTAL VELOCITY INCREASE FOR BOTH STAGES BECOMES A MAXIMUM.

EXACT FORMULA:

$$\frac{W_2}{\sqrt{PW_1}} = \frac{1}{2\pi}\left(\frac{W}{S}\right)\left[\sqrt{\frac{P}{W_1}}(\pi-1) + \sqrt{\frac{P}{W_1}(\pi-1)^2 + 4\left(\frac{S}{W}\right)^2\pi}\right]$$

LINEARIZED FORMULA:

$$\frac{W_2}{\sqrt{PW_1}} = 1 - \frac{1}{2}(\pi-1)\left[1 + \left(\frac{W}{S}\right)\sqrt{\frac{P}{W_1}}\right]$$

$\frac{S}{W} = .143$

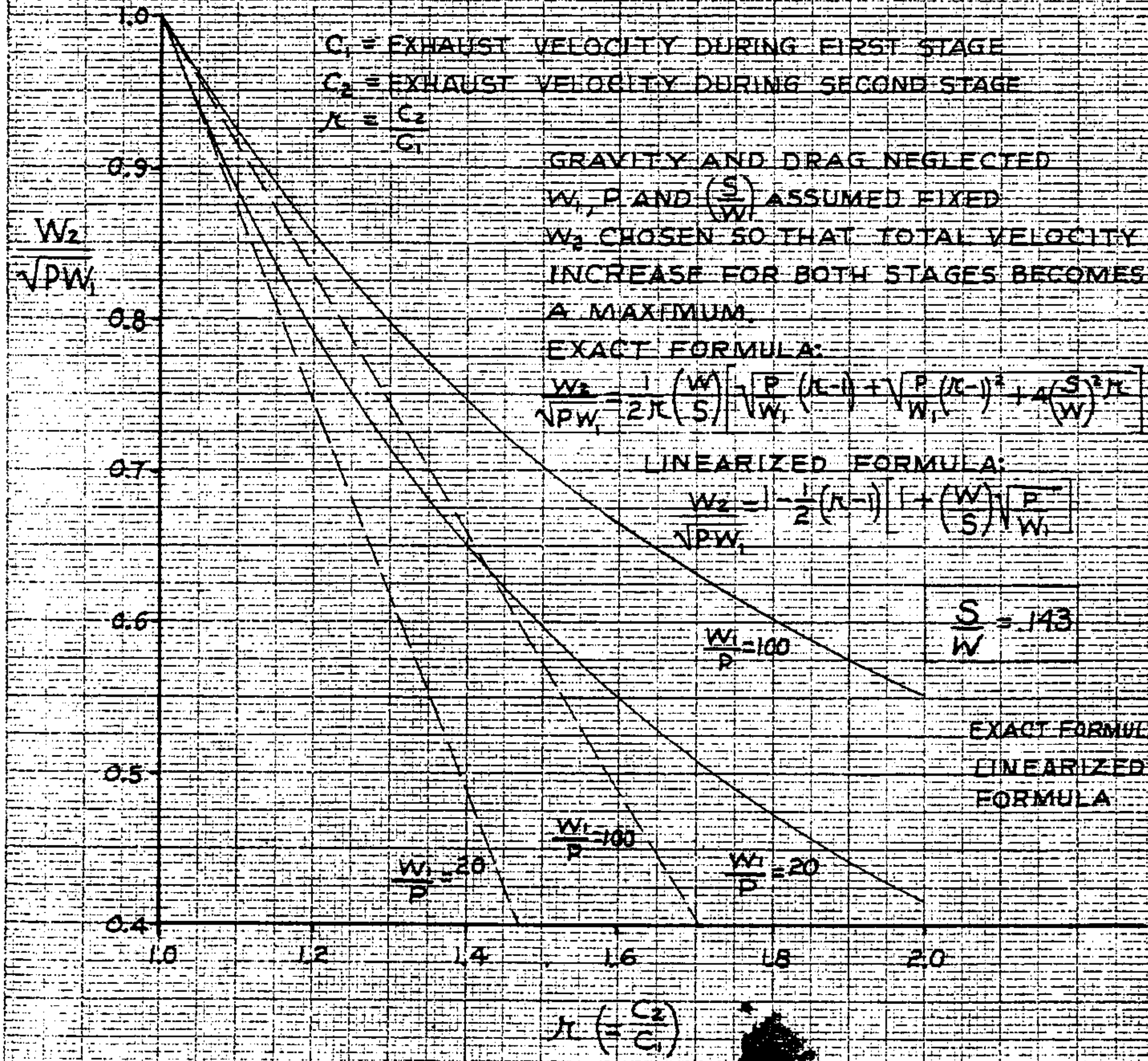

FIGURE 7A
CHAPTER 8

OPTIMAL PROPORTIONING OF STAGES OF TWO-STAGE ROCKET WHEN EXHAUST VELOCITIES OF THE TWO STAGES ARE DIFFERENT (USING LINEARIZED FORMULA)

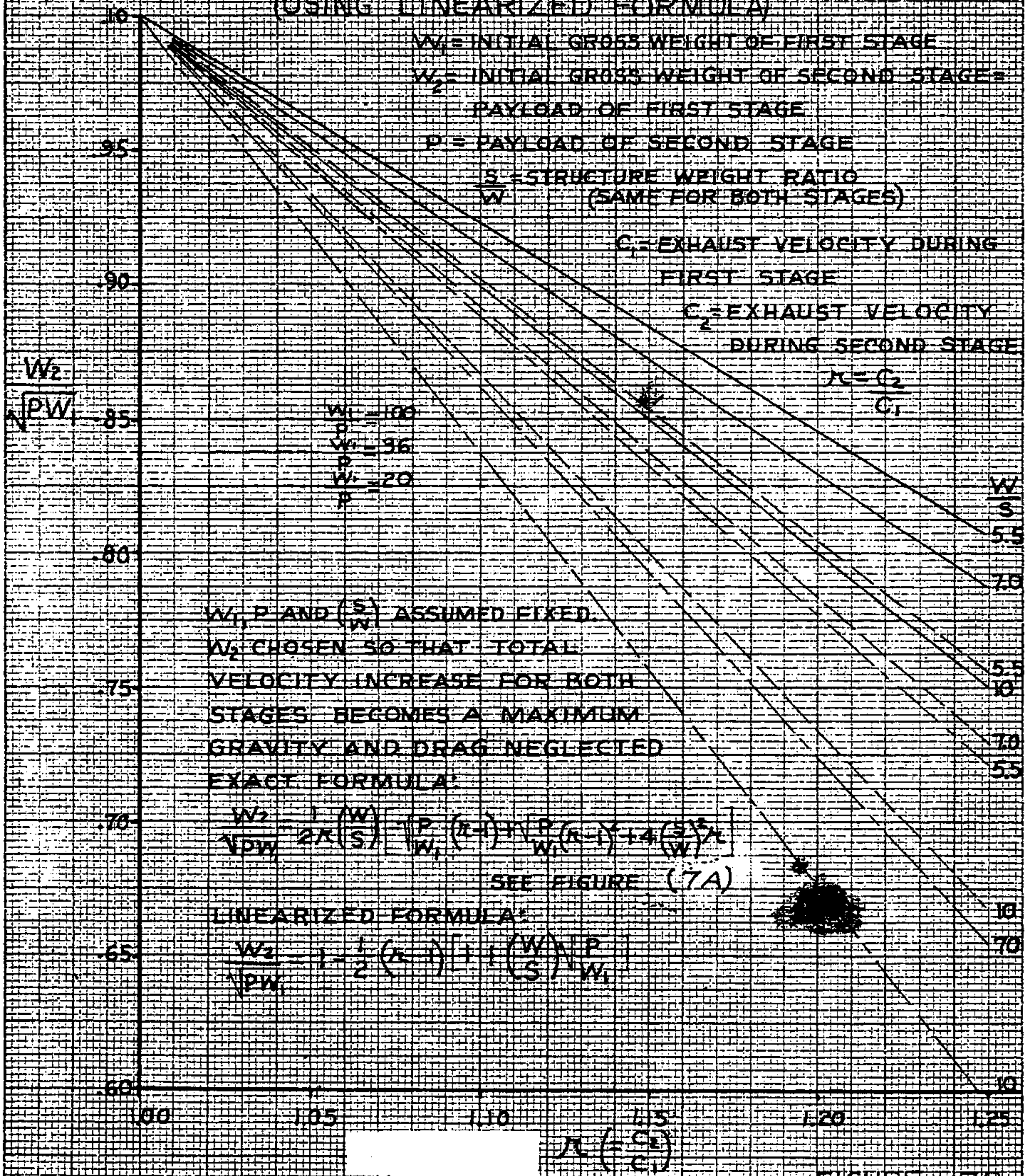

FIGURE 7B
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<u>Drag</u> - An investigation of the effect of drag on the optimum proportioning of two stages was made for the case in which the first stage had higher drag than the second (this is the normal case encountered). The results showed that the influence of drag on optimal proportioning is comparatively insignificant.

<u>General Considerations about Optimal Proportioning</u>. <u>Influence of Structural Weight Ratio</u> - The following qualitative method for determining the optimal proportioning might sometimes prove useful. Details of the method are omitted.

Consider a two-stage rocket as before. Express the performance for each stage as a function of the payload weight ratio. Denote these functions by $f_1(x)$ and $f_2(x)$. Put $\frac{P}{W_1} = a =$ constant and the payload weight ratio of the first stage $= \frac{W_2}{W_1} = x$; then the payload payload weight ratio of the second stage is $\frac{P}{W_2} = \frac{a}{x}$. The total velocity increase expressed as a function of the payload weight ratio of the first stage is then $V(x) = f_1(x) + f_2\left(\frac{a}{x}\right)$

Consider the difference $d = V\left(\frac{a}{x}\right) - V(x) = \left[\left(f_2\left(\frac{a}{x}\right) - f_1\left(\frac{a}{x}\right)\right) - \left(f_2(x) - f_1(x)\right)\right]$. If d is zero, the graph of V against [illegible] a certain type of symmetry around the value where $\frac{a}{x} = x$, i.e. $x = \sqrt{a}$. It is then geometrically evident that this point of symmetry represents a maximum. The difference d is zero when f_1 and f_2 are equal or differ by a constant. This is true for the idealized case when $f_1(x) = f_2(x) = -c \ln \left(x + \frac{S}{W}\right)$. It remains true when gravity is considered.

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Now take the case of different exhaust velocities. Then the difference d is $= (c_2 - c_1)\left[\ln\left(x + \frac{S}{W}\right) - \ln\left(\frac{a}{x} + \frac{S}{W}\right)\right]$ This is negative for $x < \frac{a}{x}$. From the geometry of the graph it then follows that the maximum has been displaced from $x = \sqrt{a}$ towards smaller x.

So far we have only verified previous findings. To obtain a new result, consider the case of variable structure weight ratio, e.g. $\frac{S_1}{W_1} > \frac{S_2}{W_2}$.

$$\text{Then } d = c \ln \frac{\frac{S_1}{W_1} + \frac{a}{x}}{\frac{S_2}{W_2} + \frac{a}{x}} - c \ln \frac{\frac{S_1}{W_1} + x}{\frac{S_2}{W_2} + x} .$$

This is also negative for $x < \frac{a}{x}$ and again we conclude that the maximum has been displaced towards the left, i.e., the second stage is smaller than the geometric mean when its structural weight ratio is less than that of the first stage.

Influence of the Various Stages on Each Other - Above we have sometimes applied the results obtained for a single stage vehicle to each separate phase of a multi-stage vehicle. However, the discussion of whether V_E or V_F is the significant performance parameter for the first stage of a multi-stage vehicle (see p. 89) showed that a certain amount of care is necessary.

Another example of how the various stages influence each other is this: For each separate stage one can find the optimal load factor by the methods described at the befinning of this chapter. However, each stage cannot be designed for its own optimal load factor. The reason is simply that any stage is subjected to the maximum loads of any previous stage. And in general the optimal

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load factor is lower for later stages.

Application of General Methods to Actual Design.- So far we have developed a general analysis of performance parameters. We now proceed to apply our results to actual designs of two vehicles. Our starting point will of course be the specifications of the orbital vehicle. Two quantities are revelant for our considerations: (1) orbital velocity; (2) weight of pay load.

Strictly the altitude of the orbit should likewise be given. However, it was found in Chapter 3 that the altitude had little effect on the energies required for various orbits. This is fortunate, because it implies that the shape of the trajectory will exert only a secondary influence on our choice of design proportions. A value 24,500 ft/sec was selected as the orbital velocity to be used in our present consideration. The pay load, selected on the basis of a reasonable allowance for scientific instruments, was taken as 500 lbs. However, for the purposes of the analysis in this chapter, we also have to count the 200 lbs. of "brains" as pay load. The reason is that this is a fixed weight item which occurs only in the last stage and which is not included in a normal estimate of the structure-weight ratio $\frac{S}{W}$. Thus, for the remainder of this chapter, the weight of the pay load will be considered to be 700 lbs.

Next we need a value of the parameter c (exhaust velocity) which specifies the performance of the power plant, in other words we have to select the fuel. Two different vehicles will be considered, one powered

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by oxygen-alcohol, the other by oxygen-hydrogen.

Alcohol-Oxygen Vehicle - If liquid oxygen and alcohol are used as propellants, the value $c = 8{,}500$ fps. is a reasonable average value for our initial work. In selecting this value consideration was given to the fact that most of the operation of the rocket will be at altitude. Using this value of c, we find that $\frac{V}{c}$ is 2.88. In the preliminary work we shall use a value of $\frac{S}{W} = .16$ which is an average value obtained from the results of Chapter 7. For our present purpose it would be extremely convenient to have a correction factor to take care of these items. A good value of such a factor can only be based on long experience in designing, building and testing orbital vehicles. Since this experience is lacking, we use the following estimate. The V-2 was designed for a load factor of 6.5. A previous estimate in this chapter showed that with this load factor the losses due to gravity in a vertical trajectory amount to 25%. Inclination of the last portion reduces this to about 20%. Furthermore, preliminary calculations showed that drag will reduce the final velocity by 10%. Thus we arrive at a correction factor of 1.30. This increases the value of $\frac{V}{c}$ required from 2.88 to 3.74. A look at Figure 2 of Chapter 5 tells us that a single-stage rocket and a two-stage rocket are impossible. A three-stage rocket would have to have a weight ratio $\frac{W_1}{P}$ of 450 and a four-stage and a five-stage rocket would have about equal weight ratios of 330. Thus a three-stage vehicle would necessitate a gross weight of

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450 x 700 = 315,000 lbs. and a four-stage vehicle 330 x 700 = 230,000 lbs. This weight saving seems to warrant the additional complications of having four stages instead of three, whereas nothing is gained by adding a fifth stage. The weight of the other stages will form a geometrical progression. The ratio of the weights between two consecutive stages is $(330)^{1/4} = 4.27$. At this stage in the analysis, the detailed weight study of Chapter 7 was undertaken. Hand in hand with this structural weight study, an analysis of the optimum design proportions was made, using the methods previously explained in this chapter. This combined study resulted in the set of values presented in Chapter 7.

The next step in our design study will be to determine more rigorously the actual trajectory of the vehicle, taking into account the variation of drag, exhaust velocity and inclination. This more detailed computation will be done in the next chapter. The results obtained there will give us an indication of how accurate the preliminary analysis of this chapter has been.

It is clear that it is impossible to maintain a strict logical order in determining the proportions of the vehicle. Actually one has to repeat the process described here several times, just as when solving a problem by successive approximations. As a "first approximation" for W_1 we found the value 230,000 lbs. above. In the course of the revisions indicated above, this value was changed to 233,669. Hence, we may conclude that the factor 1.30

used above for correcting $\frac{V}{c}$ was a reasonably good estimate. The values for $\frac{S}{W}$ finally established were .168, .18, .157 and .156 for stages 1, 2, 3 and 4 respectively, which are likewise in satisfactory agreement with the assumed value of .16. It was unnecessary to alter the value of 6.5 for the maximum ratio of thrust to weight.

Hydrogen-Oxygen - When the analysis was first made for the liquid hydrogen-liquid oxygen rocket, the structural weight ratio was estimated to be 0.20. In addition, it was erroneously assumed that higher accelerations would be used than were used for the alcohol oxygen rocket with a consequent reduction in the correction factor to 1.2. Under these conditions, it appeared that a three stage hydrogen-oxygen rocket would give slightly smaller overall gross weights than a two-stage rocket. However, the gain was so small that it was decided to avoid the complications of the three-stage rocket and proceed with the design of a two-stage rocket.

As the work on structural weight analysis progressed, it became apparent that the acceleration would have to be reduced to the value used for the alcohol-oxygen rocket. Making allowance for this decreased acceleration and also for the higher drag of the hydrogen rocket, a revised correction factor of 1.32 was obtained. In addition, it was found that the structural weight

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ratio would increase to about 0.25. When these later figures were used, they showed that it was advantageous to use three stages instead of two. However, the design study for this vehicle had proceeded so far that it was inadvisable to alter the number of stages.

If we use an exhaust velocity of 13,500 ft/sec and a correction factor of 1.32 we find $\frac{V}{c} = 2.4$. For $\frac{S}{W} = 0.25$ figure 5D shows that a value of $\frac{W_1}{P}$ of 400 is necessary for a two-stage rocket. This corresponds to a gross weight of 280,000 lbs.

The final design values resulting from the combined structural weight and performance study are tabulated in Chapter 7. The overall gross weight was 291,564 lbs. The values of $\frac{W}{S}$ were .238 and .245 for the 1st and 2nd stages respectively.

It is worthwhile at this point to say a few words about the possibilities of a three-stage hydrogen-oxygen rocket. As mentioned above, the study had proceeded too far for alteration when the design values had crystallized sufficiently to show the definite advantages of using three stages. Let us examine this case at greater length. Using $\frac{V}{c} = 2.4$ and $\frac{S}{W} = 0.25$, we find from figure 5D that $\frac{W_1}{P} =$ 120 for three stages. This implies that the overall gross weight of this vehicle would be 84,000 lbs. which is considerably less than the weight of either the two-stage hydrogen-oxygen rocket or the four-stage alcohol-oxygen rocket. From this we may conclude that the three stage liquid hydrogen-liquid oxygen should be given serious consideration in any further studies of satellite vehicles.