Chapter 9

9. FINAL ESTABLISHMENT OF SIZES AND TRAJECTORIES

In the preceding chapter, we have discussed the choice of burning time for a single stage rocket and the optimum proportioning of weights between two successive stages for multi-stage rockets. In order to proceed farther in our analysis, it is necessary to have an integrated picture of the variation of altitude, speed, inclination and mass at all points along the trajectory. In order to obtain these data, we shall use the design values obtained in the last chapter and carry out detailed calculations of the entire trajectory. The results of these calculations will show how much these design values are in error and it will give us considerably more reliable values which could be used in repeating the design studies of chapter 8. Ideally, this process of iteration should be continued until a satisfactory set of final design data are obtained. For the present study, however, our attention was confined to an investigation of the trajectories for the two proposed designs, without a repetition of the calculations of chapter 8.

The vehicles which were selected for study and which served as a basis for the calculations of this chapter are tabulated below.

Vehicle Powered by Alcohol-Oxygen Rockets

Stage	1	2	3	4
Gross Wt. (lbs.)	233,669	53,689	11,829	2,868
Weight less fuel (lbs.)	93,669	21,489	4,729	1,148
Payload (lbs.)	53,689	11,829	2,868	500
Max. Diameter (in.)	157	138	105	90

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Vehicle Powered by Hydrogen-Oxygen Rockets

Stage	1	2
Gross Wt.	291,564	15,364
Weight Empty	84,564	4,464
Payload (lbs.)	15,364	500
Max. Diameter (in.)	248	167

In all of the vehicles, the rate of fuel consumption was maintained constant at a value calculated to give a maximum ratio of thrust to weight of 6.5. The drags were calculated according to the methods given in Appendix B. The variations of exhaust velocity with altitude were taken from the graphs in Chapter 6.

In order to avoid a mass of distracting details, none of the lengthy and involved trajectory calculations will be presented in this chapter. We shall simply indicate the methods used and present the final results that were obtained within the time available, which was, to say the least, insufficient to answer all the questions that will inevitably arise.

The mathematical developments necessary for this work, as well as samples of the calculation methods used, are presented in appendices C and D at the end of this report.

In the early stages of the work, we set for ourselves the goal of establishing a 500 lb. payload on an orbit approximately 100 miles above the surface of the earth, hoping it could remain there for a period of 5 to 10 days before its energy was dissipated in the rarefied atmosphere. The 500 lb. figure was chosen as a reasonable estimate of the weight of

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scientific apparatus necessary to obtain results sufficiently far reaching to make the undertaking worthwhile.

The altitude figure was a compromise between the desire to increase the altitude in order to reduce the drag and give larger limits of error in establishing the orbit on the one hand and to keep the orbital energies low and to aid the short wave radio control and tracking problem on the other. Preliminary calculations of energy dissipation in the orbit, based on conventional evaluation of drag coefficients and an isothermal atmosphere indicated that the vehicle could remain aloft for at least 5 to 10 days at an altitude of 100 miles. However, closer examination revealed that the conventional drag predictions were inadequate and that the assumption of an isothermal atmosphere was in error. Revised methods of drag prediction were developed and better estimates of the structure of the atmosphere* were obtained from an extensive search of the literature. The results of this revised study showed that it would be advisable to use a altitude of 300 to 400 miles in order to obtain the durations desired. The calculations were revised correspondingly but unfortunately time was not available for the work to be completed in detail.

In approaching the problem of launching a vehicle into a satellite orbit we see that consideration must be given to the following:

1. Obtaining time-velocity-acceleration-distance-mass-relationship for the trajectory used in placing the vehicle on a satellite orbit.
2. Devising a means of stably controlling the vehicle so that

*The methods developed for drag prediction are given in Appendix B and the data accumulated on the atmosphere are given in Appendix A.

it reasonably well follows the desired trajectory and is established within permissible limits of error on the chosen orbit. The present chapter concerns itself with the first topic. The second will be the subject of the following chapter.

In our present work, we shall find that it will be necessary to apply forces of control normal to the flight path to obtain desirable trajectories. These forces cannot be applied without incurring losses. Consequently it will be necessary to anticipate the methods of control proposed in the following chapter in order that the losses incurred by this control may properly be included in the calculations. This method of control consists of movable vanes in the rocket jet stream by means of which the entire vehicle is rotated so that a component of the thrust is applied in any direction normal to the flight path. If T is the rocket thrust and α is the angle between flight path and rocket jet axis (for brevity, α will be referred to as the "tilt") then the control force produced is $T \sin \alpha$ and the effective thrust along the flight path is reduced by $T(1-\cos\alpha)$. If α can be kept less than 15°, the reduction in effective thrust is less than 4%. When the desired orbital altitude was 100 miles, it was found that the tilt could be kept within this limit. However, to achieve an altitude of 300 miles, tilts of greater than 15° were required and a revised scheme was found to be necessary as will be seen presently.

In considering possible trajectories, we see that air resistance, starting from its initial value of zero, will first rise rapidly as the speed increases, then less rapidly as altitudes of reduced density are reached. We shall see later that, for the vehicles we have considered, the drag reaches a maximum at altitudes of about 10,000 ft. to 20,000 ft.

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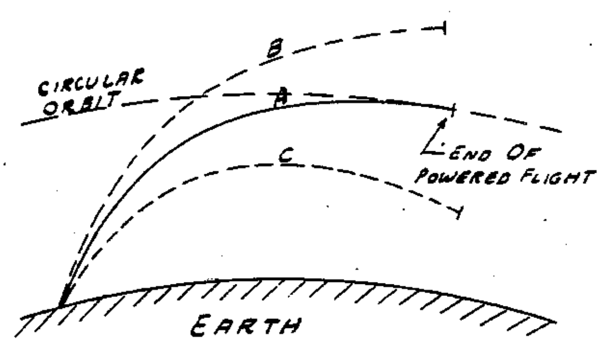
Beyond this the decrease in density reduces the drag faster than the corresponding increase in speed. By the time 150,000 ft. has been reached the drag has become a factor of minor importance. It is apparent that the initial portion of the trajectory should be nearly vertical so as to reduce as much as possible the portion of the flight path affected by drag.

First let us consider the following trajectory. The vehicle is launched nearly vertically. As it accelerates upward, gravity will curve the trajectory toward the horizontal in the direction of the initial inclination*. If this initial condition of launching is correctly selected, it is seen that the trajectory will be sufficiently curved without the application of control forces so that the vehicle is on a circular orbit at the end of powered flight as shown by trajectory A in the accompanying figure. It is clear that only one such initial condition of launching exists for a given program of acceleration. If the initial launching is too nearly vertical, the trajectory will end up at a higher altitude, inclined out into space as indicated by B.



If insufficiently near vertical, it will end up at a lower altitude, inclined toward the earth as indicated by C. We are forced to conclude that if our vehicle characteristics are already chosen (i.e. weights, thrusts, etc.) there is only one altitude for the orbit into which it can be launched by this method. Initial calculations showed that for the vehicles

*Mathematically, the initial point of launching is a complicated singularity. Mechanically, this means that a short set of guiding rails will have to be provided.

considered in this report, this altitude was about 35 miles, considerably short of our two successive goals of 100 miles and 300 miles.

We are next faced with the question of how we should apply control forces so that the trajectory* will end up on a circular orbit of greater altitude. Of the three trajectories shown in the figure above, B offers the most obvious possibilities. If downward forces are applied during the late portions of B, it is conceivable that this trajectory can be curved sufficiently to end tangent to a circular orbit. A rigorous examination of the equations of motion shows that this is the best way of obtaining the desired increase in altitude. One might be inclined to question this result on the ground that an application of downward control forces is inefficient when attempting to gain more altitude. Actually, the control forces have little direct effect on the altitude, which is gained primarily by the increased steepness of the earlier portions of the trajectory. The control forces serve primarily to insure a horizontal tangent at the trajectory's end point.

By the use of tilt, it was found possible to determine satisfactory trajectories for orbits at altitudes of 100 miles without exceeding 15°, angle of tilt. However, when the desired altitude was increased to 300 miles, the tilt angles became so large that the losses in effective thrust were excessive. In seeking means of avoiding these losses it was found that the judicious insertion of an extended period of coasting in the thrust program would accomplish the desired result. A little study showed that this coasting could be most effectively used if it came late

*For convenience, we shall refer to that portion of the path traversed before the end of powered flight simply as the trajectory. After powered flight, the path will be called the orbit.

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in the thrust program. In order to avoid the necessity of shutting down a rocket motor and firing it up again, the coasting period was always inserted in the interval between the discarding of the next to the last stage and the firing of the last stage. Unfortunately, insufficient time was available to pursue this study as far as was desirable; however the tentative conclusions indicate that optimum conditions for coasting correspond to a long, slightly inclined coasting trajectory during which altitude is gained surprisingly slowly, followed by a final stage of rocket power during which very little tilt is used.

The equations governing the motion of the vehicle during its acceleration along the trajectory are derived in Appendix C. They are

$$\frac{dV}{dt} = -g\left(1 - \frac{2h}{R}\right)\sin\theta + \frac{T}{m}\cos\alpha - \frac{D}{m},$$

$$\frac{d\theta}{dt} = \frac{V}{R}\left(1 - \frac{h}{R}\right)\cos\theta + 2\Omega - g\,\frac{\left(1 - \frac{2h}{R}\right)\cos\theta}{V} + \frac{T\sin\alpha}{mV}$$

where V is the velocity along the flight path,

T is the rocket thrust,

D is the drag,

m is the mass of the vehicle,

h is the altitude above the earth,

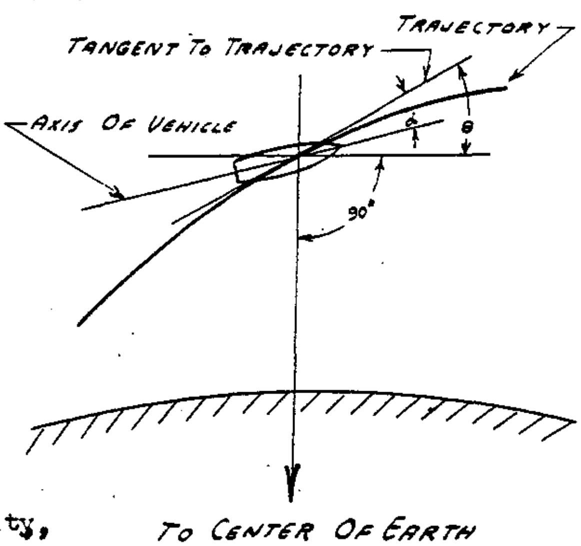
R is the radius of the earth,

Ω is the angular velocity of the earth,

t is the time,

g is the acceleration of gravity,

θ and α are angles explained in the figure



Chapter 9

It is impossible to obtain explicit analytic solutions of these equations for the cases we are considering. Instead we resorted to a step by step method of solution in which the intervals were chosen with sufficient care to insure that an accuracy of better than 1% was maintained in the final values. A detailed set of sample calculations is give in Appendix D.

When the desired altitude was 100 miles, the calculations were carried out in the following manner:

Four Stage Alcohol-Oxygen Rocket-No Coasting - The calculations were made for each of three ratios of fuel weight to gross weight so that by interpolation, the amount of fuel necessary to attain the correct final velocity could be predicted. The trajectory for the first half(in time) of the first stage is taken as a vertical path. At this point a constant angle of tilt is applied and this is carried through to the end of the second stage. This calculation was carried out for each of three fixed angles of tilt, so that the results could be interpolated for any intermediate tilt. In the meantime, an independent set of calculations had been proceeding in which the equations were worked backwards, beginning at the end of the last stage, with the vehicle on the orbit, and computing the reverse history along the trajectory back to the beginning of the third stage, where these calculations were connected up with those proceeding the other way. These reverse calculations were also carried out for three fixed angles of tilt. When all these calculations were complete, cross plots of trajectory inclination and altitude at the junction point were made. From these plots, values of tilts for both sets of calculations could be selected so that the juncture was continuous for both altitude and inclination. It will be remembered that each of these sets of calculations had been made for a series of fuel weight ratios. The final results were cross plotted so that the velocity at the juncture was also continuous.

In Figures 1 and 2 are shown the flight characteristics and trajectory for our proposed design of an alcohol-oxygen rocket. It will be noticed, that for the particular weight ratios used in this design, the final altitude was 165 miles. For this altitude, the tilt required during the last two burning periods was 35°, which was so large that significant losses in effective thrust occured.

Two Stage Hydrogen-Oxygen Rocket - No Coasting - The method used for the hydrogen-oxygen rocket was substantially the same as that described above. However, when it became apparent from the alcohol-oxygen rocket results that it would be impossible to reach altitudes of 300 miles without the use of excessive tilt, further effort along these lines was discontinued. Instead, attention was concentrated on the use of coasting as a more efficient means of obtaining altitude. The calculations were revised as follows:

Four Stage Alcohol-Oxygen Rocket - Coasting - Instead of the juncture occurring at the end of the second stage, it was now placed at the beginning of the fourth stage. A constant angle of tilt was maintained from the middle of the first stage to the end of the third stage. This was followed by a variable amount of coasting. A set of calculations, working backward from the end of the last stage, was made with several fixed values of tilt. A sufficient number of valmes of all parameters

FORM 25 BF

Analysis PRELIMINARY DESIGN OF SATELLITE VEHICLE

Prepared by . . Kraeger, . . Gunkel

Date May 2, 1946

DOUGLAS AIRCRAFT COMPANY, INC.

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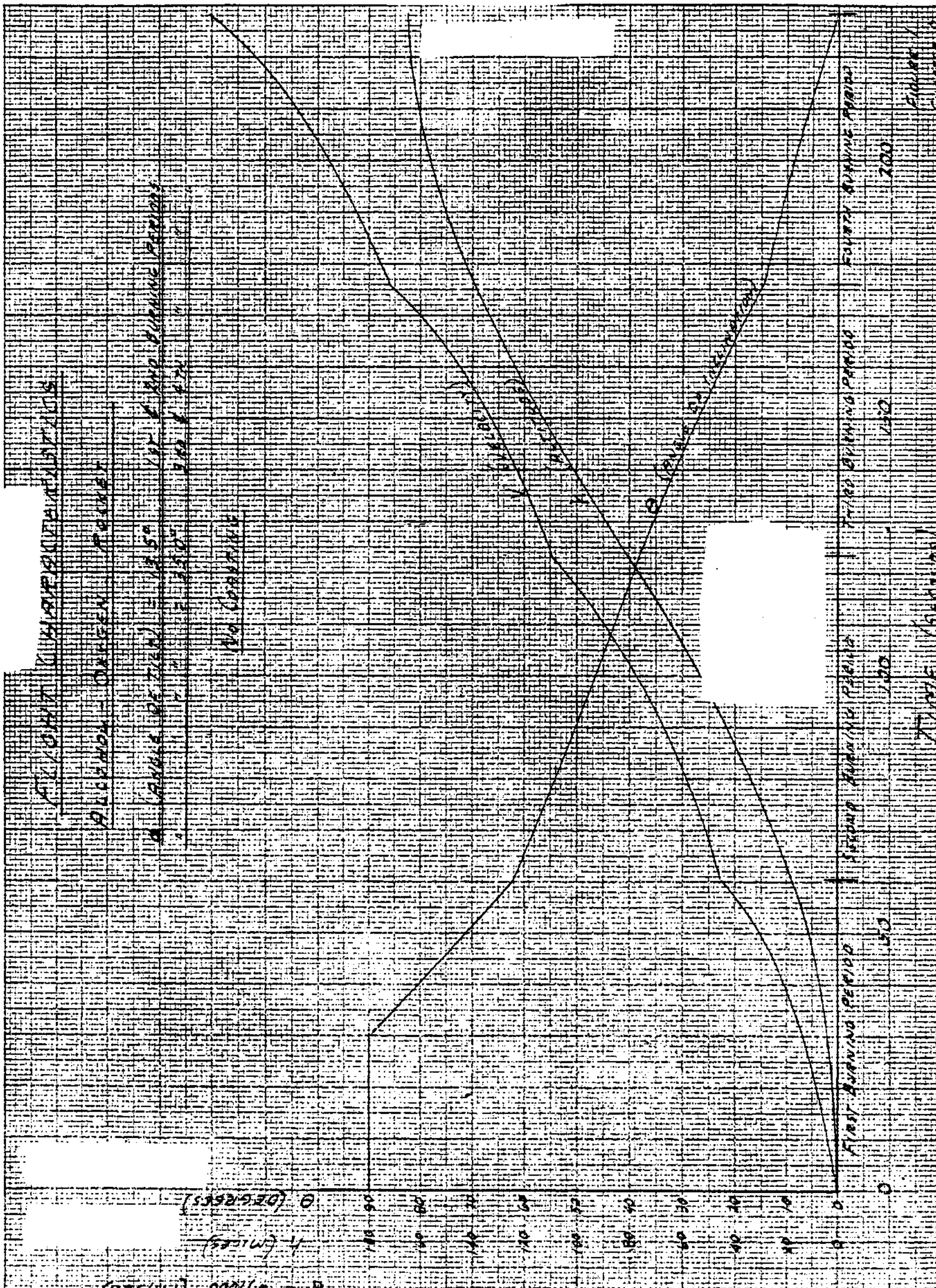
Report No. 11827



FORM 25 BP

Analysis PRELIMINARY DESIGN OF SATELLITE VEHICLE

Prepared by [illegible] F. H. Clauser

Date [illegible] 1946

DOUGLAS AIRCRAFT COMPANY, INC.

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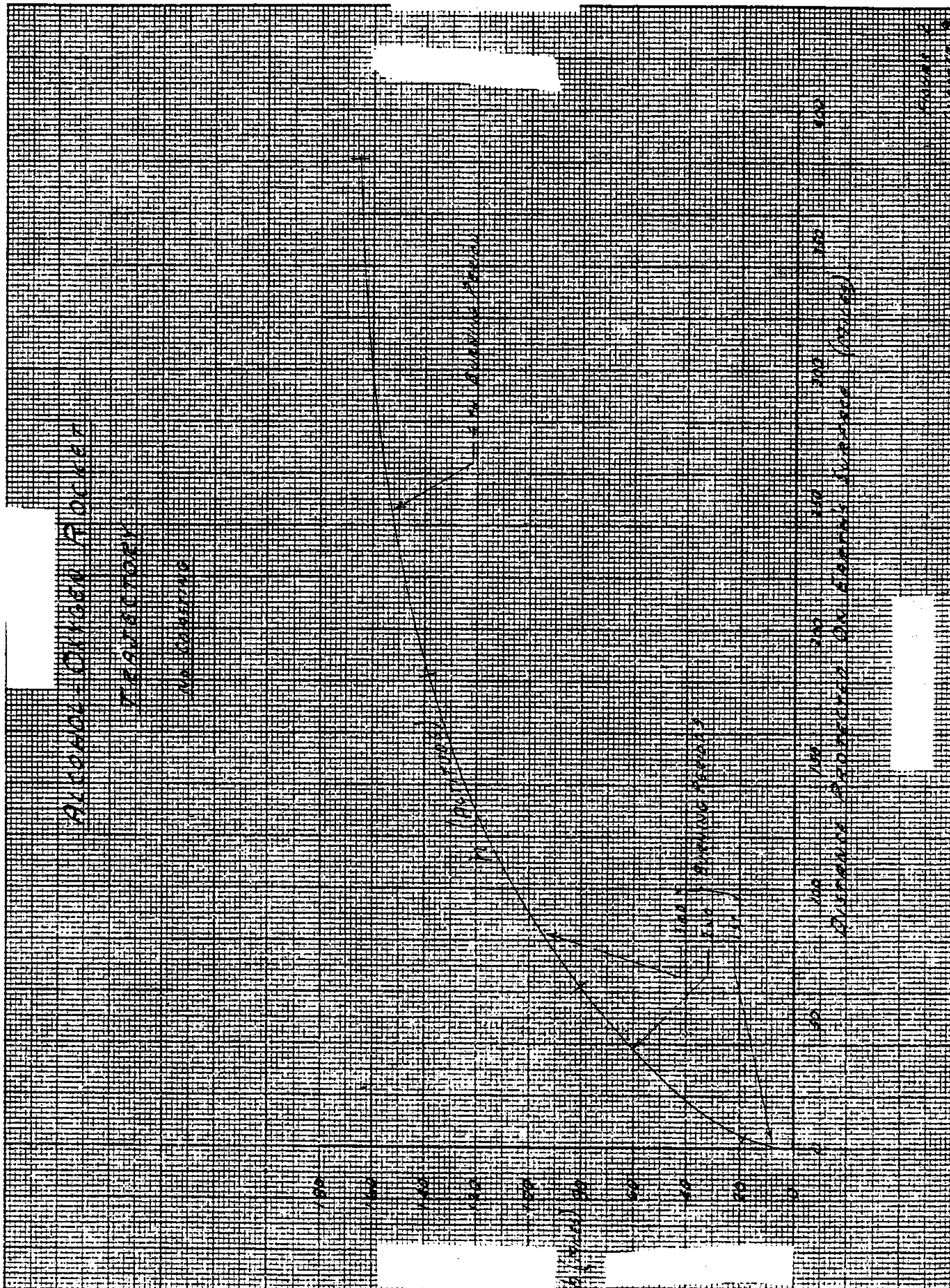
Model [illegible]

Report No. [illegible]



was used so that a continuous juncture could be made at the beginning of the fourth stage. Since a variable amount of coasting has been added to the other variations possible, the choice of values to affect a smooth juncture is not unique. Although time was not available for an exhaustive investigation, it is believed that the optimum trajectory is that discussed a few paragraphs above. The results of these calculations for our proposed alcohol-oxygen rocket are shown in figures 3 and 4. It will be seen that the introduction of coasting has increased the altitude to approximately 480 miles. The greatest angle of tilt required was only 13.5°.

Two Stage Hydrogen-Oxygen Rocket.- For this case, the coasting was inserted between the two stages. In other details, it was the same as the alcohol-oxygen rocket. The results are shown in figures 5 and 6. It will be seen that for the particular weight ratios chosen, the fuel was insufficient to give an altitude greater than 150 miles even with the greater efficiency obtainable from coasting. To achieve an altitude of 400 miles, it would have been necessary to approximately double the weight of the vehicle. The reason for this is not that the altitude has a large effect on performance, but that, with two stages, the hydrogen-oxygen rocket is so far from being an optimum design that the gross weight is highly sensitive to changes in performance requirements. A three stage vehicle would have shown substantially superior performance and weight figures.

Analysis PRELIMINARY DESIGN OF SATELLITE VEHICLE

Prepared by R. W. Krueger

Date May 2, 1946 F. H. Clauser

DOUGLAS AIRCRAFT COMPANY, INC.

Plant

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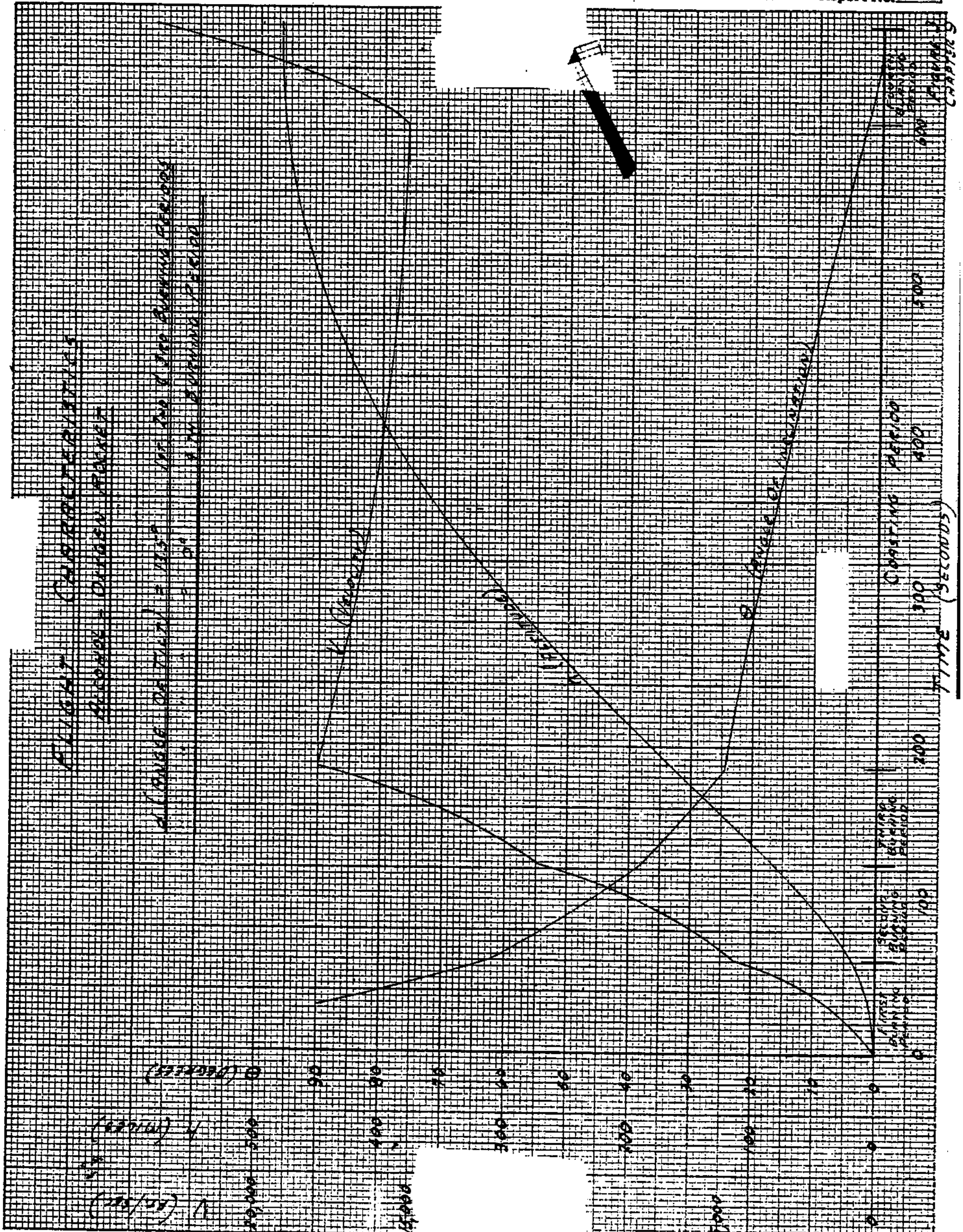
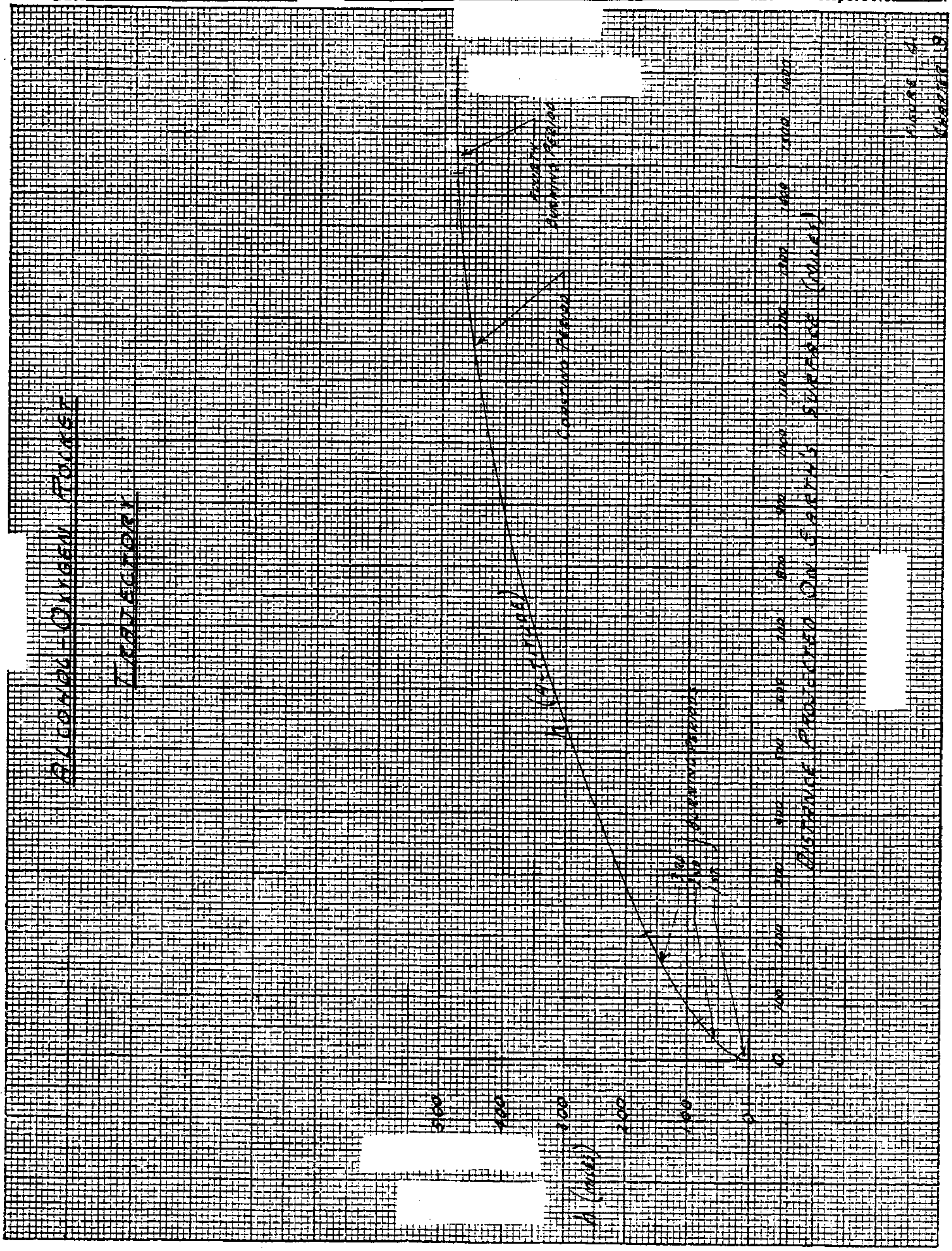
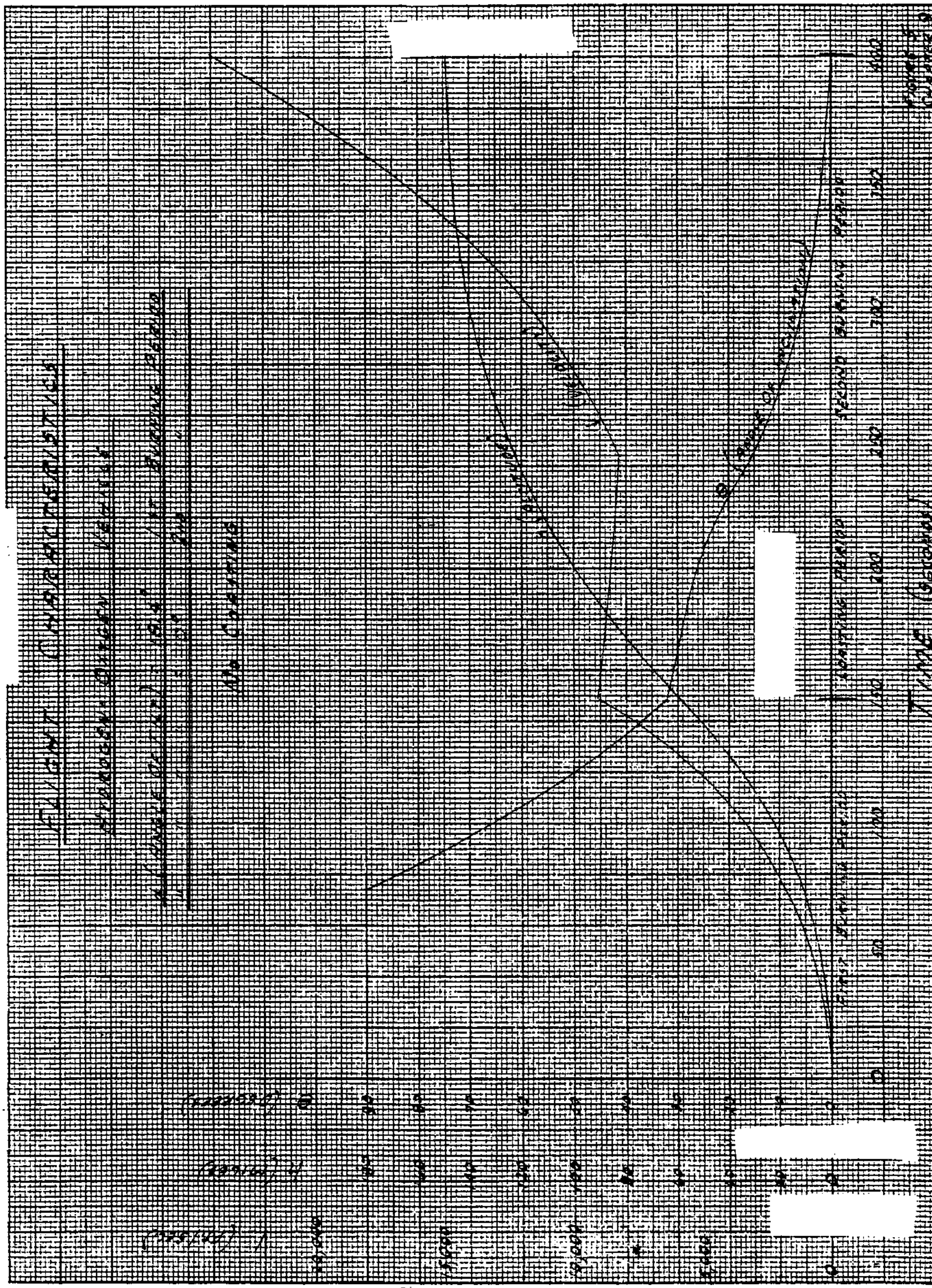
Model #1033

Report No. 11827




ALCOHOL-OXYGEN ROCKET
TRAJECTORY
FOURTH BURNING PERIOD
COASTING PERIOD
h (ALTITUDE)
BURNING PERIODS
DISTANCE PROJECTED ON EARTH'S SURFACE (MILES)
h (MILES)
0
100
200
300
400
500
FIGURE 4


FLIGHT CHARACTERISTICS
NO COASTING
TIME (SECONDS)
0
50
100
150
200
250
300
350
400

FORM 25 BS (REV. 7-42)

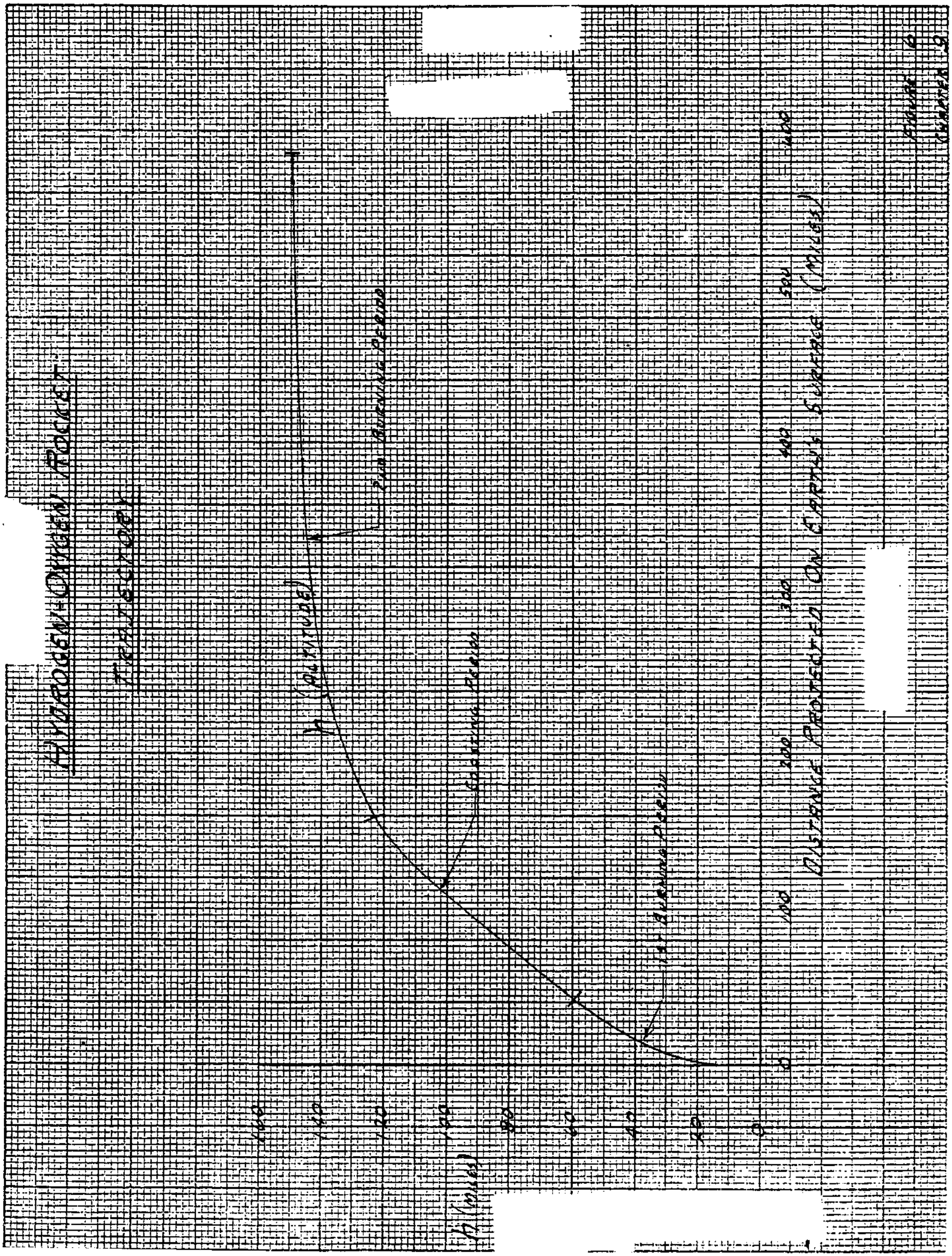
Analysis PRELIMINARY DESIGN OF SATELLITE VEHICLE

Prepared by R. W. Krueger F. H. Clauser

Date May 2, 1946

DOUGLAS AIRCRAFT COMPANY, INC.

Santa Monica Plant



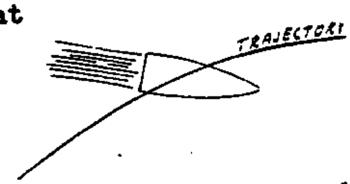
Chapter 10

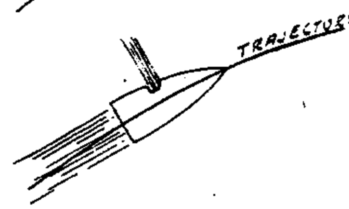
10. METHOD OF GUIDING VEHICLE ON TRAJECTORY

Up to this point, our analysis has considered the design of a vehicle and the selection of its trajectory without regard to the means of guidance to insure that the vehicle follows the prescribed trajectory. In the following paragraphs attention will be devoted to this guidance problem.

In the latter three quarters of the trajectory, the density of the air is so low that in spite of the very great speeds, the dynamic pressures are incapable of adequately guiding the vehicle. Consequently, we are led to the conclusion that we must use reaction motors to obtain forces for guidance.

Two means of obtaining such forces are at once apparent. In the first, the vehicle is rotated (e.g. by means of vanes in the main rocket stream) so that a component of the main rocket thrust is applied in the desired direction. In the second, a small auxiliary rocket oriented normal to the axis of the vehicle is used to obtained the desired guidance forces (several such rockets would have to be provided for control in all directions).





If c is the exhaust velocity available from rockets, T the thrust along the trajectory and L the guidance force normal to the trajectory, then in the first case

$$T = c \frac{dm}{dt} \cos\alpha ,$$

$$L = c \frac{dm}{dt} \sin \alpha,$$

where $\frac{dm}{dt}$ is the mass ejection rate of the main rocket and α is the angle between the trajectory and the vehicle axis. Eliminating α, we have

$$\frac{c\frac{dm}{dt}}{T} = \frac{\frac{dm}{dt}}{\left(\frac{dm}{dt}\right)_o} = \sqrt{1 + \left(\frac{L}{T}\right)^2},$$

where $\left(\frac{dm}{dt}\right)_o$ is the rocket fuel consumption required to produce the thrust if the guidance force were not present.

For the second case, when a small auxiliary rocket is used,

$$T = c \frac{dm}{dt}1,$$

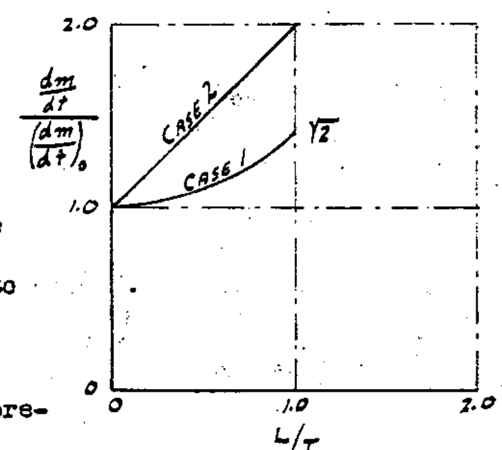
$$L = c \frac{dm}{dt}2,$$

and

$$\frac{\frac{dm}{dt}}{\left(\frac{dm}{dt}\right)_o} = 1 + \frac{L}{T}$$

where $\frac{dm}{dt}1$, $\frac{dm}{dt}2$ and $\frac{dm}{dt}$ are the consumptions of the main rocket, the auxiliary rocket and the total.

In the adjoining figure, the ratio of the consumptions with and without guidance force have been plotted against the ratio of guidance force to thrust. It is at once apparent that case 1 is markedly superior to case 2. In fact with case 1, substantial guidance forces may be obtained without appreciable penalty in thrust. Case 1 will be the method of guidance considered in what follows.



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If the orbit is not entered with precision it becomes necessary to apply corrections to flight path angle and velocity after the starting trajectory has been completed.

It can easily be demonstrated that, if a thrust is applied normal to the flight path in order to correct the angle, then

$$\frac{\Delta W_{fuel}}{W} = \left(\frac{V}{c}\right) \theta,$$

where

ΔW_{fuel} = fuel weight required,

W = gross weight,

V = flight velocity,

c = exhaust velocity,

θ = change in angle measured in radians.

$\frac{V}{c}$ is approximately equal to 3, so for a one degree correction of angle a weight of fuel equal to 5% of the gross weight is required.

Similarly it can be shown that if resultant velocity is to be corrected, then

$$\frac{\Delta W_{fuel}}{W} = \frac{\Delta V}{c}$$

where ΔV = the velocity increment. A 1% change in resultant velocity requires a fuel weight equal to 3% of the gross weight.

Since these corrections optimistically assume normal rocket efficiency for short period operation, it is evidently very costly in fuel to make corrections of any magnitude.

As an alternative to applying corrections assume that an eccentric orbit can be tolerated, if the eccentricity can be kept within certain specified limits. It is then necessary to know the relationships existing between the various orbital parameters. These parameters are as follows:

V_o = the correct velocity for a circular orbit at the starting altitude,

ΔV = the velocity increment above V_o,

β_o = the allowable variation from horizontal (measured positively downward in radians) of the path at the starting altitude,

R_o = the starting radius from the center of the earth,

Δh = the allowable drop in altitude from the starting point,

ΔH_{max} = the maximum minus the minimum distance of the orbit from the earth's surface.

Fig. 1 gives the relationships between Δh, ΔH_{max}, β_o and $\frac{\Delta V}{V_o}$. The curves have been plotted from exact equations but for small deviations in angle and velocity the following approximate relations can be used:

$$\beta_o \doteq \sqrt{\left(\frac{\Delta h}{R_o}\right)\left[\left(\frac{\Delta h}{R_o}\right) + 4\left(\frac{\Delta V}{V_o}\right)\right]},$$

or

$$\left(\frac{\Delta h}{R_o}\right) \doteq -\frac{2\Delta V}{V_o} + \sqrt{4\left(\frac{\Delta V}{V_o}\right)^2 + \left(\beta_o\right)^2},$$

or

$$\left(\frac{\Delta H_{max}}{R_o}\right) \doteq 2\sqrt{4\left(\frac{\Delta V}{V_o}\right)^2 + \left(\beta_o\right)^2},$$

or

$$\left(\frac{\Delta H_{max}}{R_o}\right) \doteq 2\left[\left(\frac{\Delta h}{R_o}\right) + 2\left(\frac{\Delta V}{V_o}\right)\right].$$

Assume that the altitude of the vehicle should not drop more that 50 miles below the design starting altitude to be attained at the end

FORM 25 BS

Analysis SATELLITE VEHICLE

Prepared by C. V. STURDEVANT

Date 5 - 3 - 46

DOUGLAS AIRCRAFT COMPANY, INC.

SANTA MONICA Plant

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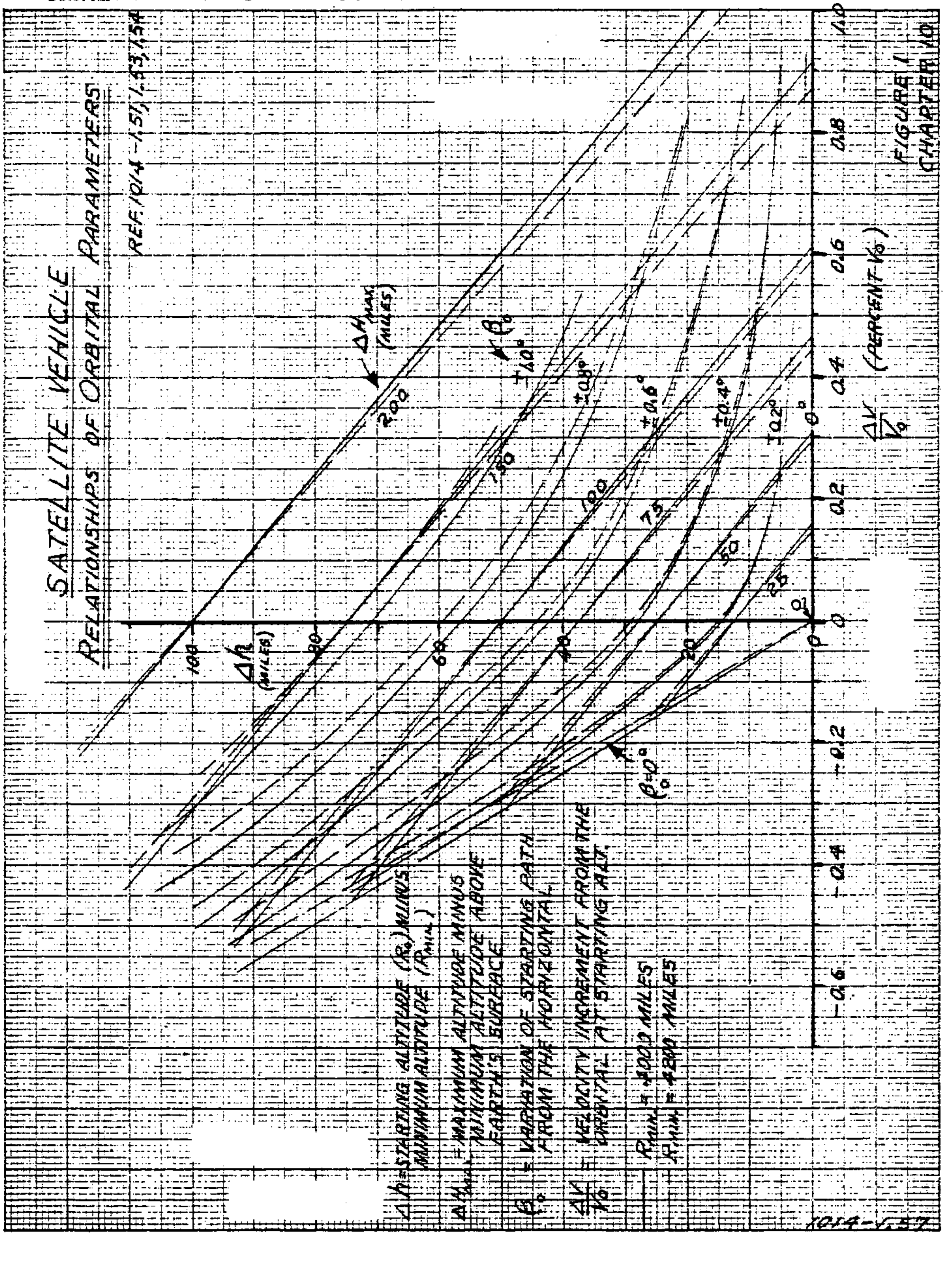
Model #1033

Report No. SM 11827



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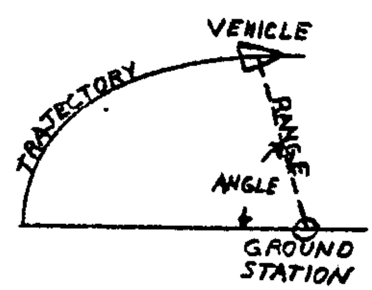
of the starting trajectory, and that the total variation of altitude should not be more than 100 miles. Then if the starting angle is exactly correct ($\beta_o = 0$) the starting velocity may drop 0.3% below the correct value or rise 0.6% above it. However, if the angle can only be established to within $\pm$ 1/2 degree the velocity must fall within the range -0.15% to + 0.40%. The above limits are somewhat arbitrary but illustrate the orders of magnitude involved.

The evident necessity for maintaining close tolerances on the starting conditions for the orbit leads to a preliminary investigation of stability and control requirements.

In the latter part of the starting trajectory a direct control of flight path angle is desirable since this angle must be maintained with extreme accuracy. This requires a restoring moment proportional to deviation of flight direction from the horizontal. It is anticipated that this deviation can be measured by means of a radar equipped ground station which measures both range and angle, computes rate of change of altitude and sends a corresponding control impulse to the vehicle. A beacon will be used in the vehicle to act as a "transponder" and can also be used to convey information from the vehicle to the ground.



In addition to the restoring moment proportional to deviation of flight direction from the horizontal it is necessary to apply either a damping moment depending on flight direction or a restoring moment depending on pitch. The latter would probably be simpler to use. To correct for an unknown eccentric thrust (so that the vehicle will not only be stable but also approach the exact flight angle desired) it is also necessary to apply a restoring moment proportional to the integral of flight direction over a period of time. This corresponds actually to an altitude control. Use of integral terms in control problems has been discussed by Weiss* and such a control is used on constant speed propellers.

From stability considerations it can be shown that the restoring moment depending on pitch (or damping moment depending on flight direction, whichever is used) should be large. However, further investigation is required to determine desirable magnitudes for the other terms.

Velocity control can probably be obtained by using an integrating accelerometer which operates a fuel cut-off valve. Such an accelerometer was used on the V-2 with accuracy of 1/2 % over a 60 second period, and it is believed that this accuracy can be improved. At present radar techniques involving radar ranging or Doppler effect do not appear to offer adequate accuracy.

In the early stages of the starting trajectory it should be sufficient to control pitch as a predetermined function of time. For stability, restoring and damping moments can be applied as functions of the difference between actual and desired pitch angles (determined with the aid of a pre-set gyro). In order to approach the desired trim condition a moment should also be applied as a function of the integral of deviation in pitch angle over a period of time, and it may also be necessary to apply a predetermined moment as a function of time to compensate for the calculated curvature of the trajectory.

* "Dynamics of Constant-Speed Propellers" Herbert K. Weiss, Journal of the Aeronautical Sciences, Vol. 10, No. 2, Feb. 1943.

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The areas of the control surfaces in the jet have been briefly studied. The control moments required depend upon the angular accelerations required and upon the fixed disturbing moment, such as that due to a displaced thrust line. The control surface areas shown on the drawings are quite arbitrary but serve to show, in conjunction with figure 2 that design of sufficiently powerful surfaces should not be difficult. This figure shows the variation of control surface area with displacement of the thrust line from the center of gravity in inches for each of the four stages. Actually the maximum error is expected to result from a rotation of the thrust line about the throat of the engine by about 0.5 deg. The resultant displacement is indicated in the figure as the maximum probable displacement. Also shown are the areas required to produce a pitching or yawing acceleration of 10 deg./sec^2. It is seen that the areas shown in the drawings are adequate to overcome the moment due to the maximum probable thrust line displacement and to produce in addition a 10 deg./sec^2 acceleration if a 10 deg. control surface angle is used. Since these areas are far from excessive from a mechanical standpoint, they could be increased if more thorough control studies showed the desirability of obtaining higher angular accelerations.

Analysis CONTROL
Prepared by R.S. SHEVELL
Date 5-7-46

DOUGLAS AIRCRAFT COMPANY, INC.

S.M. Plant

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CONTROL SURFACE AREA REQUIRED TO BALANCE DISPLACEMENT OF THE THRUST LINE

CONTROL SURFACE ANGLE = 10 DEG.

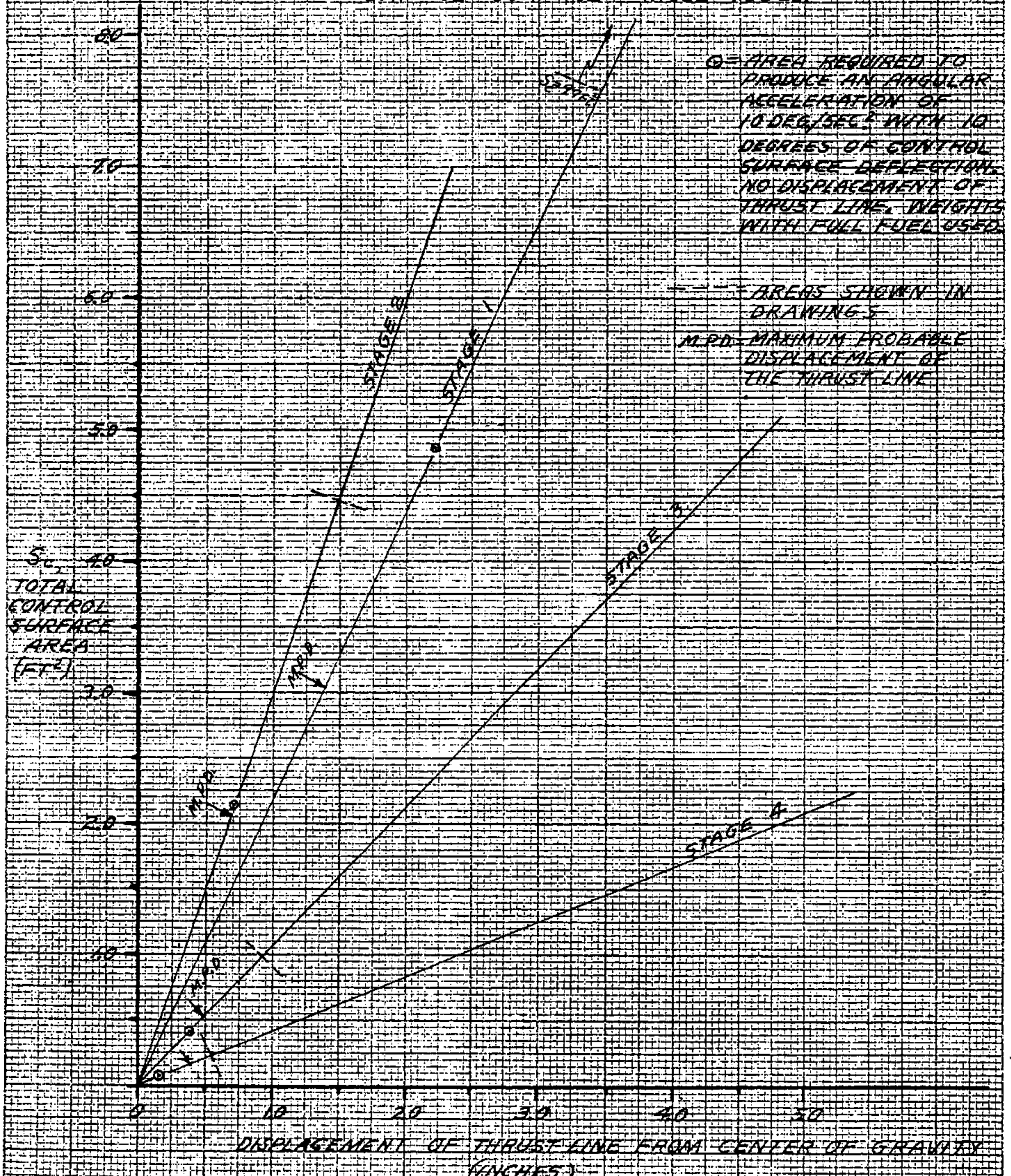

FIGURE 2
CHAPTER III

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The preceding investigations are preliminary only, but suggest the following:

1. It is desirable to rotate the entire vehicle to obtain a component of the jet thrust for purposes of control.
2. Corrections on the orbit are undesirable after it has once been established
3. At the end of the starting trajectory the flight path angle should be accurate to $\pm \frac{1}{2}$ degree and the velocity should be accurate to $\pm$ 0.3%.
4. This accuracy and the necessary stability can probably be attained.

Chapter 11

11. PROBLEMS AFTER ORBIT IS ESTABLISHED

Once the vehicle has been established in its orbit at the desired altitude various other problems arise in connection with the satisfactory operation of the vehicle. For example, the vehicle will be constantly exposed to the possibility of being hit by meteorites of all sizes and some of which will be travelling at very high speed. Also, at such high altitudes the intense heating of the vehicle by the sun is a problem to be considered. Radio contact must be maintained. These and other problems connected with the satisfactory operation of the vehicle are discussed below.

Meteorites. (The Probability of a Meteorite Hitting a Satellite Vehicle Traveling in the Upper Atmosphere.) It is well known that a great many meteorites enter the earth's atmosphere each day. If a body should be situated in the upper atmosphere at altitudes where meteorites are observed with high frequency, the question arises as to what are the chances that the body will be struck by a meteorite and if a strike does occur what are the probabilities that the meteorite will seriously damage or otherwise interfere with the motion of the body.

Meteorites* are discrete masses of matter from outer space which enter the earth's atmosphere. Judging from those which are large enough to survive the journey through the air and reach the ground, and which are then called fallen meteorites, they are composed mainly

* Leonard, F. C.: Meteorites: Immigrants From Space. Publications of the Astronomical Society of the Pacific, Vol. 57, No. 334; p. 1, Feb. 1945.

of stony matter (similar to igneous rock) and metallic nickeliferous iron. Like fallen meteorites, the relative amounts of iron and stony matter in the meteorites may be expected to vary greatly ranging from almost all iron to almost all stone. However, it is quite likely that the stony meteorites are more prevalent than iron meteorites by a factor of more than ten, although a stony meteorite itself may contain some 25 percent iron by mass. It will be assumed that the meteorites consist mainly of stony matter.

Meteorites vary greatly in size ranging from something smaller than a pin head or grain of sand up to the large meteoritic masses found on the earth which weigh 10 or 20 tons or more. (According to Leonard, Reference 1, meteorites may be of any magnitude whatever, from the size of the tiniest solid particle to that of a mass of planetary dimensions, and are the smallest discrete astronomical bodies. The term meteor is properly used to denote the luminous phenomenon which results from the motion of a meteorite through the earth's atmosphere.) It is estimated that the weight of the average fallen meteorite is 220 pounds before entering the atmosphere and that this is reduced to about 44 pounds by the time the earth's surface is reached. However, meteorites which are large enough to reach the earth's surface occur with such low frequency, 5 or 6 a day for the whole earth, that they need not be considered here.

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Typical values of velocity and altitude as determined by observations of certain bright meteors are given in Table 1 which is taken from Hoffmeister.(2)

As one might expect, it is seen from Table I that the higher the meteorite makes its appearance the greater is its velocity. Velocities ranging from 80,000 to 250,000 ft. per sec. are quite common. According to Watson(3) most meteors appear at a height of about 300,000 feet regardless of their brightness and may be taken to have an average atmospheric velocity of about 150,000 feet per second. Thus, at an altitude of 500,000 feet, where the body is assumed to be situated, most all of the meteorites will be intact and will not have suffered complete dissipation. At this altitude the body will therefore be exposed to practically all of the meteorites which enter the atmosphere.

The number, size and mass of meteorites entering the atmosphere each day is given in Table 2 which is based on a table given by Watson (ibid., p. 115).

The visual magnitude of a meteor is expressed in terms of a scale in which numerically large magnitudes represent faint bodies. Two meteors which differ by five magnitudes have a hundred-fold difference in brightness and, since the brightness is directly proportional to

(2) Hoffmeister, C.: Die Meteore. Probleme Der Kosmischen Physik, Band XVII; p. 71, 1937.

(3) Watson, F. G.: Between the Planets. The Blakiston Company, Philadelphia; p. 93, 1945.

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TABLE 1

VELOCITY AND ALTITUDE OF BRIGHT METEORS

(From Hoffmeister, Ref. 2)

Velocity		Mean Height of Appearance			Mean Height of Disappearance		
km/sec.	ft./sec.	km	ft.	Number	km	ft.	Number
10	32,810	66	216,000	4	28	91,900	4
20	65,600	92	302,000	35	44	144,200	34
30	98,500	112	368,000	93	47	154,000	93
40	131,200	131	430,000	107	47	154,000	107
50	164,000	144	472,000	74	47	154,000	75
60	196,800	138	452,500	54	55	180,500	54
70	229,600	149	489,000	31	63	206,500	31
80	262,500	183	600,000	11	74	243,000	12
90	295,000	197	645,000	15	98	321,500	15
100	328,100	217	712,000	3	96	315,000	3
110	361,000	226	741,000	6	136	446,000	6

1 km. = 3,281 ft.

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TABLE 2

THE NUMBER, MASS, AND SIZE OF METEORITES ENTERING THE ATMOSPHERE EACH DAY

(Based on Watson,

Visual Magnitude	Observed Number	True Number N	Mass Grams	Weight lbs. w	Diameter of Equivalent Sphere, Ft.* d
-3	28,000	28,000	4.0	8.72×10^{-3}	$.427 \times 10^{-1}$
-2	71,000	71,000	1.6	3.53×10^{-3}	$.317 \times 10^{-1}$
-1	180,000	180,000	.630	1.39×10^{-3}	$.232 \times 10^{-1}$
0	450,000	450,000	.250	5.51×10^{-4}	$.1705 \times 10^{-1}$
1	1,100,000	1,100,000	.100	2.20×10^{-4}	1.257×10^{-2}
2	2,800,000	2,800,000	.040	8.72×10^{-5}	$.922 \times 10^{-2}$
3	6,400,000	7,100,000	.016	3.53×10^{-5}	$.683 \times 10^{-2}$
4	9,000,000	18,000,000	.0063	1.39×10^{-5}	$.500 \times 10^{-2}$
5	3,600,000	45,000,000	.0025	5.51×10^{-6}	$.367 \times 10^{-2}$
6		110×10^{6}	.0010	2.20×10^{-6}	$.2705 \times 10^{-2}$
7		280×10^{6}	.00040	8.72×10^{-7}	$.1986 \times 10^{-2}$
8		710×10^{6}	.00016	3.53×10^{-7}	1.471×10^{-3}
9		18×10^{8}	.000063	1.39×10^{-7}	1.078×10^{-3}
10		45×10^{8}	.000025	5.51×10^{-8}	$.793 \times 10^{-3}$
15		45×10^{10}	2.5×10^{-7}	5.51×10^{-10}	1.705×10^{-4}
20		45×10^{12}	2.5×10^{-9}	5.51×10^{-12}	$.367 \times 10^{-4}$
25		45×10^{14}	2.5×10^{-11}	5.51×10^{-14}	$.793 \times 10^{-5}$
30		45×10^{16}	2.5×10^{-13}	5.51×10^{-16}	1.705×10^{-6}

lbs. = grams $\times 2.205 \times 10^{-3}$

*Based on a specific gravity of 3.4.

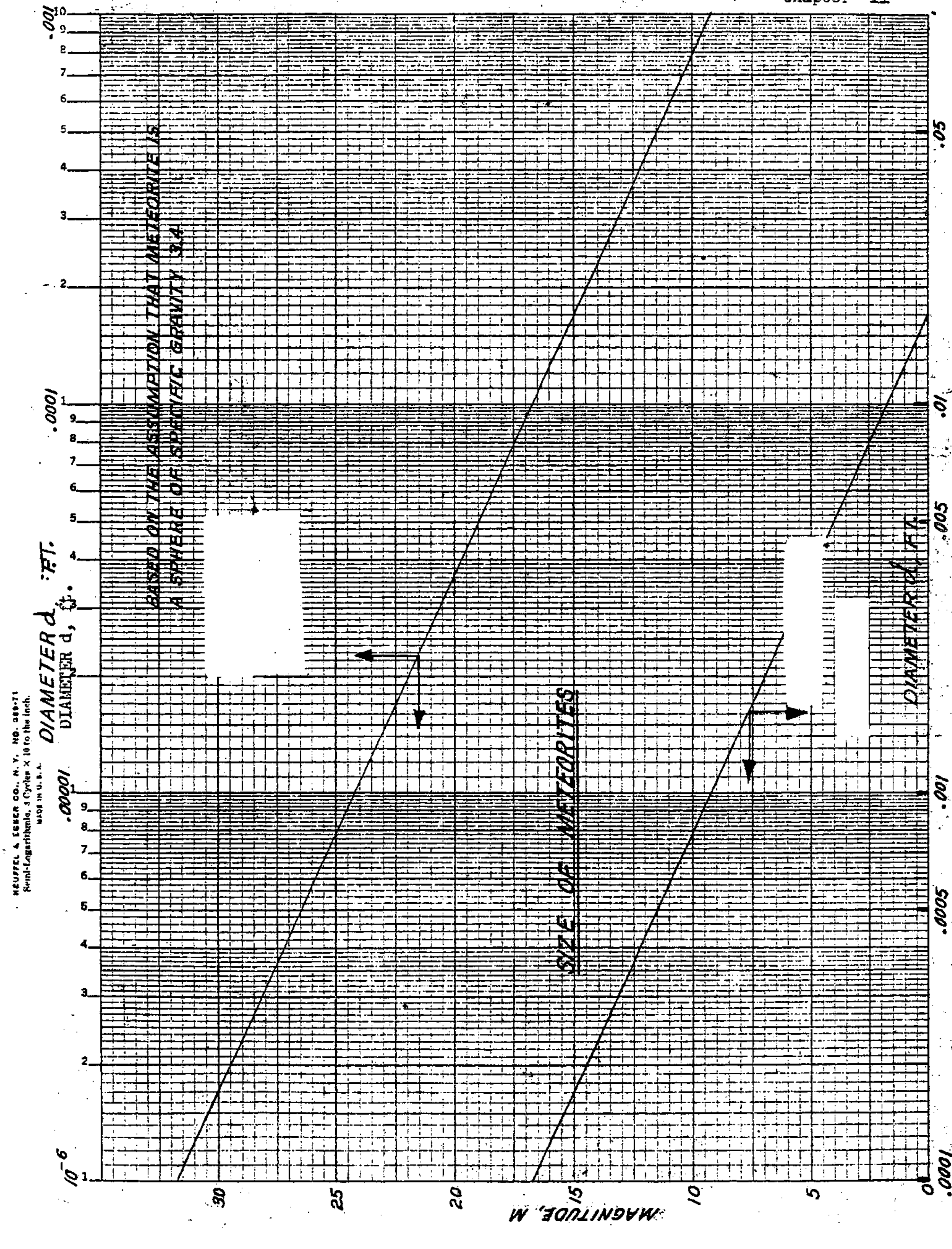
the mass, they represent a hundred-fold difference in mass. A meteor just visible to the naked eye has a magnitude of 5 while the full moon has a magnitude of -14. Also, it will be noticed from Table 2 that when the magnitude differs by five units, the number of meteors changes by a factor of 100. In this way the table may be extended to include smaller and smaller meteorites (numerically larger magnitudes).

However, there is a limiting magnitude beyond which there can be few, if any, meteorites, and according to Watson, p. 116, this limiting size is a meteorite of magnitude 30. This is explained by the fact that for a particle smaller than this the solar radiation pressure is sufficient to repel any particle to such an extent that it could not remain in the solar system.

In Table 2, figures are included for meteorites down to the smallest possible size, magnitude 30. The sizes have been computed on the basis that the meteorite is a sphere composed mainly of stony matter which according to Whipple,[(4)] has a specific gravity of 3.4. The variation of size with magnitude is presented in Fig. 1.

It is seen that a great range in size and mass is represented in the table and the question immediately arises, especially for the very small particles, as to what velocities are to be associated with the various sizes. If it is assumed that the meteorites move in parabolic orbits at the same distance from the sun as the earth, a meteorite

(4) Whipple, F. L.: Meteors and the Earth's Upper Atmosphere. Reviews of Modern Physics, Vol. 15, No. 4; p. 252, Oct. 1943.



See Table 2.

Fig 1 Size of Meteorite Corresponding To Meteor Magnitude.

encountering the earth head-on will enter the outer atmosphere with a speed of 250,000 ft. per sec., while one overtaking the earth will enter the atmosphere with a speed of only 43,300 ft. per sec. These speeds would be the same for meteorites of all sizes since there has not yet been any deceleration resulting from air resistance. However, once the meteorite has entered the atmosphere, deceleration must take place and this must certainly be greater for a small meteorite than for a larger one, assuming that both enter the atmosphere with the same speed. Whipple, loc. cit. p. 252 gives as a universally accepted expression for the deceleration,

$$\frac{dV}{dt} = \frac{-\gamma a}{m^{1/3}} \rho V^2, \text{ --------------------- } (1)$$

where V = velocity,

m = mass,

ρ = density of the atmosphere,

$a(m)^{2/3}$ = effective cross sectional area,

γ = non-dimensional form factor depending on the shape of the meteorite but independent of velocity.

For a sphere, $\gamma = 1/3$ and if the density is 3.4, $a = 0.53$.

The rigorous investigation of the deceleration would involve a study of the variations of γ, a, ρ, and especially m, as a function of time or distance. Since time is not available to carry on such a study at the present and since at this stage of the project, approximate values will be satisfactory, we therefore adopt the following approximate method.

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Assume that γ, a, and m are independent of the motion of the meteorite through the air and since values for density and its variation in the very high atmosphere are lacking, replace ρ * by an average or effective value $\bar{\rho}$.

The differential equation may then by written

$$\frac{d\,(V^2)}{V^2} = \frac{2\gamma a\bar{\rho}}{m^{1/3}}\,dh$$

and therefore

$$V_f = V_i\,(e)^{-\frac{\gamma a\bar{\rho}\,\Delta h}{m^{1/3}}} = (e)^{-\frac{k\,\Delta h}{m^{1/3}}} \qquad \text{- - - - - - - - - - -(2)}$$

In this expression V_i is the velocity of the meteorite when it enters the atmosphere, and V_f its velocity after it has fallen a vertical distance Δh through the air. As a maximum condition it will be assumed that the meteorite enters the atmosphere head-on so that its speed is given by $V_i \doteq 47.4$ mi./sec. $\doteq$ 250,000 ft./sec. See p. 9 of Leonard, loc. cit.

From Table 1 it is estimated that bright meteors (about magnitude 2) have a velocity of about 200,000 ft./sec. at 100 miles altitude (528,000 ft.) so that V_f = 200,000 ft./sec. Although the height of the approximate upper limit of the atmosphere is not known, auroral

* At this stage of the analysis, the density values derived in Appendix A were not yet available. However, it is believed that a more exact treatment of the density variation with altitude would not appreciably change the results derived here by the approximate method.

observations indicate the pressure of atmosphere at 1000 km. (622 miles) and this figure will be used to represent the altitude of the effective limit of the atmosphere. Since we are interested in the meteorite velocity at 100 miles altitude,

$$\Delta h = 522 \text{ miles} = 840 \text{ km} = 8.40 \times 10^7 \text{ cm.}$$

For a meteor of magnitude 2 it is found from Table 2 that $m_2 = .040$ gram, and hence $m_2^{1/3} = .343$. The constant k may now be evaluated giving $k = 0.9 \times 10^{-9}$.

It was found in Table 2 that a change of one magnitude corresponded to a change in mass by a factor of 2.5. Thus if all masses are referred to that of a meteor of magnitude 2 one may write

$$m_M = \frac{m_2}{[2.5]^{M-2}} = \frac{.040}{[2.5]^{M-2}}, \quad \text{- - - - - - - - - - - - -(3)}$$

and Eq. (2) may then be written

$$V_M = 2.5 \times 10^5 \, [e]^{-.22 \times [2.5]^{\frac{M-2}{3}}} \text{ ft. per sec., - - - (4)}$$

at 100 miles altitude. This function is plotted in Fig. 2 where it is seen that for magnitudes greater than 5, the velocity decreases rapidly.

Since the vehicle may also operate at altitudes higher than 100 miles, say up to 400 miles, it becomes necessary to obtain an

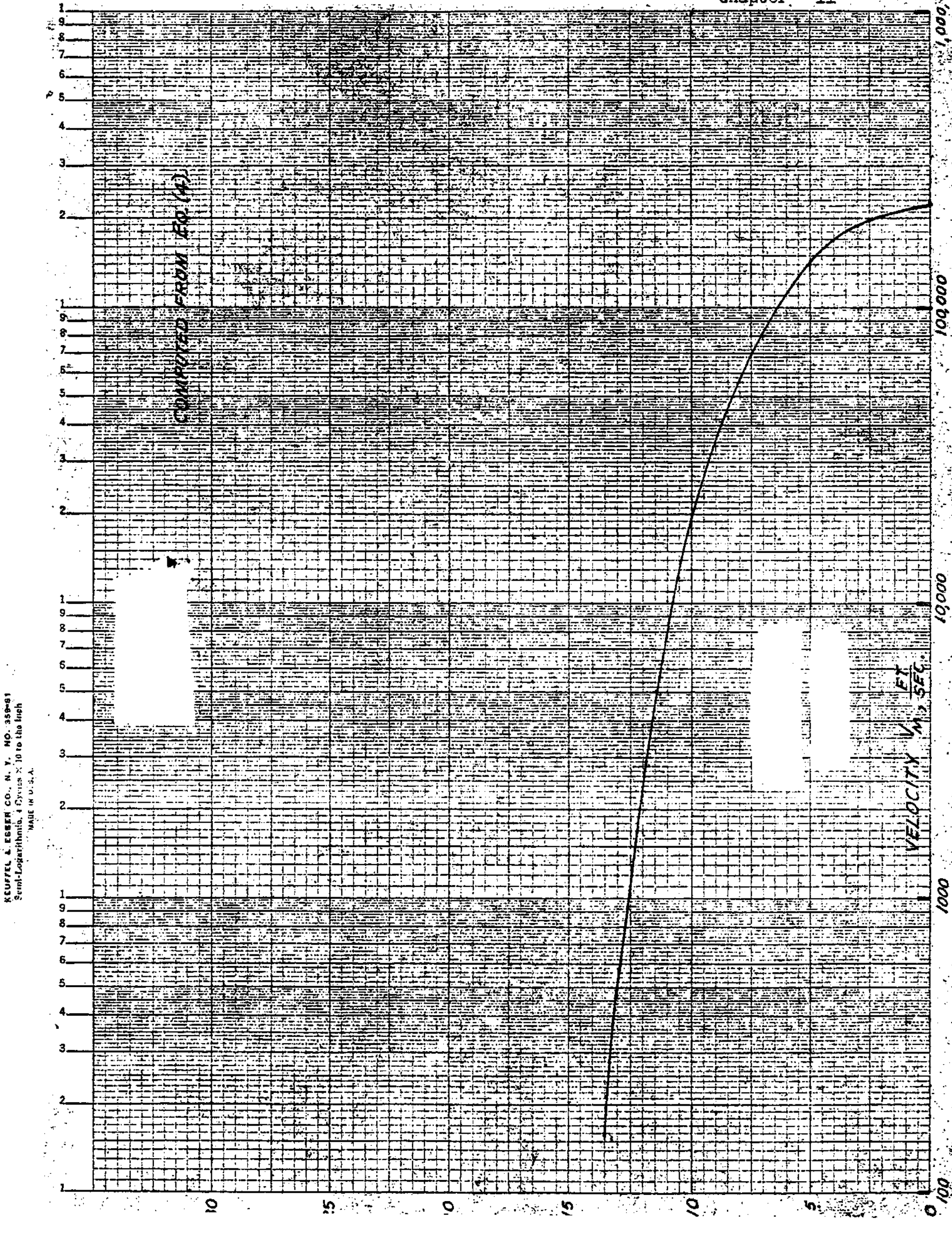

FIG. 9 ESTIMATED METEORITE VELOCITY AT 100 MILES ALTITUDE.

estimate of V_M for this upper altitude limit also. At an altitude as high as 400 miles it is not possible to obtain any estimate of the deceleration from Table 1. Furthermore, the effective density value $\bar{\rho}$ from 400 to 622 miles is certainly much different than that from 100 to 622 miles and therefore the value found above for the deceleration factor k would not apply to this much higher altitude. In fact at such a high altitude it is not entirely unlikely that the deceleration would be negligible. We thus have the two extremes within which the velocity must lie, that of no change in velocity, and that with velocity given by using the value $k = 0.9 \times 10^{-9}$ found above for the 100 mile altitude.

The velocity corresponding to this latter limit is given by the equation

$$V_M = 2.5 \times 10^5 \left[e \right]^{-.094 \times [2.5]^{\frac{M-2}{3}}} , \qquad (4A)$$

corresponding to $\Delta h = 622 - 400 = 222$ miles.

These two extremes are shown in Fig. 2A where the solid curve which has been drawn in to represent a compromise between the two extremes will be used as an estimate of the velocities prevailing at 400 miles altitude. It is seen that at these extremely high altitudes, the smaller meteorites maintain a fairly high velocity.

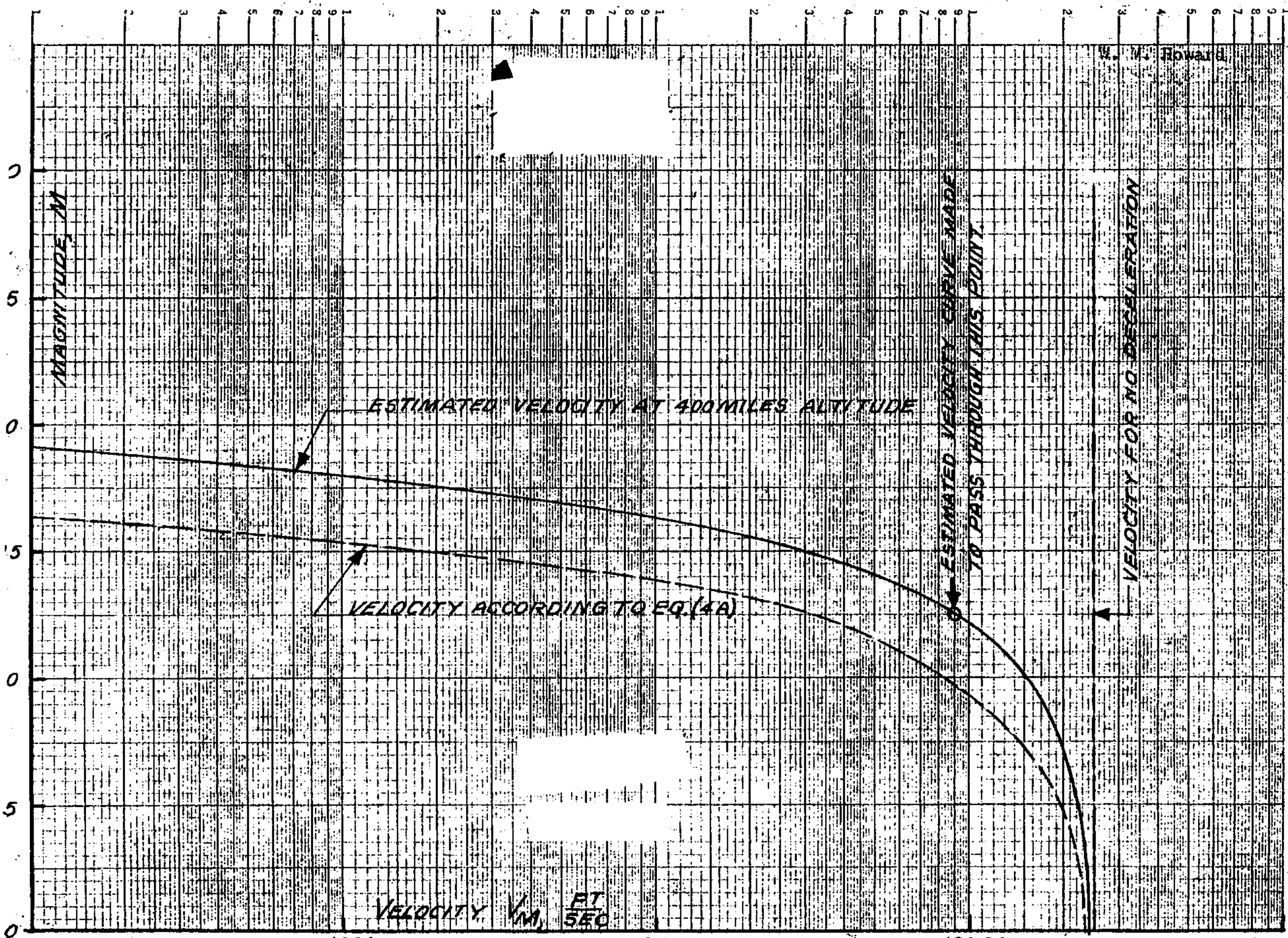

FIG. 2A. ESTIMATED METEORITE VELOCITY AT 400 MILES ALTITUDE.

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Having the relation between $\bar{V}_M$ and M given by Figs. 2 and 2A it is now necessary to determine what is the smallest meteorite which will penetrate through (perforate) the skin of the satellite vehicle. There seems to be very little, if any, information available, either theoretical or experimental, on the penetration of metal plate by very small but extremely high speed particles. Bethe,(5) has apparently worked on this problem to some extent but unfortunately his paper is not available.

However, the indications are that in the case of normal impact of a small but very high speed particle on a metal plate in which the speed of the particle is large compared to the velocity of propagation of plastic deformation in the plate, the particle penetrates as though the plate were perfectly deformable like a fluid. In this case the differential equation for the motion of the particle through the plate is

$$m\frac{dv}{dt} = \frac{m}{2}\frac{d(V^2)}{ds} = -C_D \frac{\rho_o}{2}\pi\frac{d^2}{4}V^2, \quad \text{-------------(5)}$$

where

m = mass of particle $= \frac{4}{3}\pi\frac{d^3}{8}\rho_M$ (assumed spherical),

d = diameter of particle,

s = distance of penetration into the plate,

ρ_o = density of the plate $= 2.8 \frac{gm}{cm^3}$ for dural,

(5): Bethe, H. A.: "Attempt at a Theory of Armor Penetration", Unnumbered Report of the Frankfort Arsenal, 1941.

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ρ_M = density of particle = $3.4 \frac{\text{gram}}{\text{cm}^3}$ for a meteorite,

$\sigma = \frac{\rho_o}{\rho_M} = .825$,

C_D = drag coefficient = 2/3, see Epstein.(6)

The equation becomes

$$\frac{d(V^2)}{V^2} = -\sigma\frac{ds}{d},$$

which when integrated gives

$$s = \frac{d}{\sigma} \log \frac{V_M^2}{V^2},$$

where V_M is the velocity of the particle as it first strikes the plate and V is its velocity after it has penetrated a distance s. When the speed of the particle has dropped to about 5 times the plastic deformation velocity V_1, it will be assumed that this law of penetration is no longer to be used. In the range in which it is to be used the equation is then written

$$s = 2\frac{d}{\sigma} \log \frac{V_M}{5V_1} \text{ - (6)}$$

When the speed of the particle is less than 5 times the plastic deformation speed it will be assumed that the penetration takes place according to one of the armor penetration formulas.

(6): Epstein, P. S.: Proc. Nat. Acad. Sci.. Vol 17, p. 532, 1931.

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The well-known DeMarre armor penetration formula for plain wrought[7] iron is

$$t^{.65} = \frac{.333 \times 10^{-4}\, w^{\frac{1}{2}}\, V}{d^{.75}}, \text{ - - - - - - - - - - - - - - - (7)}$$

where

t = penetration in ft.,

d = diameter of particle in ft.,

w = weight of particle in pounds,

v = velocity of particle in ft./sec.

In this equation, V is the velocity necessary to perforate a thickness t of wrought iron by a particle of weight w and diameter d.

The Watertown Arsenal[7a] uses the formula

$$t = F\,\frac{mV^2}{d^2}, \text{ - (8)}$$

where

t = thickness penetrated,

m = mass of projectile,

F = Thompson coefficient,

d = diameter of projectile,

V = velocity of projectile.

The work of Duwez and Clark[8] on penetration of copper by high

(7): The United States Naval Institute: Naval Ordnance, Annapolis, Md. 1939, p. 301.
(7a): Sullivan, J.F.: An Empirical Approach To The Efficient Design of Armor For Aircraft. Watertown Arsenal Laboratory, Watertown, Mass. Experimental, Report No. WAL 710/506, Jan., 1944, p. 9.
(8): NDRC Report Number M-317

speed small caliber bullets directly verifies the velocity squared relation for speeds of the order of 4000 ft./sec. Their results can be expressed by

$$\frac{t}{d} = 5.9 \times 10^{-7} V^2. \qquad (9)$$

Thus the experimental data give rise to penetration formulas proportional to $V^{1.5}$ or V^2. Since the results of Duwez and Clark are considered the best to use in the present study, a formula corresponding to (9), which refers to copper, will be used in the ballistic range. Since the ultimate strengths of dural and copper are of the same order of magnitude and since the ballistic penetration is closely connected with this property of the metal, it will be assumed that the results for copper also apply to dural within the degree of approximation of the equations.

Eq. (9) was obtained from experiments with 0.224 inch diameter projectiles. If it is assumed that the energy loss per unit penetration is proportional to the frontal area A of the projectile and also constant over the ballistic range (as shown in ref. (8)) it follows that

$$\frac{\Delta KE}{t} = \frac{\frac{1}{2} mV^2}{t} = CA,$$

where ΔKE = change in kinetic energy, and

C = a constant

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In the tests, the projectile weight was 69 grains and was brought to rest in about 2 inches of copper. This gives

$$\frac{1}{2} mV^2 = 13.8 \times 10^3 \ \frac{\text{ft.-lb.}}{\text{ft.}}$$

and

$$C = \frac{\frac{1}{2} mV^2}{tA} = 5.03 \times 10^7 \ \frac{\text{ft.-lb.}}{\text{ft.}^3}.$$

The corresponding penetration formula for a sphere would be

$$\frac{C \pi d^2}{4} = \frac{\frac{1}{2} \rho_M \frac{\pi}{6} d^3 V^2}{t}, \text{ or}$$

$$\frac{t}{d} = \frac{\rho_M}{3C} V^2.$$

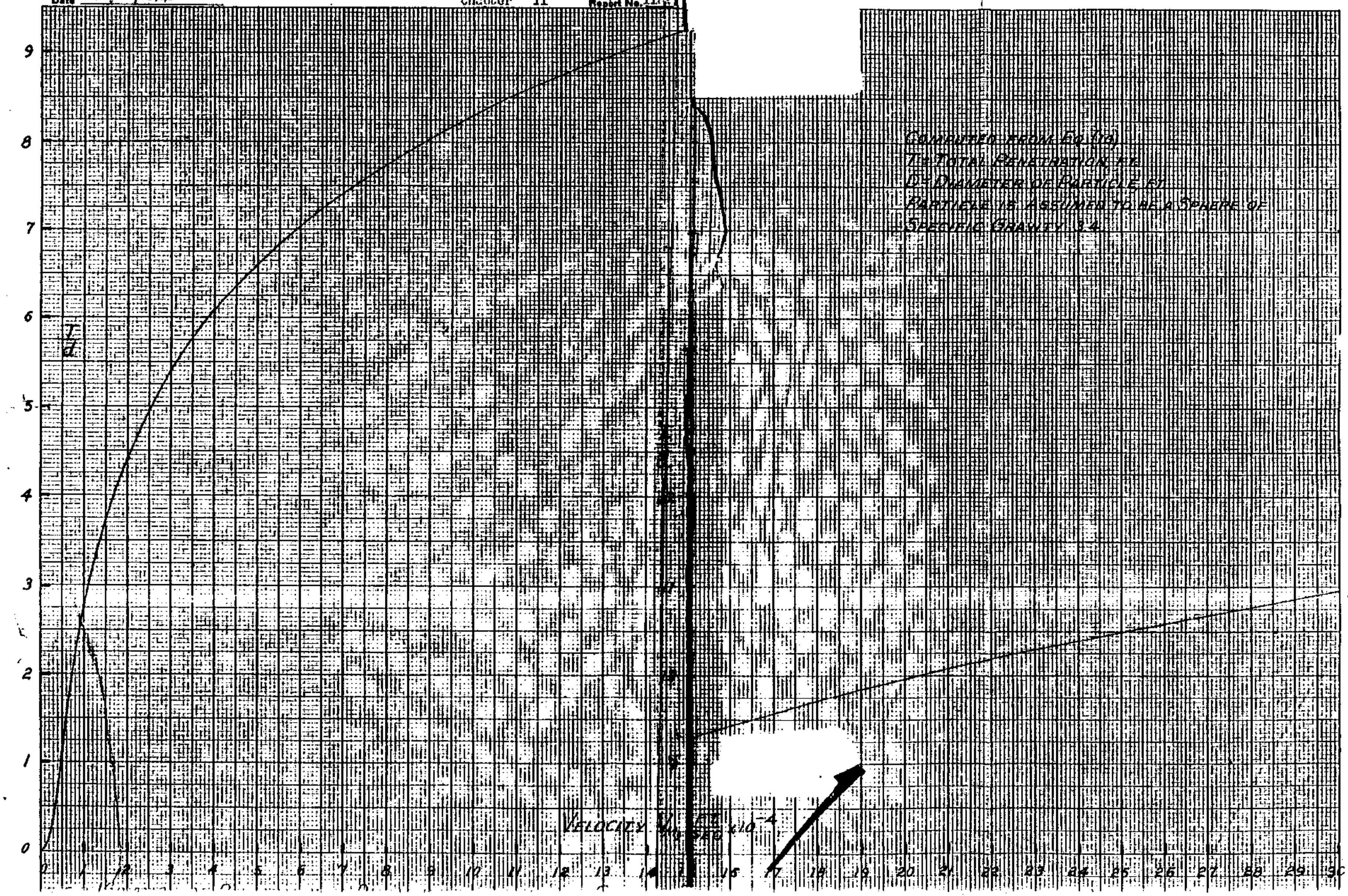
Taking $\rho_M = 3.4 \frac{\text{gram}}{\text{cm}^3} = 6.6 \frac{\text{slug}}{\text{ft.}^3}$ and $C = 5.03 \times 10^7$

as determined from the firing tests on copper, the penetration formula for a sphere becomes

$$\frac{t}{d} = 4.4 \times 10^{-8} V^2 \ . \ \text{-----------------} \ (9A)$$

The velocity of plastic deformation in compression is known to be around 1000 ft. per sec.(9) so that the limiting speed used in the fluid-flow equation (6) will be taken to be 5,000 ft. per sec..

(9) NDRC Report Number M-302.


COMPUTED FROM EQ (10)
T = TOTAL PENETRATION, FT.
D = DIAMETER OF PARTICLE, FT.
PARTICLE IS ASSUMED TO BE A SPHERE OF
SPECIFIC GRAVITY 3.4
T/d
VELOCITY, FT/SEC × 10^-4
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

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This corresponds to a plastic deformation Mach number of 5. Thus to compute the total penetration for a particle (meteorite) having a speed greater than 5000 ft. per sec., Eq. (6) is first used to compute the penetration s at which the speed is slowed down to 5,000 ft. per sec. The remainder of the penetration t is then computed from Eq. (9) using V = 5,000 ft. per sec. The total penetration is then given by the sum s + t. Letting T=s+t, the computation can be simplified by joining Eqs. (6) and (9) at V_M = 5,000 ft. per sec. and the total penetration T is then given by

$$\frac{T}{d} = \begin{cases} 4.4 \times 10^{-8} V_M^2, & \text{for } 0 \leq V_M \leq 5{,}000 \text{ ft./sec.} \\ 1.1 + 2.4 \log_e \dfrac{V_M}{5{,}000}, & \text{for } V_M \geq 5{,}000 \text{ ft./sec.} \end{cases} \qquad (10)$$

The variation of $\frac{T}{d}$ with V_M is shown in Fig. 3.

These equations are essentially empirical and neglect effect of shape of projectile and influence of the physical properties of the metal on the critical velocities. Since the basis for the formulas lies in extrapolating rather meager ballistic data and theories to the small sizes of meteorites, considerable error may be expected. The present results may, however, serve to give an indication of the order of magnitude of the impact effects.

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Using the results contained in Figs. 1 and 2 which connect the diameter and velocity of a meteorite with the magnitude of its meteor, the penetration at an altitude of 100 miles can be expressed directly as a function of meteor magnitude. This is tabulated in Table 3 and presented graphically in Fig. 4.

In a similar fashion, using Fig. 2A, the penetration at 400 miles altitude is obtained as a function of meteor magnitude, and this is presented in Table 3A and Fig. 4A.

From Tables 3 and 3A or Figs. 4 and 4A, one may see immediately how thick the skin (assumed to be of dural) of the satellite vehicle must be to withstand perforation by meteorites of different sizes (magnitudes). Thus at 100 miles altitude, for a meteor of magnitude 0, the skin, according to the analysis, would have to be at least 2.08 inches thick in order to resist perforation. For a skin thickness of .05 in. = .00416 ft. which represents the order of thickness of metal commonly used in aircraft design, it is seen that the smallest meteorite which will perforate corresponds to about magnitude 9 or 10. For velocities less than about 1000 ft. per sec. the particles would probably not penetrate the plate at all, but simply bounce off. In view of the penetration results presented in Table 3 for the 100 mile altitude, it is seen that, as far as presenting perforation hazard is concerned, meteorites of corresponding magnitude greater than 12 can certainly be completely disregarded.

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TABLE 3

PENETRATION OF DURAL PLATE BY METEORITES - ALTITUDE 100 MILES

Magnitude of Meteor M	Velocity of Meteorite V_M, ft./sec.	Penetration Ratio $\frac{T}{d}$	Diameter of Meteorite d, ft.	Total Penetration Distance T, ft.	Total Penetration Distance T, inches
0	220,000	10.2	.017	.1734	2.08
1	213,000	10.12	.0126	.12751	1.53
2	203,000	10.00	.0092	.09200	1.104
3	189,000	9.83	.0068	.06684	.8021
4	170,000	9.58	.0050	.04790	.5748
5	143,000	9.17	.0037	.03393	.40716
6	112,000	8.58	.0027	.02317	.27804
7	82,500	7.83	.0020	.01566	.18792
8	56,500	6.90	.00147	.01014	.12168
9	36,000	5.84	.00107	.006249	.074988
10	19,500	4.32	.00079	.003413	.040956
11	8,200	2.28	.00058	.001322	.015864
12	2,500	0.25	.00042	.000105	.001260
13	550	0.03	.00032	.0000096	.000115

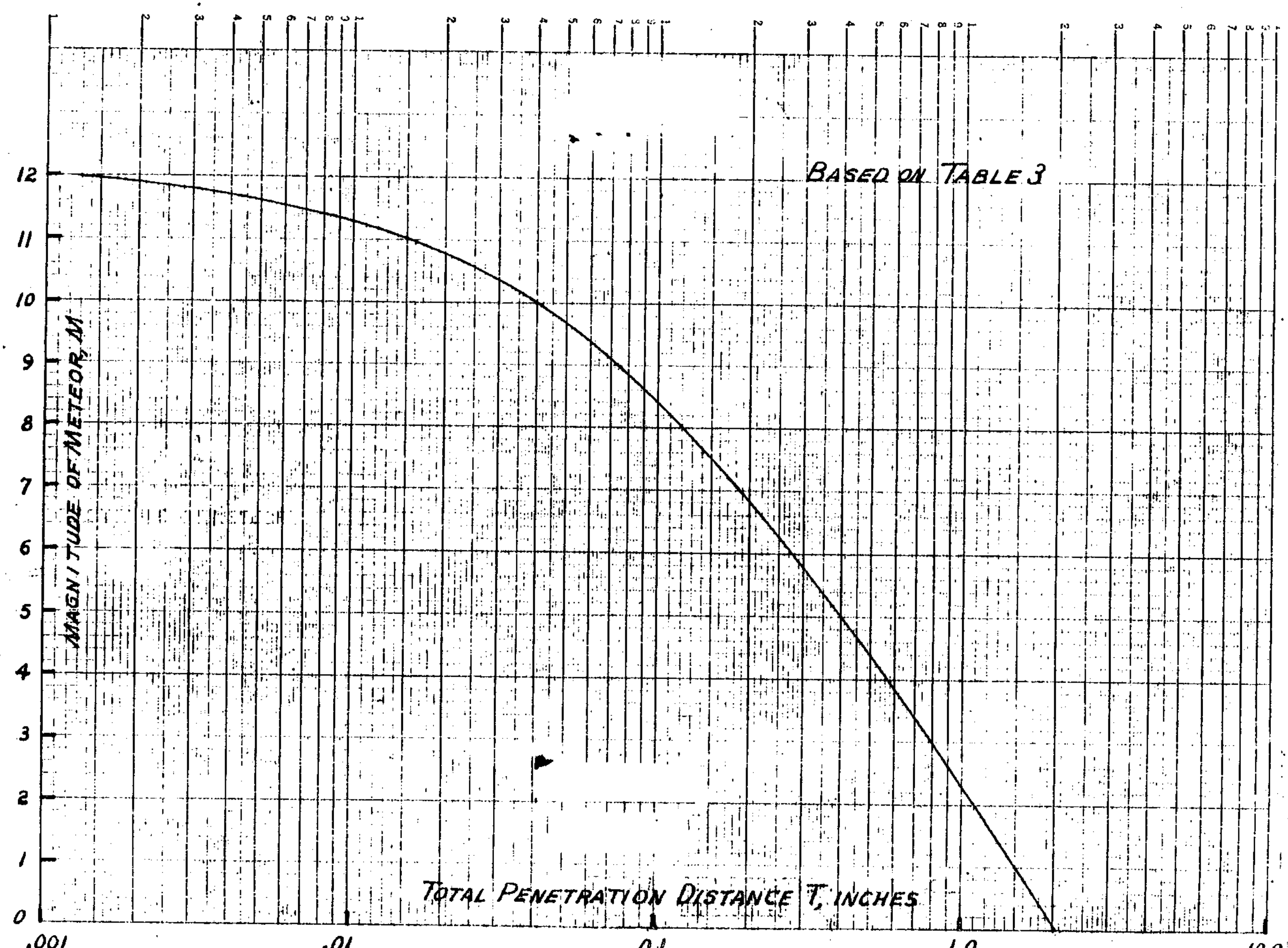

FIG 4 PENETRATION OF DURAL PLATE BY METEORITES ALTITUDE 100 MILES

TABLE 3A

PENETRATION OF DURAL PLATE BY METEORITES - ALTITUDE 400 MILES

MAGNITUDE OF METEOR M	VELOCITY OF METEORITE V_M FT./sec	PENETRATION RATIO $\frac{T}{d}$	DIAMETER OF METEORITE d,ft.	TOTAL PENETRATION DISTANCE T,ft.	TOTAL PENETRATION DISTANCE T,inches
0	244,000	10.43	.017	.1775	2.130
1	242,000	10.41	.0126	.1312	1.572
2	240,000	10.39	.0092	.0955	1.145
3	236,500	10.35	.0068	.0704	0.844
4	231,000	10.30	.0050	.0515	0.618
5	225,000	10.24	.0037	.0379	0.454
6	215,000	10.14	.0027	.0274	0.329
7	203,000	10.00	.0020	.0200	0.240
8	190,000	9.84	.00147	.0145	0.174
9	173,000	9.62	.00107	.0103	0.124
10	154,300	9.35	.00079	.0074	0.089
11	131,000	8.96	.00058	.00520	0.0623
12	105,400	8.43	.00042	.00354	0.0424
13	78,000	7.68	.00032	.00246	0.0295
14	52,000	6.70	.00023	.00154	0.0185
15	30,000	5.40	.00017	.00092	0.0110
16	12,450	3.35	.00013	.00046	0.0055
17	4,400	0.90	.00009	.00008	0.00096
18	1,020	0.05	.00007	.000004	0.000048

Table 3A
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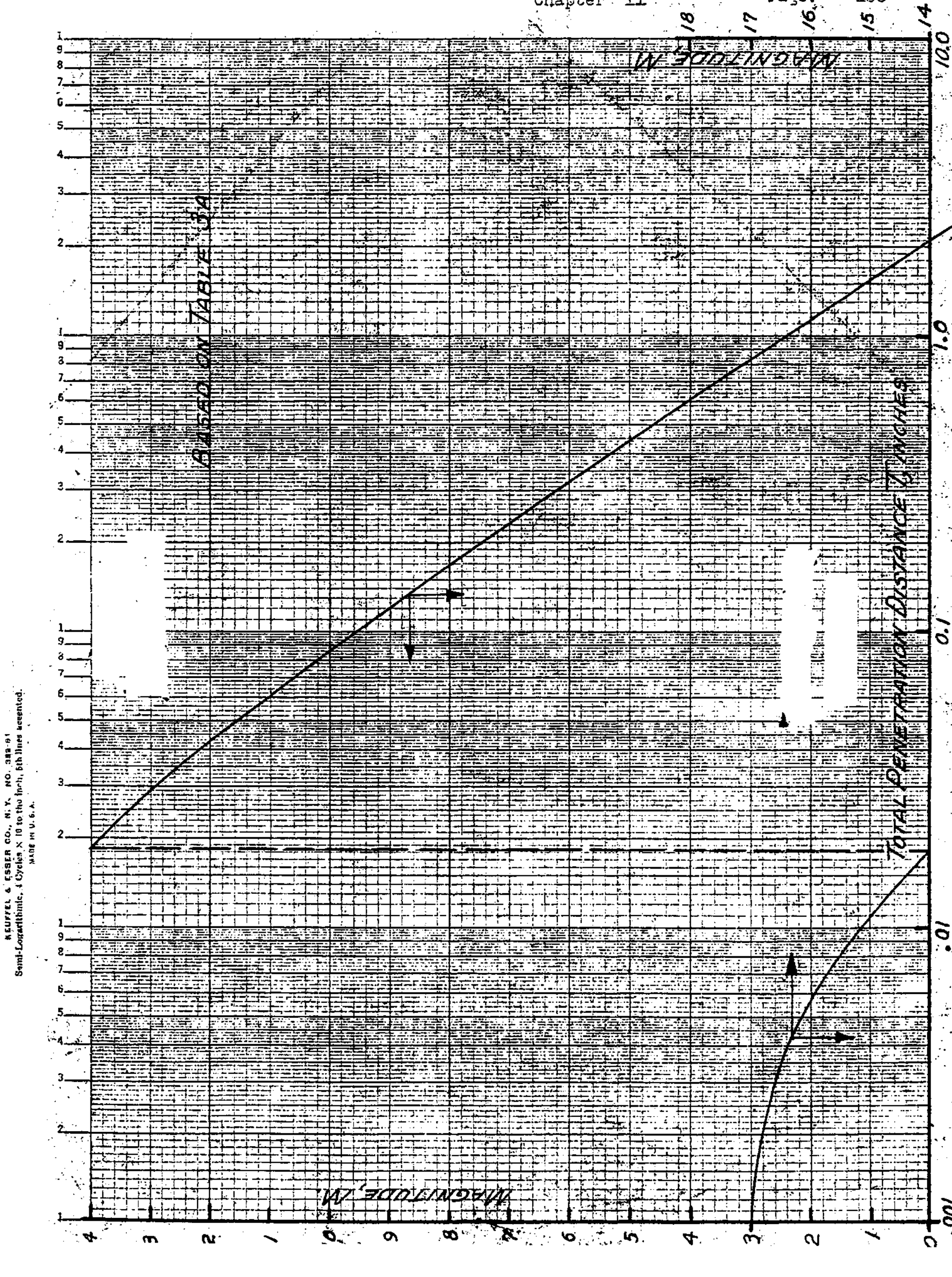

FIG. 4A. PENETRATION OF DURAL PLATE BY METEORITES, ALTITUDE 400 MILES.

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Comparing the values in Tables 3 and 3A it is found that for magnitudes of 5 or less the increase in altitude from 100 to 400 miles does not appreciably affect the penetration. For the smaller sizes the difference in penetration at the two altitudes becomes more marked, although for magnitudes greater than 15, the penetration becomes negligible.

In case the skin is made of material of greater strength and hardness than dural, stainless steel for example, the penetration would be expected to be correspondingly less. In this connection, however, it is worth noting that according to the experiments reported in ref. 8, when the projectile size was much smaller than the plate thickness, dural gave greater resistance to perforation than face hardened steel when the comparison is made on the basis of the weight per unit area of plate.

Aside from the problem of perforation by a meteorite, the question also arises as to what sort of average impact force (averaged over a long interval of time) is to be expected as a result of meteorite hits. For a given magnitude (size) M, if $\bar{n}$ is the average number of hits per hour, W the weight in pounds, V the velocity in ft. per sec., the average impact force $\bar{F}$ is simply

$$\bar{F} = \frac{\bar{n}}{3600} \frac{W}{g} V, \text{ lbs.} \qquad (10A)$$

From Table 2 it will be found that the product of $\bar{n}W$ is constant and equal to 1.73×10^{-12}. Thus the average force of impact is given by

$$\bar{F} = 1.5 \times 10^{-17} V, \text{ lbs.},$$

which shows that even with the highest velocities considered here, this average force would be far too small to in any way affect the performance of the vehicle.

Having arrived at figures for the penetration by the meteorites of different sizes, it now remains to find the probabilities that the vehicle will be struck by these particles. The following notation will be used.

N = number of meteorites (either total or of specified size) entering the earth's atmosphere in each 24 hour period.

A_e = number of square feet of atmospheric surface at a height of 500,000 feet. The number which will be used here is $A_e = 4\pi \times (21.39)^2 \times 10^{12} = 57.4 \times 10^{14}$ sq. ft. Radius of earth = 20.89×10^6 ft.

A_b = planform area of the satellite vehicle, sq. ft. The number which will be used is for a triangular planform 60 ft. x 32 ft. which gives the value, $A_b = 960$ sq. ft.

p_{1+} = probability that <u>at least one</u> hit will occur in the time T.

p_o = probability that <u>no</u> hit will occur in the time T. ($p_o = 1 - p_{1+}$).

p_1 = probability that <u>exactly one</u> hit will occur in the time T.

T(0.5) = time interval such that the vehicle has a 50 to 50 chance of not being hit.

T(0.99) = time interval such that the vehicle has a 100 to 1 change of not being hit.

T(0.999) = time interval such that the vehicle has a 1000 to 1 chance of not being hit.

BLE 4

OBABILITIES OF HIT OF A METEORITE AND A SATELLITE VEHICLE
OBABILITIES AND TIME INTERVALS BASED ON NUMBER OF METEORITES OF ONE SIZE ONLY

gnitude, M	-3	0	2	5	6	7	8	9	10	12	15	20	25	30
verage number of hits per our, N	2.0×10^{-10}	3.14×10^{-9}	2.0×10^{-8}	3.14×10^{-7}	7.9×10^{-7}	2.0×10^{-6}	4.95×10^{-6}	.0000125	.0000314	.0001[illegible]	.00314	.314	31.4	314
verage number of hours between hit	5.0×10^{9}	3.18×10^{8}	5.0×10^{7}	3.18×10^{6}	1.27×10^{6}	5.0×10^{5}	2.02×10^{5}	80,000	31,800	50[illegible]	318	3.16	.0318	.000318
1+ for 24 hours	4.8×10^{-9}	7.54×10^{-8}	4.8×10^{-7}	7.54×10^{-6}	1.9×10^{-5}	4.8×10^{-5}	1.19×10^{-4}	3.0×10^{-4}	7.54×10^{-4}	4.73×10^{-3}	.073	$1-5.49\times10^{-4}$	$1-10^{-326}$	$1-10^{-3}$
for 24 hours	$1-4.8\times10^{-9}$	$1-7.54\times10^{-8}$	$1-4.8\times10^{-7}$	$1-7.54\times10^{-6}$	$1-1.9\times10^{-5}$	$1-4.8\times10^{-5}$	$1-1.29\times10^{-4}$	$1-3.0\times10^{-4}$	$1-7.54\times10^{-4}$	$1-4.73\times10^{-3}$	9.27×10^{-1}	5.49×10^{-4}	10^{-326}	10^{-32600}
for 24 hours	4.8×10^{-9}	7.54×10^{-8}	4.8×10^{-7}	7.54×10^{-6}	1.90×10^{-5}	4.8×10^{-5}	1.19×10^{-4}	3.0×10^{-4}	7.54×10^{-4}	4.73×10^{-3}	6.98×10^{-2}	4.13×10^{-3}	10^{-326}	10^{-32600}
for 120 hours	2.4×10^{-8}	3.77×10^{-7}	2.4×10^{-6}	3.77×10^{-5}	9.5×10^{-5}	2.4×10^{-4}	6.0×10^{-4}	.002	.004	[illegible]24	.313	$1-2.0\times10^{-16}$	$1-10^{-1630}$	$1-10^{-163,00}$
for 120 hours	$1-2.4\times10^{-8}$	$1-3.77\times10^{-7}$	$1-2.4\times10^{-6}$	$1-3.77\times10^{-5}$	$1-9.5\times10^{-5}$	$1-2.4\times10^{-4}$	$1-6.0\times10^{-4}$	9.98×10^{-1}	9.96×10^{-1}	9.76×10^{-1}	6.67×10^{-1}	2.0×10^{-16}	10^{-1630}	$10^{-163,0}$
for 120 hours	2.4×10^{-8}	3.77×10^{-7}	2.4×10^{-6}	3.77×10^{-5}	9.5×10^{-5}	2.4×10^{-4}	6.0×10^{-4}	2.0×10^{-3}	4.0×10^{-3}	$2.3[illegible]\times10^{-2}$	2.59×10^{-1}	2.54×10^{-15}	10^{-1630}	$10^{-163,0}$
(0.5), hours	3.46×10^{9}	2.2×10^{8}	3.46×10^{7}	2.2×10^{6}	8.8×10^{5}	3.46×10^{5}	1.4×10^{5}	55,400	22,000	[illegible]20	220	2.20	.022	.00022
(0.99), hours	5.0×10^{7}	3.2×10^{6}	5.0×10^{5}	3.2×10^{4}	1.27×10^{4}	5000	2020	800	318	[illegible].7	3.18	.0318	.000318	3.18×10
(0.999), hours	5.0×10^{6}	3.2×10^{5}	5.0×10^{4}	3.2×10^{3}	1.27×10^{3}	500	202	80	31.8	[illegible]07	.318	.00318	3.18×10^{-5}	3.18×10

efinition of symbols: p_{1+} = Probability of at least 1 hit

p_o = Probability of no hit

p_1 = Probability of only one hit

$T(0.5)$ = Time interval to give a 50 to 50 chance of no hit

$T(0.99)$ = Time interval to give a 100 to 1 chance of no hit

$T(0.999)$ = Time interval to give a 1000 to 1 chance of no hit

ABLE 5

ROBABILITIES OF HIT OF A METEORITE AND A SATELLITE VEHICLE
ROBABILITIES AND TIME INTERVALS BASED ON NUMBER OF METEORITES OF A GIVEN SIZE PLUS ALL THOSE OF LARGER SIZE

agnitude, M	-3	0	2	5	6	7	8	9	10	12	15	20	25	30
verage hits per hour, $\bar{n}$	$2.0x10^{-10}$	$5.07x10^{-9}$	$3.28x10^{-8}$	$5.20x10^{-7}$	$1.31x10^{-6}$	$3.28x10^{-6}$	$8.22x10^{-6}$	$2.07x10^{-5}$	$5.20x10^{-5}$	$3.28x10^{-4}$	$5.20x10^{-3}$	$5.20x10^{-1}$	$5.20x10^{1}$	5.20x10
verage number of hours between hits, $\bar{t}$	$5.0x10^{9}$	$1.92x10^{8}$	$3.05x10^{7}$	$1.92x10^{6}$	$7.63x10^{5}$	$3.05x10^{5}$	$1.22x10^{5}$	$5.16x10^{4}$	$1.92x10^{4}$	$3.05x10^{3}$	$1.92x10^{2}$	1.92	$1.92x10^{-2}$	1.92x10
1+ for 24 hours	$4.8x10^{-9}$	$1.22x10^{-7}$	$7.88x10^{-7}$	$1.28x10^{-5}$	$3.14x10^{-5}$	$7.88x10^{-5}$	$1.97x10^{-4}$	$4.97x10^{-4}$	$1.25x10^{-3}$	$7.88x10^{-3}$	.117	$1-2.63x10^{-5}$	$1-15^{542}$	$1-10^{-54200}$
for 24 hours	$1-4.8x10^{-9}$	$1-1.22x10^{-7}$	$1-7.88x10^{-7}$	$1-1.25x10^{-5}$	$1-3.14x10^{-5}$	$1-7.88x10^{-5}$	$1-1.97x10^{-4}$	$1-4.97x10^{-4}$	$1-1.25x10^{-3}$	$1-7.88x10^{-3}$	.883	$2.63x10^{-5}$	10^{5542}	10^{-54200}
for 24 hours	$4.8x10^{-9}$	$1.22x10^{-7}$	$7.88x10^{-7}$	$1.25x10^{-5}$	$3.14x10^{-5}$	$7.88x10^{-5}$	$1.97x10^{-4}$	$4.97x10^{-4}$	$1.25x10^{-3}$	$7.88x10^{-3}$	$1.10x10^{-1}$	$3.28x10^{-4}$	10^{-540}	10^{-54200}
1+ for 120 hours	$2.4x10^{-8}$	$6.10x10^{-7}$	$3.94x10^{-6}$	$6.25x10^{-5}$	$1.57x10^{-4}$	$3.94x10^{-4}$	$9.85x10^{-4}$	$2.48x10^{-3}$	$6.25x10^{-3}$	.038	.465	$1-1.3x10^{-27}$	$1-10^{-2710}$	$1-10^{-27100}$
for 120 hours	$1-2.4x10^{-8}$	$1-6.10x10^{-7}$	$1-3.94x10^{-6}$	$1-6.25x10^{-5}$	$1-1.57x10^{-4}$	$1-3.94x10^{-4}$	$1-9.85x10^{-4}$	$1-2.48x10^{-3}$	$1-6.25x10^{-3}$	.962	.535	$1.3x10^{-27}$	10^{-2710}	10^{-27100}
1 for 120 hours	$2.4x10^{-8}$	$6.10x10^{-7}$	$3.94x10^{-6}$	$6.25x10^{-5}$	$1.57x10^{-4}$	$3.94x10^{-4}$	$9.85x10^{-4}$	$2.48x10^{-3}$	$6.25x10^{-3}$	$3.79x10^{-2}$	$3.34x10^{-1}$	$8x10^{-26}$	10^{-2710}	10^{-27100}
(0.5), hours	$3.46x10^{9}$	$1.36x10^{8}$	$2.11x10^{7}$	$1.33x10^{6}$	$5.28x10^{5}$	$2.11x10^{5}$	$8.45x10^{4}$	$3.58x10^{4}$	$1.33x10^{4}$	$2.11x10^{3}$	$1.33x10^{2}$	1.33	$1.33x10^{-2}$	$1.33x10^{-1}$
(0.99), hours	$5.0x10^{7}$	$1.92x10^{6}$	$3.05x10^{5}$	$1.92x10^{4}$	$7.63x10^{3}$	$3.05x10^{3}$	$1.22x10^{3}$	$5.16x10^{2}$	$1.92x10^{2}$	3.05x10	1.92	$1.92x10^{-2}$	$1.92x10^{-4}$	[illegible]
(0.999), hours	$5.0x10^{6}$	$1.92x10^{5}$	$3.05x10^{4}$	$1.92x10^{3}$	$7.63x10^{2}$	$3.05x10^{2}$	$1.22x10^{2}$	5.16x10	1.92x10	3.05	$1.92x10^{-1}$	$1.92x10^{-3}$	$1.92x10^{-5}$	[illegible]

Definition of symbols: p_{1+} = Probability of at least 1 hit

p_0 = Probability of no hit

p_1 = Probability of only one hit

T(0.5) = Time interval to give a 50 to 50 chance of no hit

T(0.99) = Time interval to give a 100 to 1 chance of no hit

T(0.999) = Time interval to give a 1000 to 1 chance of no hit

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These probabilities and time intervals are presented in Tables 4 and 5. The formulas by which they are computed are derived in Appendix G.

The meteorites entering the atmosphere are assumed to have a random distribution both as regards their surface distribution over the atmospheric layer surrounding the earth and as regards their occurrence with time. It is assumed that the meteorites travel through the atmosphere along the vertical and that the planform area of the vehicle is normal to the vertical.

The two tables are entirely similar, the only difference being that the values used for N in Table 4 are based on the total number of meteorites of one size only; whereas, the values for N used in Table 5 include the total number of meteorites of a given size plus all those of larger size. At the lower magnitudes these two numbers do not differ appreciably, but at the higher magnitudes, 9 or 10 and higher, the difference is large enough to be considered. See Table 1, Appendix G. For this reason the values in Table 5 are considered to be the more significant and this table will receive the main consideration.

Considering Table 5, it is seen, first of all, that the average time interval between hits does not attain values comparable to the contemplated time of operation of the vehicle (say from 5 to 10 days) until the meteorite size becomes as small as that corresponding to magnitude 14 or 15. Thus, it is seen from the column for $M = 15$, that on the average, the vehicle could operate for 192 hours before it would be hit by a meteorite of size corresponding to magnitude 15 or any larger size. For magnitudes greater than 15 the average time between hits becomes relatively small but by this time the meteorites are of such small size and velocity that it does not matter. The probability numbers, of course, show the same general tendencies as do the numbers for $\bar{n}$ and $\bar{t}$.

The most important probability to consider, it would seem, is the probability p_{1+} which is the probability that the vehicle will be hit at least once. Or, stated slightly differently, p_{1+} gives simply the probability that the vehicle will be hit, the number of times it will be hit not being specified. The probability scale is such that a probability of 1 means that the event is certain to occur, while a probability of 0 means the event is certain not to occur. Considering the values of p_{1+} in Table 5 for the 120-hour interval (5 days), for instance, the probability of a hit is less than 1 in a 1000 (i.e. 0.001) for all magnitudes of 8 or less. At magnitude 15, however, the probability has greatly increased and shows that there is only about a 50 - 50 chance that the vehicle will not be hit. Here again, however, the size of the particle becomes so small that even though the probability becomes high, it does not matter as far as damage to the vehicle is concerned. For magnitudes 20 and above, the vehicle is certain to be hit, but it certainly will not matter considering the small size of these particles.

Considering next the probability-based time intervals, we see, for example, that if we specify a 1000 to 1 chance of not being hit by a meteorite of corresponding magnitude 10 or less, the vehicle may operate for 19.2 hours. If the probability number is relaxed down to a 100 to 1 chance of no hit, the operating time increases to 192 hours, etc.

It is interesting to note that at around magnitude 15, the probabilities p_{1+} , p_o and p_1 , all take on comparable values, showing that somewhere in this range of magnitude the occurrence of these three events becomes more or less equally probable.

In general, the probability tables indicate that for the meteorite sizes which are large enough to present a perforation hazard, the probabilities of a hit are quite small, never exceeding about 0.001 (for a reasonable plate thickness say, of 0.10 in.) or about 1 chance in 1000.

Having the relation between T and M (Figs. 4 and 4A) and the relation between p_{1+} and M (Table 5), one may then derive a relation between T and p_{1+}, where p_{1+} is the probability that a meteorite of corresponding magnitude M will just perforate a dural skin thickness of amount T. This relationship has been derived and is shown in Fig. 5, for the two altitudes 100 miles and 400 miles. These curves represent, essentially, the net result of the perforation and probability study when presented in the most usable form.

Since the relation of the type shown in Fig. 5 has been determined only for the case of a 5 day time interval, we shall suppose that the vehicle is to operate for a period of 5 days. We then,

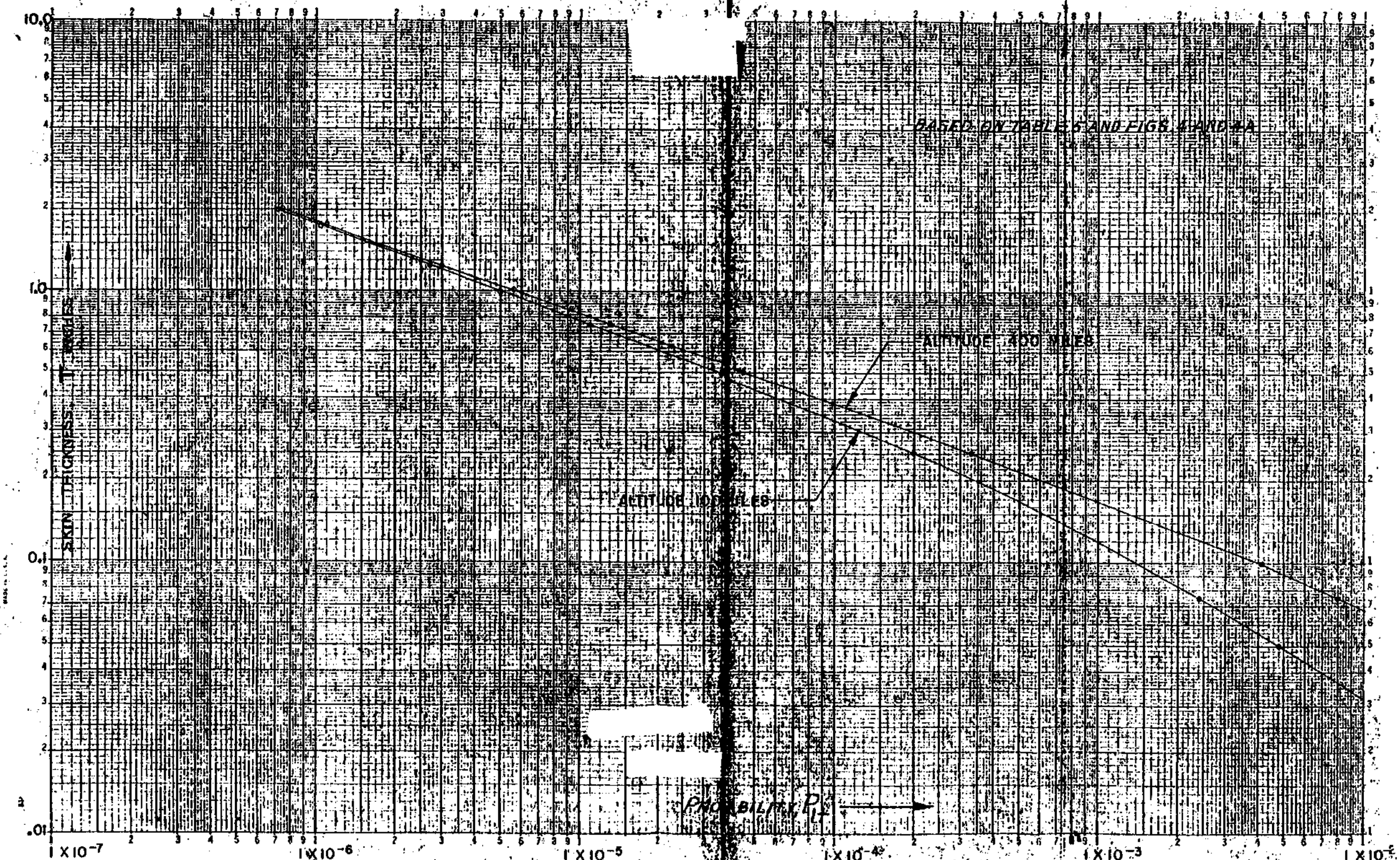

FIG.5 PROBABILITY OF BEING HIT IN FIVE DAYS BY A METEORITE WHICH WILL PERFORATE DURAL SKIN OF GIVEN THICKNESS

for example, may ask what is the probability that a skin of say 0.12 in. thick dural will be perforated by a meteorite when the altitude is, for instance, 100 miles. Referring to the 100 mile altitude curve of Fig. 5 for T = 0.12 in., it is found that $p_{1+} = 1 \times 10^{-3}$ = .001. Thus, for the chosen condition the chances are 1000 to 1 that the skin will not be perforated. For a skin thickness T = .05 in., the probability is $p_{1+} = .0048$, and in this case the chances are only 208 to 1 that perforation will not occur. There is not much point in considering values of $T < .05$ in. since at least this much thickness would be required simply from considerations of structural strength.

The use of the curves of Fig. 5 may also be considered from the reverse point of view. Assuming operation at 100 miles altitude, suppose we are willing to take a 1000 to 1 chance on the occurrence of perforation, and then ask what the skin thickness must be. For $p_{1+} = .001$ and at 100 miles altitude it is found that T = 0.12 in. These examples are sufficient to show how the perforation - probability-time curve is used.

Chapter 11

11. PROBLEMS AFTER ORBIT IS ESTABLISHED

Temperature of Vehicle when on the Orbit. - The temperatures reached by the vehicle when it is on its orbit are calculated by considering the process of radiation of heat from the sun and earth to the body, and from the body to space.

Suppose, to begin with, that the simple illustrative example of the heating of the earth by the sun is considered. It is desired to compute the average temperature of the earth's surface. The rate of heat radiation by the sun will be

$$\frac{\partial Q}{\partial t} = \sigma T_s^4 \pi d_s^2 \qquad (1)$$

where

$\sigma = 0.173 \times 10^{-8}$ BTU/(sq.ft.)(hr.)(deg.R.)^4

$T_s = 10{,}800^\circ$R., sun's surface temperature

$d_s = 864{,}000$ miles, sun's diameter

This energy travels into space on spherical surfaces, and therefore the fraction of this energy which is intercepted by the earth is equal to the percentage of the area of the sphere, of radius equal to the distance from the sun to the earth, which is blocked off by the earth. This must further be multiplied by the absorptivity of the earth. Therefore, the rate at which heat is absorbed into the earth is

$$\frac{\partial Q}{\partial t} = \sigma T_s^4 \pi d_s^2 \frac{\alpha A}{4\pi L^2} \qquad (2)$$

in which α = absorptivity

A = projected area exposed to sun's rays

L = distance from earth to sun, 93×10^6 miles

The rate at which the earth radiates heat is

$$\frac{\partial Q}{\partial t} = \sigma \epsilon T_e^4 \pi d_e^2 \tag{3}$$

Here, ϵ = emissivity

T_e = average surface temperature of earth

d_e = diameter of earth, 7920 miles

When heat is radiated out as fast as it is absorbed, equilibrium conditions exist. This is obtained by setting (2) equal to (3).

$$\sigma T_s^4 \pi d_s^2 \frac{\alpha A}{4\pi L^2} = \sigma \epsilon T_e^4 \pi d_e^2$$

From this is found, after setting $A = \pi d_e^2/4$, and $\epsilon = \alpha$ by Kirchoff's law,

$$T_e^4 = \frac{T_s^4 d_s^2}{16L^2} \tag{4}$$

$$T_e = \frac{T_s}{2}\sqrt{\frac{d_s}{L}} \tag{5}$$

Using the values noted above, we find $T_e = 520^\circ R = 60^\circ F$, which is in reasonable agreement with our everyday experience.

Now let us apply a similar analysis to a satellite vehicle. The vehicle will alternately be in front and behind the earth. Supposing that it is in front of the earth for a sufficiently long time to reach equilibrium, the governing relation is

$$\sigma T_s^4 \pi d_s^2 \frac{\alpha A_{vs}}{4\pi L^2} + \sigma T_e^4 \pi d_e^2 \frac{\alpha A_{ve}}{\pi d_e^2} = \sigma \epsilon T_{v_f}^4 S_v \tag{6}$$

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where the symbols are as noted previously except for

A_{ve} = projected area of vehicle as seen from earth

A_{vs} = effective projected area of vehicle as seen from sun

T_{vf} = temperature of vehicle, in front of earth

S_v = surface area of vehicle

Again assume $\alpha = \varepsilon$, and simplify (6). T_e can be eliminated by the use of (4). This leads to

$$T_{vf}^4 = \frac{T_s^4 d_s^2}{16L^2}\left[4\frac{A_{vs}}{A_v} + \frac{A_{ve}}{S_v}\right] \tag{7}$$

Now it is necessary to determine the ratio of projected to surface areas. The vehicle is conical in shape, with an altitude of 16 2/3' and a base diameter of 3 1/3'. If the vehicle axis remains tangent to its orbit, then

$$\frac{A_{ve}}{S} = \frac{.5 \times 3.33 \times 16.67}{.5\pi \times 16.67 \times 3.33 + .25\pi\, 3.33^2} = .292$$

The projection of the vehicle as seen from the sun is a circle at "dawn", gradually changing to a triangle at "high noon" and then going back to a circle at "dusk". The mean fourth root of the projected area raised to the fourth power is

$$\frac{A_{vs}}{S_v} \doteq \frac{19.3 \times .292}{.5 \times 3.33 \times 16.67} = .203$$

Hence

$$T_{vf}^4 = 1.10\,\frac{T_s^4 d_s^2}{16L^2} = 1.10\, T_e^4 \tag{8}$$

or

$$T_{vf} = 1.02\, T_e = 530^\circ R = 70^\circ F.$$

If the vehicle were behind the earth for a time great enough to produce equilibrium temperatures, then the equation governing the situation is like (6), but without the first term.

$$\sigma T_e^4 \pi d_e^2 \frac{\alpha A_{ve}}{\pi d_e^2} = \sigma \epsilon T_{vB}^4 S_v \tag{9}$$

This is readily reduced to

$$T_{vB}^4 = .292\, T_e^4 \tag{10}$$

It follows directly that

$$T_{vB} = .732\, T_e = 381^\circ R = -79^\circ F.$$

Thus we have shown that the temperature of the vehicle must lie between the limits 70°F. and -79°F. To find just where, between these limits, the temperatures lie, it is necessary to set up the differential equation relating temperatures with time. The net rate of heat transfer for the body in front of the earth with respect to the sun is

$$\left.\frac{\partial Q}{\partial t}\right]_F = \sigma T_s^4 \pi d_s^2 \frac{\alpha A_{vs}}{4\pi L^2} + \sigma T_e^4 \pi d_e^2 \frac{\alpha A_{ve}}{\pi d_e^2} - \sigma \epsilon T_{v_F}^4 S_v$$

using (4), putting $\alpha = \epsilon$, and simplifying, we find

$$\left.\frac{\partial Q}{\partial t}\right]_F = \sigma \epsilon S_v \left[1.10 \frac{T_s^4 d_s^2}{16L^2} - T_{v_F}^4\right] \tag{11}$$

For the values used previously,

$$\left.\frac{\partial Q}{\partial t}\right]_F = \sigma \epsilon S_v \left[530^4 - T_{v_F}^4\right] \tag{12}$$

The specific heat is defined to be

$$c = \frac{\Delta Q}{W \Delta T} \tag{13}$$

in which W is the weight of the vehicle. From this, it is seen that

$$\frac{\Delta Q}{\Delta t} = cW \frac{\Delta T}{\Delta t} \tag{14}$$

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(12) can then be reduced to

$$\frac{dT_{V_F}}{dt} = \frac{\sigma \epsilon S_v}{cW}\left[530^4 - T_{V_F}^4\right] \qquad (15)$$

There are several calculations which can be immediately made using (15). First, for equilibrium, $dT_{VF}/dt = 0$, and $T_{VF} = 530$. This checks the calculation made previously. Second, the greatest rate of heating occurs when T_V is least, and the least value of T_V is 381°R. Using that value, $\epsilon = 1.0$, $C = .12$ BTU/lb.°F, and $W = 300$ lb*, we find $\partial T/\partial t = 265$°F/Hr.

An expression similar to (15) can be set up for the time when the vehicle is behind the earth. For that case

$$\frac{dT_{V_B}}{dt} = -\frac{\sigma \epsilon S_r}{cW}\left[T_{V_B}^4 - 381^4\right] \qquad (16)$$

Here the maximum rate of increase in T_V is also 265°F./hr. Since the half-period is about .75 hr., it can be seen that the temperature changes will tend to be large.

The mean temperature, T_{Vm}, can now be obtained by assuming that the rate of change of temperature is constant, and then

$$T_{V_m}^4 - 381^4 = 530^4 - T_{V_m}^4$$

From this we find that

$$T_{V_m} = 471\,°R = 11\,°F$$

An estimate of the limits to the temperature variation on the orbit can now be found by calculation of the rate of temperature change at this mean temperature.

* Structural weight only. Payload assumed to be insulated from structure.

$$\frac{dT_{V_m}}{dt} = -\frac{\sigma\epsilon S_V}{cW}\left[471^4 - 381^4\right]$$

and using the previously noted numerical values, we find the rate of change of T_V, at the mean T_V, is 132°F. per hour, or about 100°F in the time required to make a half revolution.

To obtain the graph of temperature as a function of time, it is necessary to return now to (18) and (16). Inserting the values for the known constants, we have

$$\frac{dT_{V_F}}{dt} = (.821 \times 10^{-2})^4 (530^4 - T_{V_F}^4) \tag{17}$$

and

$$\frac{dT_{V_B}}{dt} = -(.821 \times 10^{-2})(T_{V_B}^4 - 381^4) \tag{18}$$

For (17), we have

$$\int_{t_0}^{t_1} dt = \int_{T_{V_{F_0}}}^{T_{V_{F_1}}} \frac{dT_{V_F}}{(.821 \times 10^{-2})^4 (530^4 - T_{V_F}^4)}$$

Now

$$\frac{1}{a^4 - b^4} = \frac{1}{2a^2}\left[\frac{1}{a^2 - b^2} + \frac{1}{a^2 + b^2}\right]$$

$$= \frac{1}{2a^2}\left[\frac{1}{a^2+b^2} + \frac{1}{2a}\left(\frac{1}{a-b} + \frac{1}{a+b}\right)\right]$$

$$= \frac{1}{4a^3}\left[\frac{2a}{a^2+b^2} + \frac{1}{a-b} + \frac{1}{a+b}\right]$$

Hence

$$t_1 - t_0 = \frac{1}{4(.321\times10^{-2})^4\, 530^3}\left[2\tan^{-1}\frac{T_{vF}}{530} + \ln\frac{530+T_{vF}}{530-T_{vF}}\right]_{T_{vF_0}}^{T_{vF_1}}$$

$$= .860\left(\tan^{-1}\frac{T_{vF_1}}{530} - \tan^{-1}\frac{T_{vF_0}}{530}\right)$$

$$+ .430\left(\ln\frac{530+T_{vF_1}}{530-T_{vF_1}} - \ln\frac{530+T_{vF_0}}{530-T_{vF_1}}\right) \qquad (19)$$

when the vehicle is in the sun.

Equation (18) leads to

$$\int_{t_1}^{t_2} dt = \frac{-1}{(.821\times10^{-2})^4}\int_{T_{vB_1}}^{T_{vB_2}} \frac{dT_{vB}}{T_{vB}^4 - 381^4}$$

and

$$\frac{1}{x^4-a^4} = \frac{1}{2a^2}\left(\frac{1}{x^2-a^2} - \frac{1}{x^2+a^2}\right) = \frac{1}{4a^3}\left(\frac{1}{x-a} - \frac{1}{x+a} - \frac{2a}{x^2+a^2}\right)$$

so

$$t_2 - t_1 = -1.00\left(\ln\frac{T_{vB_2}-381}{T_{vB_2}+381} - \ln\frac{T_{vB_1}-381}{T_{vB_1}+381}\right)$$

$$+ 2.00\left(\tan^{-1}\frac{T_{vB_2}}{381} - \tan^{-1}\frac{T_{vB_1}}{381}\right)$$

when the vehicle is in the earth's shadow.

Equations (19) and (20) were used to obtain fig. 6 , which shows the time variations in temperature.

The calculations made up to this point have considered only radiation phenomenon. It is necessary also to investigate the heating due to friction. As in the case of the drag calculations, various regimes of heat transfer arise, depending primarily on the speed and altitude. The orbit will be of necessity chosen to be at an altitude so great that kinetic theory methods may be used here for computing heat transfer. We will begin with simple assumptions which will tend to make the heat transfer larger than it actually is.

FORM 25 BS (REV. 7-42)

Analysis TEMPERATURES
Prepared by HAROLD LUSKIN
Date 5-6-46

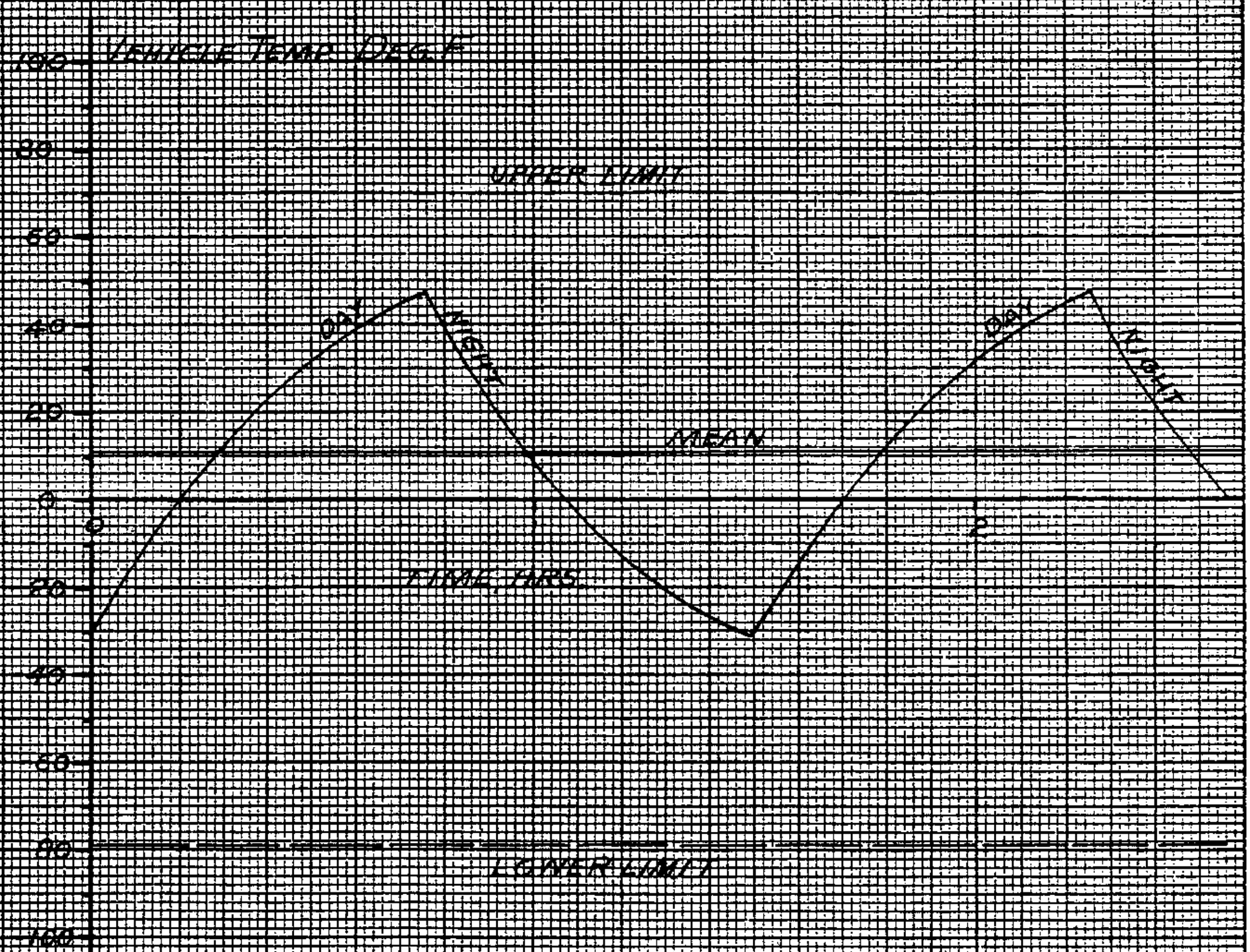
DOUGLAS AIRCRAFT COMPANY, INC.

SANTA MONICA Plant

Page 177
Model 1033
Report No. 11327

VEHICLE SKIN & STRUCTURE
TEMPERATURE VARIATIONS
WHEN VEHICLE IS ON ITS
ORBIT AT GREAT ALTITUDE

VEHICLE ORIENTED WITH
AXIS PARALLEL TO FLIGHT
PATH.



Chapter 11

Consider a surface of the vehicle of unit area. Then if N is the number of molecules in unit volume, V is the vehicle speed, and α the angle between the surface and the direction of motion, the number of molecules colliding with unit area in unit time is $NV\alpha$. Each molecule has an energy $(1/2)\mu V^2$, where μ is the mass of one molecule, and where we disregard the random velocity because it is small compared to V. The energy of all the molecules which strikes unit area in unit time is evidently

$$E_{in} = \frac{1}{2}\mu N V^3 \alpha = \frac{1}{2}\rho V^3 \alpha \tag{21}$$

The same number of molecules will leave unit area in unit time, but their momenta will be different. We will make the assumption that the air molecules enter the skin surface, come to thermal equilibrium with the skin molecules, and then are discharged in random fashion. The energy leaving the skin unit area in unit time is just

$$E_{out} = \frac{1}{2}\rho V c_W^2 \alpha \tag{22}$$

where C_W is the molecular mean square speed corresponding to the skin temperature. The energy left in the wall is

$$\Delta E = \frac{1}{2}\rho V^3 \alpha \left[1 - \frac{c_W^2}{V^2}\right] \tag{23}$$

Now by assuming equipartition of energy at the skin surface, we have

$$\frac{1}{2} M c_W^2 = \frac{3}{2} R T_W \tag{24}$$

where M is the molecular weight of air, R is the universal gas constant and T_W is the temperature of the wall. Eliminating C_W between these equations leads to

$$\Delta E = \frac{1}{2}\rho V^3 \alpha \left[1 - 3\frac{R}{M}\frac{T_W}{V^2}\right] \tag{25}$$

The most pessimistic case is that in which $3\frac{R}{M}\frac{T_W}{V^2}$ is ignored compared to unity. If, after making that assumption, the rate of heat transfer is found to be small, then since taking the additional term into account makes it yet smaller, the aerodynamic heating can be ignored. This assumption has the physical meaning that all of the energy of the on-coming molecules is left in the skin. Converting to BTU, we have the heat transfer into unit area in unit time, as

$$\frac{\Delta Q}{\Delta t} = \frac{1}{2}\rho V^3 \alpha \frac{1}{J} \tag{26}$$

The following table gives values of the heat transfer at several altitudes and at orbital speeds. α is 0.1 radians. J is 778 ft. lbs. per BTU

ALTITUDE Miles	DENSITY Slugs/cu.ft.	SPEED F.P.S.	HEAT TRANSFER RATE BTU per sq.ft.per sec.
0	2.4×10^{-3}	25,900	3×10^{6} *
100	1.07×10^{-10}	25,600	1×10^{-1}
200	1.7×10^{-13}	25,300	3×10^{-4}
300	1.3×10^{-16}	25,000	1×10^{-7}
400	6.1×10^{-19}	24,700	6×10^{-10}

It is obvious that tremendous changes in heat transfer rate take place as altitude is changed. In order to get a scale with which to determine the importance of the aerodynamic heating, we calculate the radiation output of the same unit skin area that was considered above. This is

$$\frac{\Delta Q}{\Delta t} = \sigma \epsilon T^4$$

* Strictly speaking, (26) is not valid at sea level.

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which has a magnitude of 2 x 10^{-2} BTU per sq. ft. per second. It can be seen that the radiating power is extremely large compared to the rate of heat input, so long as the altitude is not below 200 miles. It seems justified, then, to neglect the aerodynamic heating for the cases of the cruising altitudes.

At the lower altitudes, where aerodynamic heating is important, more detailed expressions for aerodynamic heating are available, and these are discussed in other portions of this report.

It was assumed in these calculations, that day and night for the vehicle occupied equal times. That statement is approximately true only if the altitude is not great. The variation of length of daylight and night is shown in fig. 7.

The temperatures computed here are, in general, low compared to what we ordinarily think of as being "normal". There are several methods of controlling the temperature level, and raising it, if desired. First, if the missile is steered around its orbit so as to present its maximum projected area to the sun at all times when it is in the sunlight, a gain of 30°F. in mean temperature can be had. Second, reducing the emissivity will proportionately reduce the magnitude of temperature fluctuations, although the mean temperature will be unchanged. Third, surfaces of different values of emissivity (lower, away from the sun) will rise the mean temperature. Fourth, changing the plane of flight to one including the earth's axis and at the same time perpendicular to the sun at all times and raise the mean temperature to 70°F.

HOURS OF SUNLIGHT AND DARKNESS FOR VEHICLE REVOLVING IN EARTH'S EQUATORIAL PLANE AT ORBITAL SPEEDS

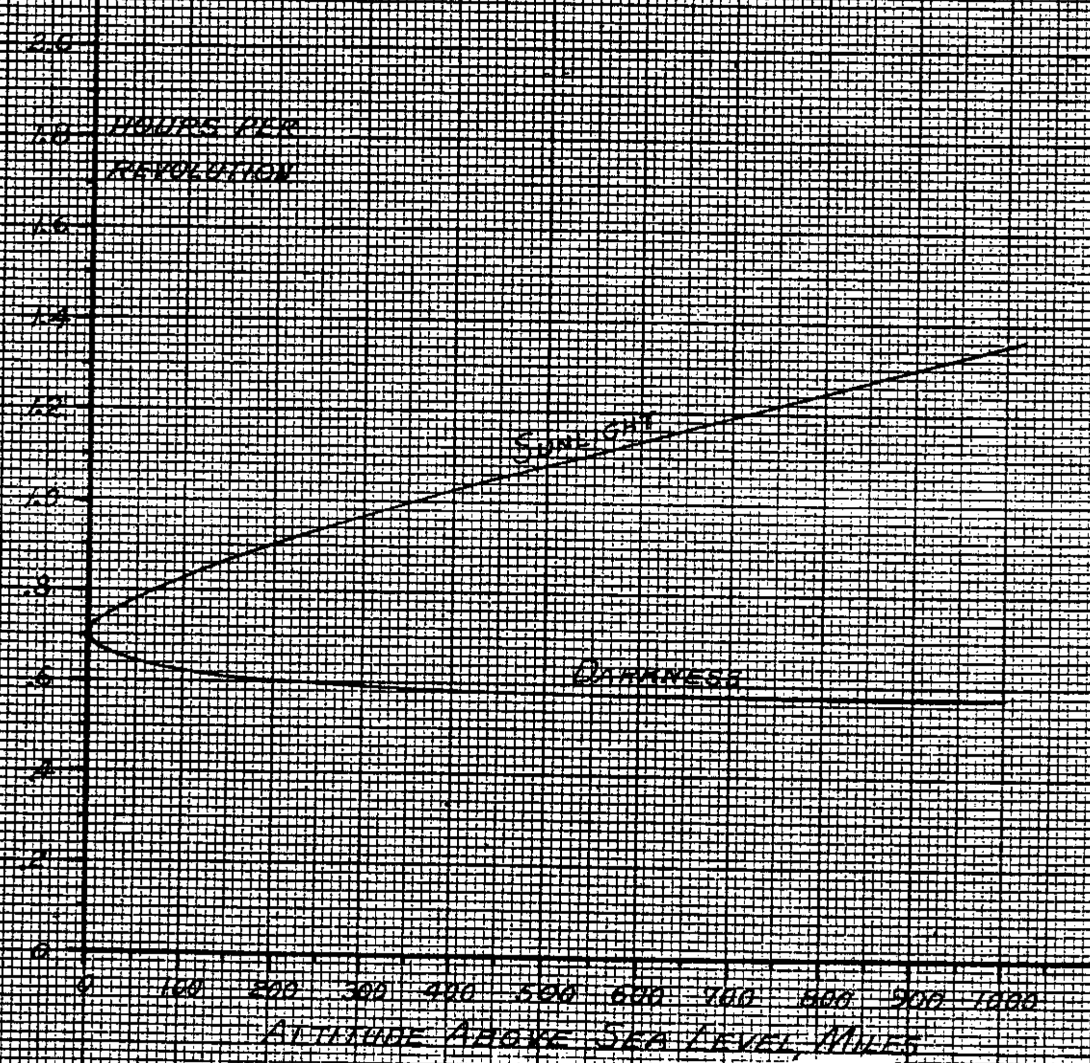

FIGURE 7 CHAPTER II

Chapter 11

11. PROBLEMS AFTER ORBIT IS ESTABLISHED - (Cont'd)

Aerodynamic Control while on Orbit. - The use of aerodynamic control is dependent on the ability to develop stagnation pressures (or indicated airspeed) of reasonably large order of magnitude. The stagnation pressure in turn, depends on the atmospheric density and the vehicle speed. In the present case, the orbital speeds are high, but at the desirable cruising altitudes the densities are extremely low. This follows directly from the fact that the densities must be kept low so that the drop per revolution is also low. The following table gives values of the indicated speed, V_i, for several altitudes, where

$$V_i = \sqrt{\rho/\rho_o}\ V$$

and where the stagnation pressure q, is

$$q = (1/2)\rho V^2 = (1/2)\rho_o V_i^2$$

Here ρ_o is standard sea level density.

ALTITUDE Miles	SPEED f.p.s.	DENSITY Slugs/cu.ft.	INDICATED SPEED f.p.s.
0	25,900	2.4×10^{-3}	25,900
100	25,600	1.07×10^{-10}	5
200	25,300	1.7×10^{-13}	0.2
300	25,000	1.3×10^{-16}	0.006
400	24,700	6.1×10^{-19}	.0000004

Now the force per unit area which can be developed by aerodynamic means is in the order of

$$\frac{F}{S} \approx .001\ V_i^2$$

Chapter 11

At 100 miles altitude, F/S is about .025 lb., and for an area of 50 sq. ft. a force of 1.25 lb. would be developed. Thus, since the weight of the vehicle is about 1000 times this force, we see that it is hopeless to obtain accelerations or balancing of weight moments through aerodynamic means. The only use of such controls at high altitude would be to balance other aerodynamic forces or moments which, of course, would be of equally small magnitude.

Another factor of importance is that the lift drag ratios at high altitudes are in the order of one. To avoid losing altitude while maneuvering, it would be necessary to counteract the drag by thrust - the thrust being of same magnitude as the lift. It is obviously more economical to use the thrust force for lifting to begin with, and to dispense with the aerodynamic control.

Chapter 11

11. PROBLEMS AFTER ORBIT IS ESTABLISHED

Attitude Control by Recoil - After the rocket fuel is exhausted the jet rudders become inoperative and other means must be adopted to control the attitude of the missile on its orbit. In the extremely rarefied air of the ionosphere it may seem unnecessary to head the missile in the direction of its path because drag is negligible. However, if a definite orientation of the missile is desired, be it for purposes of orienting missile-borne instruments, of regulating temperature aboard or of guiding the missile on a descent into the lower atmosphere towards a slow-down and eventual landing, then means to turn the missile deliberately must be provided. Two such means have been proposed - viz, (1) missile-borne flywheels which when impelled will impart an equal and opposite angular momentum to the missile and (2) small torque rockets.

The feasibility of accommodating adequate flywheels in the satellite missile can be gleaned from the following study. Either in the instrumentation head or in the ring space around the thrust nozzle there appears to be room to accommodate three flywheels; one each for pitch, yaw and roll. The projectile weighing about 1000 lbs. empty is estimated to have a radius of gyration of 3 ft. An 8" diameter flywheel weighing about 2 lbs. would have to be rotated 50,000 times as much and as fast as the missile is to be turned in the opposite direction by reaction, provided no extraneous moments interfere. Spinning this flywheel at 2500 RPM would turn the projectile end for end in ten minutes. The power necessary to attain this speed in say 20 seconds, and to keep it running would be less than 1/50 HP. The braking could be done by friction. Less power would be required if more time is allowed or if the flywheel is made larger.

Actually there should be no pressing need for high rates of tilting the projectile. Much smaller rates should suffice for casual corrections. The average rate of rotation necessary to maintain the heading aligned with the path is one revolution in 95 minutes, about 1/5 of what the control device envisaged above could cope with. However, there is no reason why the orbital angular momentum should not be already imparted by jet rudder action during the last phase of the powered ascent before entry into the orbit.

As to roll control, the lesser moment of inertia of the vehicle in roll probably admits of similar attitude control with a smaller flywheel device than pitch and yaw control.

It would thus appear that a flywheel type of machine to influence the attitude of the satellite projectile in the absence of extraneous moments poses no serious problem from the viewpoints of bulk, weight, and power involved nor is the apparatus complicated or delicate.

Problems however, remain to be solved, to be sure. The precision with which orientation of the missile will have to be predicted or commanded remains to be determined and the methods by which the attitude of the projectile with respect to its path over the earth can be telemetered deserve serious study. Even though the air density at the orbit altitude is very small, aerodynamic disturbances may yet be commensurable with the control moments considered. If such aerodynamic moments are present for a large part of the time then their continued counteraction may entail the accumulation of large flywheel momentum.

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It would be possible to allow much larger maximum flywheel speeds than the 2500 RPM previously mentioned. Limitations are drawn by technical considerations of permissible flimsiness, because the larger a flywheel of a chosen weight, w, is made the longer it can absorb a given torque before rupturing its rim. The time integral of the reaction moment that can be put into the flywheel is $\int M\,dt = wL/\omega = wr\sqrt{L/g}$ where w and r are weight and radius of the rim while L is the breaking length of the material and ω the angular velocity eventually acquired by the flywheel. Assuming that the largest diameter flywheel of which a pair could perhaps be accommodated in the satellite missile is 2r = 32" and that the rim is made of steel having a breaking length of 67,000', then the product of reaction moment in ft.-lb. by the time in seconds that can be derived from each pound of rim weight is 60 ft.-sec. before the wheel would fly apart at 9600 RPM. Some factor of safety would have to be provided. For instance, a well-proportioned flywheel of 2 ft. diameter weighing 20 lbs. which would be allowed to attain 10,000 RPM would cope with a systematic average aerodynamic moment of 1/100 ft.-lb. for 10 orbit periods in its plane. This amounts to 2 lbs. of wheel weight investment per orbit.

While rotors are spinning there will be gyroscopic cross influences which may require mutual corrective devices but they are not accumulative over more than half a satellite period.

The second proposal of attitude control by reaction envisages small gas exhaust recoil guns. Six or eight such devices arranged to emit jets tangentially to the skirt of the projectile can provide control of rotation in pitch, yaw and roll.

If the gun were mounted to fire radially at a leverage ℓ from the center of gravity of the projectile of weight W and radius of gyration i, it would have to develop a recoil force of $P = \pi nWi^2/30\ g\ell t$ in order to accumulate a pitching rate of n RPM in t seconds. Assuming $\ell = 4$ ft., $Wi^2/g = 300$ slugs ft.^{2} and $n = 1/20$ as before, $Pt = \pi/8$ lb.-sec. In other words, a recoil of about 6 ounces acting for one second or 1/10 ounce acting for one minute would suffice to achieve the specified effect. The amount of gas to be discharged at supersonic velocities under some pressure would be of the order of 1/10 oz. each time such control is given. Such gas could be branched off from an existing nitrogen pressure system or from an alcohol oxygen burner or even an oxygen vaporizer.

Again the question of how much aerodynamic pitching moment the control may have to cope with remains to be investigated. If again 1/100 ft.-lb. average torque were to be generated continuously, then only 1/4 lbs. of gas per circuit of orbit would have to be expended. On the first glance this looks about 8 times as favorable as the flywheels, but considering that momentum is no longer conserved for moment reversals the two schemes appear essentially on a par as far as weight investment is concerned. Other means may be considered for creating erecting moments or for keeping the disturbing moments small. One of the most drastic ones would be to house the real "payload" instruments in a spherical shell and expel it from the satellite vehicle after it is established in the orbit.

Chapter 11

11. PROBLEMS AFTER ORBIT IS ESTABLISHED. - (Cont'd)

Loss in Height Due to Resistance while on Orbit. - Preliminary calculations on the loss of altitude as time goes on showed that at altitudes of about 100 miles only small rates of drop would be encountered. Such calculations were based on the following formula, obtained by small perterbations from the basic equations of motion.

$$\frac{\Delta R}{R} = 4\pi \frac{D}{W}$$

ΔR is the loss in altitude per revolution, R is the orbital radius, D is the drag, and W the weight. For a hasty estimate of drag one is tempted to say $C_D \sim \frac{1}{\sqrt{M^2-1}} \sim \frac{1}{M}$, and then $D = \frac{1}{M} \rho \frac{V^2}{2} A$

Here M is the Mach number, ρ the air density, V the speed of the vehicle, and A its frontal area. To arrive at a magnitude for ρ, we assume an isothermal atmosphere. From the condition for equilibrium of the atmosphere, we have

$$dp = -\rho \, g \, dh$$

where p is the pressure, and h the altitude. From the gas law, we have

$$dp = d\rho \, g \, R_g T$$

where R_g is the gas constant.

Combining these two equations to eliminate the pressure yields

$$\frac{d\rho}{\rho} = -\frac{dh}{R_g T}$$

Integration then leads to

$$\log \rho = -\frac{h}{R_g T} + \text{const.}$$

The constant can be evaluated by saying $\rho = \rho_o$ when $h = o$.

Chapter 11

Then $\log \rho - \log \rho_o = -\frac{h}{R_g T}$

This can be written

$$\rho = \rho_o e^{-\frac{h}{R_g T}}$$

Suppose, now, the following numerical values are assigned to the quantities in this relation.

$R = 4,000 \times 5,280$ ft.

$W = 1150$ lbs.

$A = 8.7$ sq. ft.

$V = 26,000$ ft. per second

$M = V/\sqrt{\gamma g R_g T} = 26,000/\sqrt{1.4 \times 53.3 \times 500 \times 32} = 24$

$\rho_o = .002378$ slugs/cu. ft.

$h = 100$ miles $= 528000$ ft.

$T = 500^\circ$ R.

The density can easily be computed to be $.48 \times 10^{-11}$ slugs per cu. ft., and the drag is .00059 lb. The drop per revolution is then found to be 135 ft. Doubling the altitude, making it 200 miles, would have the effect of squaring the value of ρ/ρ_o, and finally changing ΔR to about 10^{-9} feet per revolution.

These results would seem to indicate that the loss in altitude was sufficiently small at altitudes well above 100 miles. In the first place, due to the extremely low Reynold's numbers encountered at high altitudes, the drag must be evaluated by unconventional methods. Secondly, the assumption of an isothermal atmosphere can carry with it large errors.

Chapter 11

For these reasons, a more comprehensive study was undertaken. A re-evaluation of the drag coefficients, based on the principles of the kinetic theory of gases was made and is presented in Appendix B. The revised drag coefficients were found to be considerably higher. This increase is seen to be reasonable when the known variations with Reynolds number are taken into account. In addition, a search of the literature was made in order to arrive at better estimates of the density of the atmosphere. The results of this study are presented in Appendix A. The following table sums up the corrections to the above method which can be made.

Altitude, miles	0	100	200	300	400
Temperature, deg. Rankine	519	906	1670	774	671
μ, viscosity, lb.sec./sq. ft.	$3.7+10^{-7}$	5.5×10^{-7}	8.3×10^{-7}	4.9×10^{-7}	4.4×10^{-7}
ρ, density, lb. sec.^{2}/ft.^{4}	2.4×10^{-3}	1.07×10^{-10}	1.7×10^{-13}	1.3×10^{-16}	6.1×10^{-19}
V, orbital speed, fps	25,900	25,600	25,300	25,000	24,700
V_i, $V\sqrt{\rho/\rho_o}$, indicated speed, fps	25,900	5	0.2	0.006	.0004
N_R, Reynolds no.	2×10^{9}	6×10^{1}	6×10^{-2}	8×10^{-5}	4×10^{-7}
M, Mach no.	23.2	17.5	12.7	15.7	14.5
C_D, drag coefficient	.2	1.0	2.0	2.0	2.0
Drag, lbs.	3×10^{6}	6×10^{-1}	2×10^{-3}	2×10^{-6}	1×10^{-8}
ΔR, drop per rev., ft.	7×10^{11}	1×10^{5}	5×10^{2}	5×10^{-1}	2×10^{-3}
Approx. time to drop to earth	0	1 hr.	3 weeks	23 yrs.	10 centuries
Approx. range before dropping to earth, mi.	0	1.75×10^{4}	8.7×10^{6}	3.43×10^{9}	1.47×10^{11}
Cycles before dropping to earth	0	6.9	3330	1.28×10^{6}	5.37×10^{7}
Time to drop to earth if ρ is in error by a factor of 1000	0	0	.5 hr	8.5 days	1 yr.

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The viscosity data were obtained, as functions of temperature, from McAdams, Heat Transmission, Second Edition, p. 411, and Durand, Aerodynamic Theory, Vol. 6, p. 227. Using these data, it is found that ΔR is grossly different than was calculated by the first, erroneous, set of assumptions.

The primary cause of the large difference between the two calculations is the difference between the isothermal and the actual atmosphere. The large temperatures existing at great altitudes causes the density to drop off much more slowly than would have been predicted by using lower temperatures. The following table, for example, shows the actual and isothermal ($500^{o}R$) values.

ALTITUDE	ρ ($T=500^{o}R$)	ρ(ACTUAL)
100 mi.	4.8×10^{-12}	1.07×10^{-10}
200 mi.	2.3×10^{-25}	1.7×10^{-13}

These differences are the direct result of the type of temperature variation assumed in each case. It will be noticed that the revised values shown in the table indicate that a vehicle starting in an orbit at an altitude of 200 miles would remain aloft about 3 weeks. However, when the accuracies of the assumption underlying these calculations are examined critically, particularly the values of density in the upper atmosphere, it is found that the duration figures may yet be in error by a factor of 1000. For this reason an additional line is included in the table showing the lower limit believed possible for the duration. Confronted by our present state of ignorance, one can only conclude that the minimum initial altitude for a satellite should be 200 miles while the recommended altitude would be between 300 and 400 miles.

Chapter 12

12. THE PROBLEM OF DESCENT AND LANDING

An important ultimate goal for any vehicle must be that of carrying human beings with safety. One obstacle which seems to stand in the way in the present case is the great energy stored in the vehicle, a part of which serves to heat the vehicle on descending into the lower atmosphere. The study which follows is an attempt to show the feasibility of lowering the craft without destroying it by fire, so that a safe landing can be made on the surface of the earth.

Landing introduces primarily a problem of dissipation of the tremendous energy stored in the vehicle by virtue of its speed. If all of this energy, for example, were to be converted to sensible heat of the vehicle, the temperature would be increased by

$$\Delta T = \frac{\frac{1}{2}\frac{W}{g}V^2}{cWJ} \quad \text{-------------------} \quad (1)$$

where:

W = vehicle weight, lbs.

g = acceleration of gravity, ft./sec./sec.

V = vehicle speed, ft./sec.

c = specific heat of vehicle, BTU/lb./$^\circ$F.

J = 778 ft. lbs./BTU

choosing $g = 32.2$, $V = 26{,}000$, $c = .12$, leads to $\Delta T = 112{,}000\ ^\circ$F. The time required to radiate all this energy into space can be estimated as follows:

$$\Delta t = \frac{\frac{1}{2}\frac{W}{g}V^2}{\sigma\epsilon T^4 AJ} \quad \text{-------------------} \quad (2)$$

where:

σ = Stefan-Boltzmann constant, BTU/sq. ft./hr./($^\circ$F.)^4

ϵ = emissivity

T = vehicle temperature, $^\circ$R.

A = vehicle surface area, sq. ft.

choosing $\sigma = .173 \times 10^{-8}$, $\epsilon = 1.0$, and $A = 95$, $W = 1150$, leads to $\Delta t\ T^4 = .940 \times 10^{14}$. If the vehicle is allowed to radiate at as high a temperature as 1000°R., then 94 hours would be required for the necessary removal of heat energy. The burning of meteors as they enter the earth's atmosphere is a striking example of the type of phenomenon which might be expected.

Actually the dissipation times will be only a small fraction of that computed above because a large portion of this energy is left behind the vehicle in the form of heat in the wake. The remainder is radiated from the surface, is stored in the vehicle as heat, or is retained as kinetic energy to be dissipated during the landing run on the ground. We deal here only with that energy which is fed into the vehicle as heat, and which must be radiated back into space.

The solution to the problem must come from a method of controlling the trajectory during the landing glide so that the heat input can be dissipated at a temperature sufficiently low to prevent damage to the vehicle. Such control of the glide path must be accomplished by aerodynamical means, implying that lifting surfaces must be provided to prevent the vehicle from entering the denser region of the atmosphere too rapidly. The trajectory will now be investigated by studying the heat flow balance.

The heat input from the boundary layer of a cone is

$$\frac{\partial Q}{\partial t}\frac{1}{S} = H(T_{BL} - T_s) \tag{3}$$

where:

Q = heat energy, BTU

t = time, sec.

S = surface area, sq.ft.

H = heat transfer coefficient, BTU/°F./sq.ft./sec.

T_{BL} = boundary layer temperature

T_s = vehicle surface temperature

The heat transfer coefficient has been shown to be

$$H = .0224 \frac{\lambda}{\ell} \left(\frac{V\ell}{\nu}\right)^{0.8} \beta^{1/3} \qquad (4)$$

in which

λ = heat conductivity of air at the temperature T_{BL}, BTU/sec/ft/°F.

ℓ = cone length, ft.

V = velocity of vehicle, ft/sec.

$\nu = \mu/\rho$

μ = viscosity of air at temperature T_{BL}, lb.sec/sq.ft.

ρ = density of air at temperature T_{BL}, slugs/cu.ft.

β = total cone angle, rad.

The work which follows will be much simplified by the arbitrary assumption that the exponent of the Reynolds number be changed from 0.8 to 1.0. (4) then becomes

$$H = .0224 \frac{\lambda}{c_p \mu} \; c_p \rho \, \beta^{1/3} V \qquad (5)$$

where:

c_p = specific heat of air at constant pressure, BTU/lb/°F.

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The group $\lambda/c_p\mu$ is the reciprocal of the Prandtl number, and is approximately constant. Using 0.7 for $c_p\mu/\lambda$, 0.24 for c_p, and .2 for ρ, we find

$$H = .0045\ \rho V \tag{6}$$

(6) can now be combined with (3), yielding

$$\frac{\partial Q}{\partial t} = .0045\ \rho VS(T_{BL} - T_s) \tag{7}$$

Now it is well-known that the stagnation temperature is related to the Mach number by the relation

$$T = T_h(1 + .2M^2) \tag{8}$$

Here we use T_h for the ambient air temperature. Experience based on German wind tunnel results has shown that the boundary layer temperature is somewhat less than the stagnation temperature, so we can write

$$T_{BL} = T_h(1 + .18M^2) \tag{9}$$

This relation, combined with (7) gives the final expression for the rate of heat input.

$$\frac{\partial Q}{\partial t} = .0045\ \rho VS\ (T_h + .18M^2T_h - T_s) \tag{10}$$

The effect of the sun is not important here, as is shown by comparison to the flight of meteors, which become very hot only on passing through a gaseous atmosphere.

Chapter 12

For the heat output, we have the familiar radiation equation,

$$\frac{\partial Q}{\partial t} = \epsilon \sigma S T_s^4 \qquad (11)$$

in which

ϵ = emissivity

σ = Stefan-Boltzmann constant

Now we shall define a special trajectory of descent, one in which a constant temperature of the vehicle is maintained. That assumption implies, as is shown by reference to equation (11), that a fixed rate of heat transfer is characteristic of this trajectory. This fixed rate of heat output must be matched by an equal heat input by proper choice of speed at each altitude in order that the skin temperature not change. The analytical expression of this trajectory is obtained by equating (10) and (11)

$$.0045\ \rho V\ (T_h + .18M^2T_h - T_s) = \epsilon \sigma T_s^4 \qquad (12)$$

For any given temperature of the vehicle, (12) defines a unique speed for each altitude. Curves showing these glide paths for several vehicle temperatures are on figure 1.

There are several important results to be obtained from figure 1. First of all, since the contours of fixed skin temperature are essentially horizontal, no serious heating due to friction will be encountered above the 70 mile level, at least for speeds in the general order of 25,000 feet per second. Second, at heights below

Analysis LANDING
Prepared by A.S.
Date 5-6-46

DOUGLAS AIRCRAFT COMPANY, INC.

S.M. Plant

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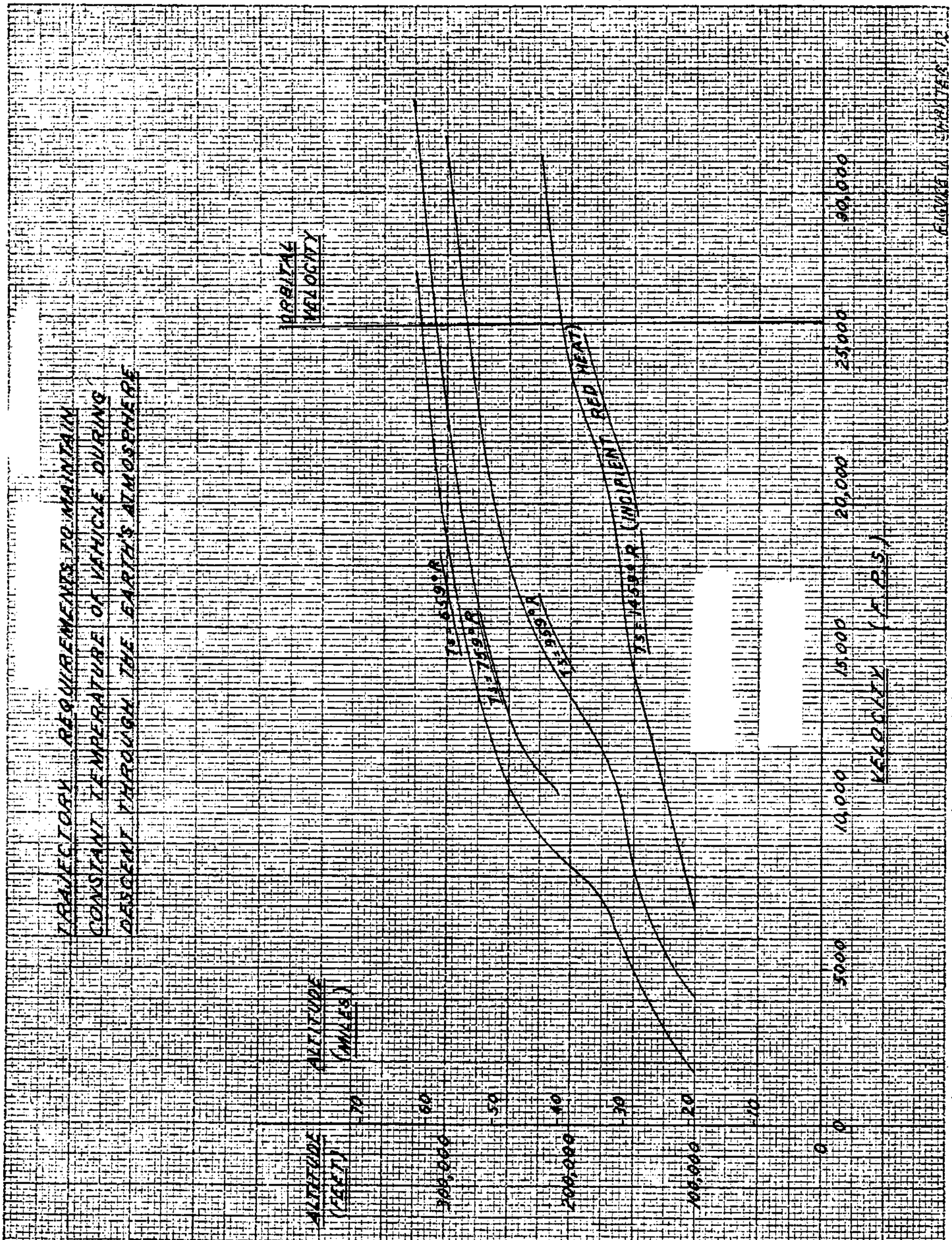
Report No. 11827



70 miles, the temperature rises to very high values, unless the speed is reduced along with the reduction in altitude. To make a successful glide, therefore, it is necessary to be able to control the speed. We shall assume, then, that wings of small size will be used for speed control during descent, and for making landings. It is important to notice at this point that once sufficient lift is provided to control the speed, the effect of high drag is to reduce the time of descent. Arrangements with poor values of lift drag ratio are not, for that reason, out of order here.

The question arises as to just what size wing is required. The answer to this question comes from considering the vertical forces acting. If wings are used to slow down below orbital speeds, then a lift on the wings must be developed which is equal to the difference between weight and centrifugal force.

$$L = (1/2)\rho V^2 C_L S' = W - \frac{W}{g}\frac{V^2}{R} \qquad (13)$$

where:

L = lift

S = wing area

R = radius of orbit $\doteq$ earth's radius

W = vehicle weight

ρ = density of atmosphere

V = vehicle speed

C_L = lift coefficient

g = acceleration of gravity $\doteq$ 32 ft/sec.^{2}

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The parameter C_LS can readily be computed now for each of the trajectories of figure 1. The results of these calculations are given in figure 2. Maximum values of C_LS as a function of skin temperature are given in figure 3. The vehicle weight was taken to be 800 lbs. for these calculations.

An additional consideration which must be taken into account, is the landing speed. Here C_LS is a function of the desired landing speed according to

$$C_LS = \frac{2W}{\rho V^2}$$

For a landing at sea level, the landing speeds corresponding to several values of C_LS have been noted in figure 3.

Inspection of figure 3 shows that wing areas compatible with reasonable skin temperatures and modest landing speeds are possible of achievement. For example, a landing speed of 100 mph, a maximum temperature of 300°F., (existing for only a brief section of the glide path) are consistent with a wing area of 30 sq.ft., (for $C_L = 1.0$) even if body lift is ignored. This corresponds to a 27 lb. wing loading.

It can be concluded as a result of this study that it appears possible to glide a space vehicle down to a landing on the earth's surface without destruction by fire or from crash landing. This gives rise to a hope of attempting space flights in man carrying craft.

Analysis LANDING
Prepared by AS
Date 5-1-46

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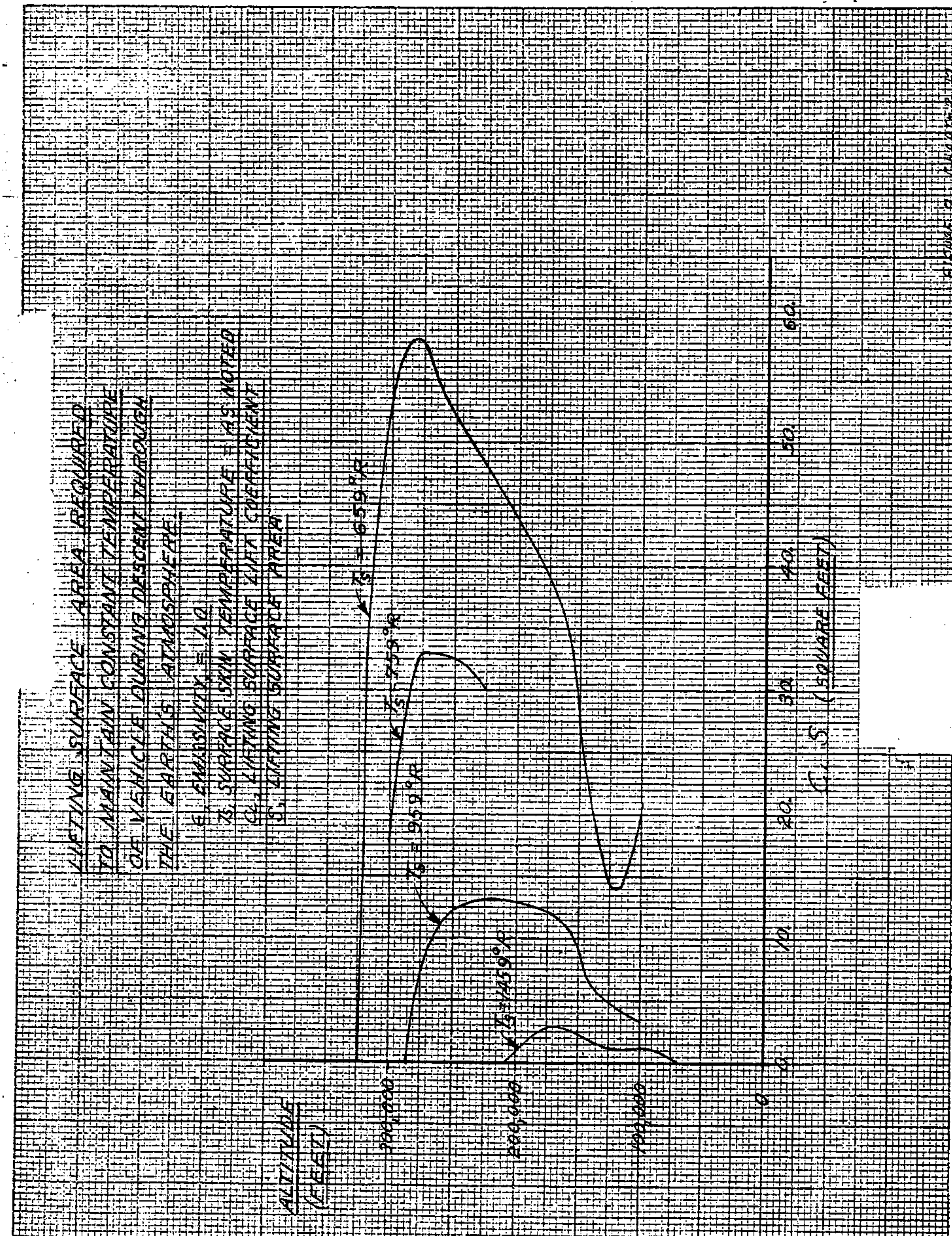

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Analysis LANDING
Prepared by AS
Date 5-7-46

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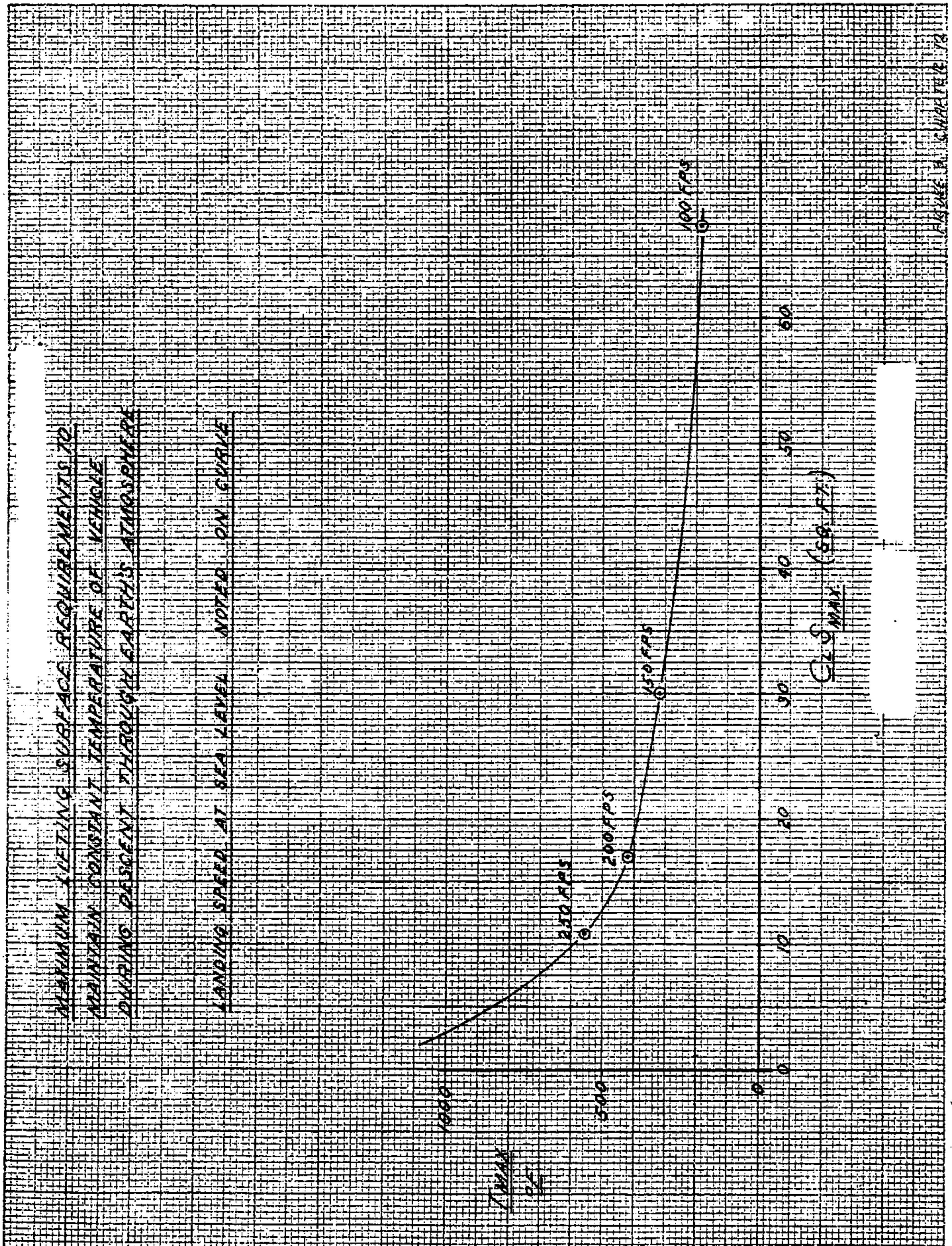

Chapter 12

The ascent to altitude differs from the descent in one important way. The climb starts at zero velocity so that the region of high densities is passed through before great speeds are built up. In the descent on the other hand, the regions of high density and of high heat transfer coefficient are traversed at very high speed. This fact makes for small gains in temperature during the climb. For a skin thickness in the order of 0.1 inches, a temperature rise of less than one hundred degrees Fahrenheit has been computed.

13. DESCRIPTION OF VEHICLE

A considerable amount of study has been devoted to the design of a practical satellite vehicle based upon the principles outlined in the preceding chapters. Two alternatives have been considered, one a four stage rocket fueled with alcohol, the other a two stage rocket fueled with hydrogen. Of these the former is lighter, smaller and by far the more conservative in terms of research requirements, safety and certainty. It will be described in some detail as to stages and technical components. The hydrogen vehicle is a much more speculative project, dependent on uncertain outcome of more ambitious and dangerous research projects. Its design aspects are therefore only briefly outlined in the last section of the present chapter for comparison. The three pictures shown at the beginning of the present chapter will give an idea of what the vehicles and their components are expected to look like. A rough weight breakdown of the two projects is presented in Chapter 7.

Shape and Dimensions. The shape will be that of a typical projectile, having a pointed nose and contoured sides which taper from a maximum width aft of the midsection to a minimum width at the base which is compatible with the motor exit diameter and space requirements for jet-vane controls. The length will be of the order of 60 to 70 feet and diameter about 12 to 14 feet, giving a fineness ratio between 4:1 and 6:1. Such a vehicle would have a density ratio, .029 pounds of loaded missile per cubic inch of volume, comparable to the German V-2 ratio of approximately .020. This indicates considerable progress towards better utilization of space.

Stages. The vehicle will be divided into four stages, the primary or first stage being nicknamed "Grandma", the second stage "Mother", the third stage "Daughter", and the final satellite vehicle "Baby". Baby will carry the payload and intelligence for all stages, in addition to its own fuel, pumps, motor and guidance, and will comprise between 1/5 and 1/4 of the length of the total vehicle. Daughter and Mother will each carry only fuel, pumps, motors, and controls, being guided by Baby, and will be about the same length as Baby. Grandma will comprise almost half the length of the total vehicle, and will also contain fuel, pumps, motor, and controls, being guided by Baby.

Structure. From the standpoint of size and applied loads, this vehicle is not out of proportion to present day large airplanes of 50,000 lb. gross and over, so it is believed entirely feasible to use airplane type of construction consisting of reinforced sheet metal. The possibility of elevated temperatures existing on the surface will probably eliminate the use of aluminum alloys for skin covering and require the use of high strength stainless steel. Motor loads in any stage may be carried into the outer skin of that stage by a truncated cone diaphragm extending from the motor base to the outer skin. The thrust load of any stage can be transmitted into the next succeeding stage by pads on either the motor base or outer covering of the next stage. The base of Grandma must be amply reinforced to withstand the dead weight of the entire assembly when mounted vertically ready for launching.

Although all flight loads are essentially directly along the axis of the vehicle, some reinforcing may be required to account for handling loads while each unit is in a horizontal position.

Tankage. In the event that the oxidizer tank does not need to be insulated from the missile outer skin, the skin will serve as both tank shell and missile shell. If insulation becomes necessary, a double skin thickness will be required, thereby increasing the length or diameter, or both, of the entire vehicle. The front diaphragm of the forward tank in each stage must be reinforced to withstand internal pressures created by excess of vapor pressure over outside air pressure at higher altitudes. When liquid oxygen is used as an oxidizer, some form of insulation will probably be required between the two tanks. This will require a double diaphragm between the two tanks. All diaphragms can be of a near-ellipsoidal shape to more nearly approach the optimum weight vs. volume ratio under the existing hydrostatic head.

Motor. The motors are designed for liquid oxygen and alcohol as the propellants with a mixture ratio of 1.5 (wt. of oxidizer/wt. of fuel.) The motor of the fourth stage is patterned after the light weight type developed by the Jet Propulsion Laboratory of C.I.T. The motors of 1st, 2nd, and 3rd stages are patterned after the German throatless motor which was intended for later use in the V-2 and Wasserfall. It consists primarily of the divergent section of the conventional motor with a short combustion section the same size as the throat added to the front. The injector is of completely new design which gave good combustion very close to the face of the injector. With but a very small penalty in gas velocity this new type of combustion chamber was adopted in the design of the motors for Stages 1, 2 and 3.

All the motors are regeneratively cooled with the Alcohol being used as the cooling medium. The Alcohol is brought to the exit section of the motor where it enters the cooling coils for circulation around the motor. The Alcohol is then directed to the injector head for injection into the combustion chamber.

The nozzle expansion ratio (area of exit/area of throat) is 6.0 for the first stage and 20 for all succeeding stages. The nozzle is shaped like a bell jar for rapid expansion near the throat and very little expansion near the exit. This will direct the jet flow straight to the rear and will permit the location of the jet rudder control inside the nozzle exit.

<u>Fuel System</u>. The fuel system is visioned as comprising a dual pump arrangement for delivering the fuel and oxidizer from the tanks to the motor through the necessary control and regulator valves. The fuel can be used as coolant for the motor by passing through coils arranged either helically or radially around the motor. Under this arrangement, both fuel and oxidizer pumps would be mounted on a common shaft and so designed that proper mixing ratios would be maintained at all operating speeds. The pump shaft would be driven by a steam turbine. Past practice has been to use the action of a catalyst on hydrogen peroxide to generate steam for the turbine. The hydrogen peroxide is delivered to

the steam generator by nitrogen gas under high pressure.

In some cases it might be necessary to use multi-stage pumps to prevent cavitation, but all pumps must be designed for extreme light weight by the use of hollow blades and other devices. They must also operate close to peak efficiency at design speed and at lower throttled speeds. The design of both turbine and pumps will be facilitated by the short duration of run.

Various on-off valves must be provided to start each turbine pump at the proper time, and some means of throttling to prevent excessive vehicle acceleration must be incorporated in these valves. A control valve operated by the motor coolant must be included in the oxidizer line to prevent the oxidizer from prematurely reaching the motor and causing an explosion.

Plumbing to the motor must be provided with expansion joints to prevent breakage of lines.

<u>Controls</u>. The control system of the missile comprises attitude and thrust controls for each stage and a common regulator system which governs the controls from a central brain station located in the final Baby unit. The attitude control is effected by a cruciform array of four vanes mounted on radial shafts near the rim of the motor nozzle. They are tilted on radial shafts by servo motors so as to deflect the hot rocket gas jet. By symmetrical deflection pitching or yawing reaction moments are developed. To create a rolling moment a cyclic differential deflection is superimposed. Thrust is controlled by regu-

lating the fuel pumping speed and governed by accelerometers.

The master regulator in the "brain station" comprises an automatic pilot system based on a tri-axial reference gyroscope system and a tilting program governor which is designed to follow a pre-set ascent trajectory calculated to enter the orbit smoothly. A radio altimeter as well as radio guide beam or remote command receivers are installed in the Baby missile. Their outputs are mixed with those of the automatic regulator so as to permit overriding the latter and applying corrections for unforeseen disturbances.

The Baby vehicle is further equipped with small reaction motors designed to exercise a moderate amount of control of orientation of the projectile when in the orbit.

Telemetering and beacon equipment will supplement the control system by maintaining transmission of tracking and intelligence information to the ground director station.

Accessories. Among accessories carried on the missile are: electric power plants to supply electric power to the instrumentation, regulators, and servosystem; stage separation devices and their controls; safety devices to interlock various functions; and appliances necessary for fueling, readying and testing before take-off.

Payload. The nose cone of the Baby vehicle is reserved for the payload and meant to house most of the useful instrumentation which is to be carried on the journey to secure information gathered there. The payload cone dimensions as now visualized are about 3 ft. diameter of the bulged base, and 7 ft. length. It houses about 20 cu. ft.; 500 lbs.

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are alotted to its content as payload proper. Some of the payload items may optionally be accommodated in the annular space around the thrust motor of the Baby vehicle.

Operation. The operational procedure in launching a satellite missile is visualized essentially in the following manner: All parts of the vehicle after having been proof tested are brought to the launching site where a base with a blast apron and suitable scaffold have been installed. Here the complete vehicle is assembled upon a launching stand by progressively hoisting the lesser stages on top of the larger ones. All systems are checked as each stage is completed. Next all containers are filled from bottom to top and eventually topped off, and the hoist moved away to clear. In the meantime all ground observation stations are placed, manned and warned. Actual firing is triggered remotely from a protected control station according to pre-arranged schedule.

Hydrogen Alternative. The alternative project of a hydrogen fueled vehicle as shown in an exploded view in the last of the three pictures heading the present chapter, would differ from the alcohol fueled version in that it would be much larger, that it would be a two-stage affair and that elaborate precautions against the dangers of hydrogen leakage and evaporation would have to be taken. This is indicated in the picture by showing a double wall around the liquid hydrogen tank. Whether the oxygen would eventually be carried above the hydrogen as shown or rather below it would have to be decided after research will have clarified the conflict between advantages and disadvantages of either system, but the choice would hardly affect the general appearance of the assembled missile. The operational problems of liquid hydrogen fuel cannot be foreseen as well as those of the alcohol fuel.

Chapter 14

14. POSSIBILITIES OF A MAN CARRYING VEHICLE

Throughout the present design study of a satellite vehicle, it has been assumed that it would be used primarily as an uninhabited scientific laboratory. Later developments could alter its capabilities for use as an instrument of warfare.

However, it must be confessed that in the back of many of the minds of the men working on this study there lingered the hope that our impartial engineering analysis would bring forth a vehicle not unsuited to human transportation.

It was of course realized that 500 lbs. and 20 cubic feet were insufficient allotment for a man who was to spend many days in the vehicle. However, these values were sufficient to give assurance that livable accommodation could be provided on some future vehicle.

The first question to be considered in determining the possibility of building a man carrying vehicle is whether prohibitively high accelerations can be avoided during the ascent. The V-2 gave hope that this was possible. Our own studies have likewise shown that the optimal accelerations do not exceed about 6.5g. A man can withstand such acceleration for the periods of time involved (several minutes) if he is properly supported with his trunk lying normal to the direction of the acceleration. In Chapter 8, it will be remembered, the analysis showed that the performance could be improved a small amount by throttling each rocket motor during the latter portion of its burning period in order to reduce the structural loads. Under these conditions, the maximum accelerations could be profitably reduced to about 4 g. All these findings confirm that ascent offers no insurmountable obstacle to the construction of an inhabited satellite vehicle.

Next we consider the safety and welfare of the man after the vehicle has been established on the orbit. Popular fiction writers have devoted considerable thought and ingenuity to means of furnishing him with air, food and water. The most ingenious of these solutions is that of the balanced vivarium in which plants and man completely supply each others needs. Leaving these problems to the inventors, we ask ourselves the engineering questions of whether we can provide livable temperatures and a reasonable protection against meteors. In Chapter 11 we have seen that the answers are tentatively in the affirmative.

Lastly we consider the problem of safely returning the vehicle's inhabitant to the surface of the earth. In Chapter 12, we have seen that, with reasonable area wings, we can control the descent sufficiently to avoid dangerously high temperatures. These same wings are adequate to accomplish the final landing on the earth's surface.

The above thoughts are far from final answers on this problem. However, they do give a note of assurance that the hope of an inhabited satellite is not futile.

Chapter 15

15. ESTIMATION OF TIME AND COST OF PROJECT

Estimate of Time Required for Project - The progress of the preliminary design study presented in this report has indicated it is extremely important that at least six months additional research and preliminary design work be done on this project before any definite design and building program be established. It is possible by doing this that a three to five year program could be planned which would take into consideration the technological advances expected during this period so that obsolescence would not overtake the development.

The above statement and those which follow in the sections on Program and Costs are based on the requirements for the alcohol-oxygen rocket units, and not on the hydrogen-oxygen alternate. Estimates on the hydrogen-oxygen units are virtually impossible at this time due to the unknown factors in the use of liquid hydrogen.

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Program of Building and Testing. In view of the complexity of the job and the increase of its magnitude with the transition to larger launching stages, which for brevity's sake will be referred to as Baby, Daughter, Mother and Grandmother, it will be desirable not to tackle all stages at once but rather to progress from the smaller to the larger in sequence in order to reap the full benefit of experience gained as the job moves along. At present the extrapolation from prototype to unprecedently large mother stages can, by the very nature of such a process, only be delicate and groping. The actual size of the real article sensitively depends on relatively small changes in technological assumptions. This apparent uncertainty will disappear as successive tests of the lesser stage units will supply the information upon which the design of the larger ones can be solidly based. The smallest baby and daughter stages however are of conventional dimensions well within existing practice and experience, so their design can well be immediately begun together in order to accelerate solution of the birth-aloft problem which is the most drastic innovation over the existing practices. The manufacturing program will be overlapping as dictated by the testing program. It is anticipated that more of the smaller stage units will be built than of the larger; and that the stages will be tested individually and also in combination. Flight testing of any torso part of the entire 4 stage aggregate requires some provision of fairing, otherwise the high air resistance in the lower atmosphere is likely to seriously impair the behavior of the missile. This difficulty is proposed to be overcome by firing the smaller stage units with faired after bodies and the larger stage units with dummy heads. An estimate of the number of units to be built and the various combinations of the four stages proposed to be tested is ventured in the following tables in which capital letters indicate "live" propelled specimens, small letters unpowered dummy specimens.

	live	dummy
Grandmother	8 G	1 g
Mother	18 M	2 m
Daughter	23 D	7 d
Baby	23 B	12 b

A dummy specimen of each may have to be devoted to static tests.

Number of flight tests	12	8	4	4	4	4	2	3	3
Combination	B	Db	DB	Md	MDb	MDB	Gm	GMd	GMDB

The DB and MDb and especially the MDB tests will offer opportunities of carrying telemetering instrumentation aloft which should furnish some of the data needed in the final phase of the project. Incidentally, the MDB aggregate may have immediate applications as a long range missile or weapon. The last item of the list, GMDB represents the firing of the final article from an equatorial location into the orbit. All preceding tests except perhaps MDB are expected to be made on domestic proving grounds with suitably expanded range facilities. The magnitude of this construction and test program is of the order of about 100 V2's, on the basis of corresponding gross weight.

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Facilities required. This program is much more in the nature of an experimental enterprise than a production job. It is therefore imperative that Development, Design, Procurement, Logistics, and Testing be closely coordinated so as to form a continuous loop.

The engineering staff will comprise a large part of the personnel engaged in the project. It will consist of aircraft and rocket power plant engineers, instrument and automatic control experts, and specialists in the sciences and arts of radio, radar, telemetry, trajectory survey, artillery and flight testing.

The manufacturing facilities required will be in the nature of the experimental shop of a large aircraft factory. As there will be little need for quantity production methods, tooling may be best devised by jigging and in many respects by improvisation and adaptation. Provision for static tests of the structures and best provided at the manufacturing plant; static firing test facilities for the rocket engines in existence and under construction will suffice to take care of baby, daughter and mother stages. A suitable test stand to run the grandmother motor would still have to be created. The mother and grandmother units are probably too big to be transported to the flight test site in assembled condition. Provisions for part assembly shipment and reassembly at the test site will have to be made for them. This will comprise elaborate cranes, and scaffold there.

The operation of the baby and daughter can presumably be handled at an existing ordnance test range such as White Sands, N.M. with facilities now available there. Apparatus necessary to handle the mother stage which is about as big as two V2's can undoubtedly be developed in stride. When it comes to Grandmother which are of the order of 10 V2's each as far as weight and fuel is concerned, a new problem will arise and the question of whether this phase of the program should be considered at an inland firing range or located elsewhere, be it at the seashore or at the equatorial orbit emplacement site remains to be investigated.

Shop and field facilities for local erection, assembly, fueling, equipment testing, and observation may have to be largely duplicated at the equatorial site. The delivery and storage of fuel and liquid oxygen at the equatorial site may become a sizable enterprise. It may wind up with the establishment of a local oxygen liquefaction plant either land based at the site or shipborne on one of the vessels which will have to be assigned to the entire project.

Since the experimental orbit missile emplacement will have to be chosen within 1 or 2 degrees of latitude, only the following locations are geographically eligible: Ecuador, N.W. Coast of Brazil around the Amazon delta, French African Congo Coast; Kenya Colony in Central East Africa; Straight Settlements and Singapore, Borneo, Celebes, and finally any of the numerous equatorial Pacific islands between Halmahera and Howland or Baker. Of these Ecuador and Kenya offer possibilities of accessible mountain sites. Politically, however, it would be preferable to stay in American controlled territory. For reasons of radio altimetry a site near an East coast is desirable. Islands for several hundred miles east

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of the emplacement site are desirable as fixed observation stations. Most of these considerations point to the archipelago north of New Guinea as the logical region in which to look for islands on which utilizable war installations may be available. Adequate living facilities for the staffs and crews will have to be found or provided at the emplacement site. Rapid communication and transportation facilities between the site and the project headquarters will be a necessity. For the orbital observation and telemetering some 20 to 50 stations may have to be installed or positioned in a belt around the equator, across the Pacific Ocean, Ecuador, Brazil, Atlantic Ocean, French Congo, Kenya, Indian Ocean and Malaya. All these stations may have to be linked with each other and/or a central director station by a rapid communication system if continuous tracking and telemetering of the satellite missile is to be maintained, and particularly if its return to earth is to be guided.

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Estimate of Cost - In order to obtain an estimate of the cost of this project, the following assumptions were made:

(a)	All development, engineering, and fabrication on the basic vehicle in quantities outlined in the section Program for Building and Testing, estimated in accordance with standard practices in the aircraft industry	$50,000,000
(b)	All development, engineering, and construction on the following items, arbitrarily assuming to be equal to the work done under (a) above: power plant, propellant pumps, turbines, controls.	50,000,000
(c)	All development, engineering, and construction on the following items; instrumentation (payload), establishment of special launching facilities, transportation and analysis of test data assumed to cost	50,000,000
	Total Cost of Project.	$150,000,000

It should be emphasized that this estimate is very rough, was hurriedly prepared and is without benefit of any experience or actuarial records in this new field. It is possible that due to the large volume of fuel tanks in this vehicle as compared to standard aircraft, this cost estimate is conservative; however, the unknown difficulties which arise in any new field of endeavor would indicate the need of a conservative estimate. One of the important phases of

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the proposed preliminary design phase of the program should be used to further investigate German cost records in the development of the V-2 as a basis for determining costs on this project.