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Chapter 2

The Origins of Inertial Navigation in Space[1]

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Modern inertial navigation is a vivid example of the realization of an idea that at first looked like pure fantasy, but presently it enjoys wide practical application. From a modern technical point of view, the concept of inertial navigation can be summarized as follows.

In some reference frames $\xi^*\eta^*\zeta^*$ can be considered as inertial, in a gravitational field $\overline{G}(\overline{R},t)$ known for any point $\overline{R}$ and every instant t. There is a standard mass of an object moving in this field, and it is possible to measure continuously the components of an external force, applied to the mass, in projections on axes of coordinate trihedron XYZ, realized on the object.

Projections of the absolute angular velocity of the trihedron XYZ on its own axes, measured, for example, by gyroscopic sensors, are known at every moment. If, for the initial moment, the coordinates of the standard mass and the orientation of the trihedron XYZ are known, then it is possible to determine current coordinates of the mass, and hence those of the object in the reference frame $\xi^*\eta^*\zeta^*$, by continuously solving the kinematic Euler-Poisson equation, which determines the current orientation of the trihedron XYZ, and by integrating dynamic Newton equations for the standard mass. At the same time, the velocity of the standard mass relative to the frame $\xi^*\eta^*\zeta^*$ is also obtained. Thus, the complete navigational problem is solved by autonomous means.

General features of the inertial navigation problem are well understood today. Many different types of systems exist to provide guidance for rockets, aircraft, ships, and spacecraft. The routineness with which these inertial navigation systems operate, however, masks the tortuous path leading to the development of these practical systems.

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[1] Presented at the Sixteenth History Symposium of the International Academy of Astronautics, Paris, France, 1982.

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The pioneers of inertial navigation research—the Americans M. E. Carrie and F. R. Sweeny, and the Russian B. Alecseev—in the first two decades of the twentieth century made significant strides in resolving most of the fundamental problems of inertial navigation.[3] Their work presented a description of the method for solving navigational problems autonomously for the motion along the Earth's surface by means of two free gyroscopes and a pendulum, but it did not use the concept of double integration of accelerations with which subsequent development of the idea is closely connected. The double integration of accelerations for determining the path traversed was apparently first suggested in R. Wussow's patent, claimed in 1905, given in 1906.[4]

In 1911 in Germany, I. M. Boykov recommended measuring an acceleration by a two-coordinate pendulum and integrating it with the use of a clock mechanism with subsequent calculation of the path traversed by the usual technique.[5] He obtained a patent for this invention in 1928.

O. Dahlke's patent (Germany, 1914), concerned itself mainly with the description of a technique for horizontal leveling of accelerometers, and it mentioned the possibility of using the vertical accelerometer for calculating vertical velocity and determining the height of the object by integrating the signal of this accelerometer. He also pointed out the need for excluding gravity acceleration from this signal. This, apparently, was the first statement of the possibility, in principle, of inertial navigation in space.[6]

In the 1920s the concept of double integration of the acceleration signal, as a possible approach to the solution of the inertial navigation problem, attracted the attention of those involved in astronautics in several locations. For instance, a clear description of such a technique for determining velocity and location of rockets in space can be found in a book by Hermann Oberth in 1929. In his system, the stabilization of accelerometers, with respect to stars, was provided by free gyrostats. Moreover, the book explained the need to account for the dependence of Earth's gravity acceleration (which had been artificially excluded from accelerometer signals) on the distance from the center of the planet.[7]

A detailed description of the concept of inertial navigation in space was contained in a book by Robert Esnault-Pelterie published in 1930. He pointed out that gravity acceleration depended on the location of the spacecraft. The acceleration, measurable by mechanical devices on a moving vehicle, Esnault-Pelterie called "sensible," stressing the difference between this acceleration and full acceleration, which included the accelera-

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[3] M. E. Carrie, U.S. Patent No., 1,253,666 (1903); B. Alecseev, Russian Patent No., 28451, 30.IY.1916 (1911); F. R. Sweeny, *Geographic Position Indicator*. 1086246, 3.II.1914.

[4] R. Wussow, *Apparat zur Bestimmung von Geschwindigkeiten und Wegelangen* (Instrument to Determine Velocity and Distance), German Patent no. 179,477 (1905).

[5] I. M. Boykov, "Navigation Mittels Derivators" (Navigation by Means of a Measure of Drift), in *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, issue no. 11, 1911.

[6] O. Dahlke, *Einrichtung zum Messen von Beschleunigungen* (Equipment for Measuring Acceleration, Velocity, and Paths) (Geschwindigkeiten und Wegen).

[7] Hermann Oberth, *Wege zur Raumschiffahrt* (Road to Spaceflight) (Berlin, 1929).

tion due to gravity. He considered different approaches to integrating sensible accelerations by using a heavy mass in viscous liquid, a gyroscopic pendulum or a pendulum clock, with speed depending on the acceleration along the mean orientation of the pendulum.[8]

For the case of a vehicle moving along a line passing through the Earth's center, with fixed orientation in the inertial space, Esnault-Pelterie developed the equation:

$$\frac{d^2r}{dt^2} = \Gamma - G - \frac{\Gamma_1}{M}$$

where r is the distance from the Earth's center, Γ is acceleration provided by propulsion system, Γ_1/M is acceleration due to environment resistance, and G is gravity acceleration, which is $G = g\,R^2/r^2$.

It is interesting that Robert Esnault-Pelterie, apparently in view of a possible time lag in computing devices, formulated a problem of determining gravity acceleration on points of the trajectory with certain time prediction, and illustrated the possibility of obtaining the solution to this problem. After denoting measured "apparent" acceleration inside the vehicle by:

$$\Gamma - \frac{\Gamma_1}{M} = \gamma$$

Robert Esnault-Pelterie had written an equation for r as:

$$\left(\frac{d^2r}{dt^2}\right)_{r+\Delta r} = \gamma_{r+\Delta r} - gR^2\left(\frac{1}{r^2} - \frac{2}{r^3}\frac{dr}{dt}\Delta t\right)$$

It is now clear that progress in computers made this foresight by Esnault-Pelterie unnecessary. It can be shown by a simple analysis that the inertial navigation system, considered by Esnault-Pelterie, is unstable. An error G of determined distance r is governed by the equation

$$d^2\sigma/dt^2 - gR^2\,(2r + \sigma)\,\sigma \,/\, r^2\,(r + \sigma)^2 = 0$$

where r = r (t) is a true current distance between the vehicle and Earth's center. For the case r = R = const it is possible to write the solution:

$$\sigma(t) = \sigma_0 \,\mathrm{ch}\,(\nu t\,\sqrt{2}) + (\sigma_0/\nu\,\sqrt{2})\,\mathrm{sh}\,(\nu t\,\sqrt{2})$$

where $\nu=\sqrt{g/R}$ is the Schuler frequency and σ, σ_0 errors of initial position and velocity, respectively, are used as input data in the integrating unit.

[8] Robert Esnault-Pelterie., *L'astronautique* (Astronautics) (Paris, 1930).

In 1934 the concept of inertial navigation in space was realized in practice at the Central Aero- and Hydrodynamical Institute by V. S. Vedrov and his colleagues, who developed a technique for the analysis of aircraft motion during a non-stationary spin. Test aircraft were equipped with three accelerometers (g-sensors) and three single-axis gyroscopic angular velocity sensors. The measured g's and angular velocities were recorded by XY plotters.

Graphs of three angular velocity vector components, and of three g-vector components recorded during the flight, were used for calculating time-dependent coefficients in Euler-Poisson equations, for time evolution of the aircraft orientation, and for calculating right-hand-sides of Newton equations, which govern space motion of the point where sensors are installed. Numerical solution to the full system of equations restructured the whole pattern of aircraft motion—its trajectory as well as fuselage orientation and velocities in every point of the trajectory. Thus the full problem of inertial navigation in space was solved. According to modern terminology, it was a typical inertial navigation system without a gimbal, with only one special feature: flight data were processed not by computer but by a group of analysts.

The paper by V. S. Vedrov and his colleagues in 1935 contained equations of an ideally operating system for inertial navigation in space for the case of arbitrary rotation of the frame, connected with sensitivity axes newtonometers (accelerometers); the form of these equations was quite similar to those used presently. They had picked out this group of Euler-Poisson equations:

$$d\varphi/dt = q \sin \vartheta + r \cos \vartheta$$
$$d\vartheta/dt = p - (q \cos \vartheta - r \sin \vartheta) \operatorname{tg} \varphi$$
$$d\psi/dt = (q \cos \vartheta - r \sin \vartheta) / \cos \varphi$$

where φ, ϑ, ψ were attitude angles of aircraft, p, q, r - angular velocities, recorded by XY -plotter.

Expressions were written down for the apparent newtonometers (g,s) measures by newtonometers along axes ξ, η, ζ, connected with aircraft;

$$n_\xi - (g_\xi - j_\xi)/g \qquad n_\eta - (g_\eta - j_\eta)/g \qquad n_\zeta - (g_\zeta - j_\zeta)/g$$
$$g_\xi — g \sin \varphi \qquad g_\eta — g \cos \varphi \cos \vartheta \qquad g_\zeta -+ g \cos \varphi \sin \vartheta$$

Here g is gravity acceleration and j_ξ, j_η, j_ζ - projections of the full acceleration vector.

The set of Newton equations is written in this form:

$$du/dt = ru - qw - g (n_\xi + \sin \varphi)$$
$$dv/dt = pw - ru - g (n_\eta + \cos \varphi \cos \vartheta)$$
$$dw/dt = qu - pv - g (n_\zeta - \cos \varphi \sin \vartheta)$$

where u, v, w are the projections of the full velocity vector on the axes connected with an aircraft.

It is stated in the paper that the coordinates of an aircraft in space are obtained by direct integration of the velocity vector after reprojecting it on the immovable frame axes. As an illustration, a typical trajectory of aircraft going out of spin (in three projections) is presented; this trajectory was calculated by the above described technique. V. S. Vedrov's paper seems to contain the most complete of the earliest accounts of the theory of inertial navigation in space.