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PERCEPTUAL PERSPECTIVE AND CEZANNE'S LANDSCAPES

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Abstract — *The author has introduced a system of perspective that takes into account not only the visual properties of the human eye, but also certain effects produced by the brain during visual perception. Such a perspective, called perceptual perspective, represents the perspective of visual perception better than geometrical (linear) perspective does. The author's analysis of Cézanne's landscapes demonstrates that Cézanne intuitively employed the underlying concepts of perceptual perspective that provide a rational basis for explaining the geometrical 'oddities' noted in his paintings.*

I. INTRODUCTION

One of the recurring problems in visual art is the representation of space and objects within it on a picture plane, and this problem continues to receive considerable attention. It is sufficient to refer to works by E. H. Gombrich [1], R. Arnheim [2, 3] and J. J. Gibson [4–6]. My task is much less ambitious, for it is an attempt to examine only the geometrical aspect of the problems.

The history of art reveals different geometrical methods of depicting space used by artists of different periods and in different geographical locations. Indeed, it would be a significant contribution to find the basic root of these methods. In confronting this task one should first find what a mathematically acceptable spatial representation should look like on a plane. Many students of the problems believe they were resolved by those who during the Renaissance proposed a scientifically relevant system of perspective. However, I feel this belief to be erroneous: *Renaissance perspective* is neither perceptually nor mathematically acceptable. I employ the term Renaissance perspective, because I consider it is more specific than terms commonly used, such as *geometrical perspective* and *linear perspective*. This distinction is important, because the perspective system that I discuss below is a geometrical system that I find is superior from the perceptual and the mathematical points of view.

II. PERCEPTUAL PERSPECTIVE

Figure 1 is a diagram showing two trees of unequal height B and C at the sides of a road A and depicted by B' and C' in Renaissance perspective on plane P. I chose the heights of the trees B and C in

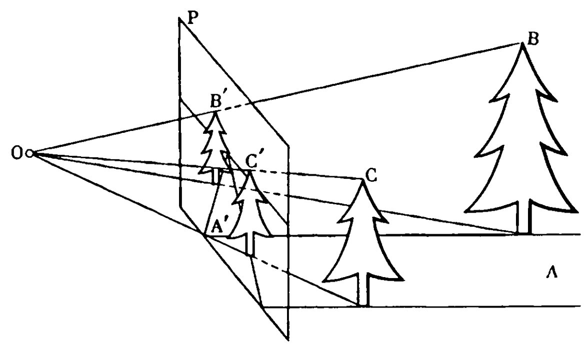

Fig. 1. A scene represented in a picture using Renaissance perspective.

the diagram so that their images B' and C' would be of equal height. Since for this situation when a viewer's eye is located at O the images projected onto the retina by light passing through the eye's lens from representations B' and C' and from trees B and C are the same, it is often presumed that Renaissance perspective and photographs made with conventional cameras result in images that are maximally close to those produced in visual perception.

But, visual perception is not the result only of the activity of human eyes; it is the result of the joint activity of the eyes and the brain. Consequently, in the above example, the presumption could be acceptable only if the brain processed in the same way both the images on the retinas resulting from viewing representations B' and C' in plane P and those resulting from viewing the real objects B and C.

Size constancy in visual perception applies, for example, when the brain produces a visual image (in response to retinal images of a viewed object) that is progressively enlarged the farther an object is from a viewer. Hence, though the image produced on the retinas by light passing through the lens of the eyes from an object C and from its representation C' in

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the picture plane are identical, the viewer will perceive the object and the representation differently, since their distances from the viewer are not identical. The representations *B'* and *C'* are at the same distance from the viewer, while the distances of the objects *B* and *C* are not. Therefore, the visual perception of the relative sizes of *B'* and *C'* in a picture will differ from the visual perception of the relative sizes of the real objects *B* and *C*. (Size constancy within pictures is not considered here because of its relative unimportance.) Thus Renaissance perspective will distort the relative sizes of objects at different distances from a viewer. These considerations show that in devising a perspective that well represents visual perception one should take into account not only the functioning of the eyes but that of the brain as well.

Psychological studies of visual perception have included many experiments concerning size constancy. The experimental results I have obtained have enabled me to represent mathematically the processing by the brain of information received on the retinas of the eyes while one is viewing objects. The mathematical functional relations included in my new system of perspective have made it possible to describe the geometrical properties of an object as perceived.

A detailed exposition of my mathematical work and a description of my experiments are given in the mathematical supplement of my book *Spatial Compositions in Painting* (In Russian) [7]. Here I shall discuss only the results obtained. A brief compendium of relevant mathematical expressions is provided in the Appendix of this article.

I have called my system *perceptual perspective*, in order to emphasize that it leads to the geometry of an object as visually perceived and not to that of the image produced on the retinas. It is interesting to compare the properties of perceptual perspective with those of Renaissance perspective.

In most general terms, one may say that representations of very distant objects appear geometrically similar in both perspective systems. These representations differ more and more, however, the closer a viewer is to an object represented. In particular for very close objects, representations in perceptual perspective approximate those in *parallel perspective*. It is to be noted that parallel perspective better presents the geometry of close objects as visually perceived than Renaissance perspective does. Thus, it is interesting to find a dominance of parallel perspective in Hellenic and medieval art. It seems that its utilization before the Renaissance indicates a deliberate attempt by artists to represent realistically objects that are intended to appear close to a viewer. My analytical method can be used to explain the geometrical 'oddities' found in many pictures in which Renaissance perspective has been employed.

A careful examination of the method of perceptual perspective leads to the important conclusion that it is impossible, in principle, to represent on a picture plane the visually perceived image of objects. For example, consider the representation of the visually perceived image of a cube (Fig. 2). Since the mathematical equations underlying my perceptual perspective permit one to calculate the visible size of each edge of the cube, the solution is straightforward. One can start the construction with point *A* and represent in sequence the perceived sizes of the edges *AB*, *BC* and *CD*. In this way one obtains a representation of the location of a corner *D*. But it can also be obtained by depicting the edges *AB*, *BE* and *ED*. If one carries out this second construction, the resultant representation of corner *D* will be at another location on the picture plane, for example *D'*. The apparent contradiction is due to the fact (it is provable) that a perspective system in which each of the depicted lengths corresponds precisely to visual perception lacks the property of *commutability*. This means that, if an artist starts to depict the geometrical elements of a spatial construction in precise correspondence to what is perceived visually, the depiction will display a 'break-up' or an overlap in the picture plane.

The only way to meet the requirement that a spatial representation in a picture plane shall not have 'break-ups' or unwanted overlaps is to introduce distortions in the representation—obvious departures from what is seen. Then it will be seen that any perspective system (Renaissance perspective included) transmits some of the depicted elements in accordance with visual perception and the remainder with distortions.

The inaccuracies of Renaissance perspective (departures from what is seen) stem directly from its method of construction, and artists employing this perspective exclusively are provided with no means to correct them. By applying perceptual perspective, artists may deliberately shift the inevitable distortions onto those geometrical elements that may be considered to be secondary. As an illustration of the idea, I present below an example that will also permit a new interpretation of the

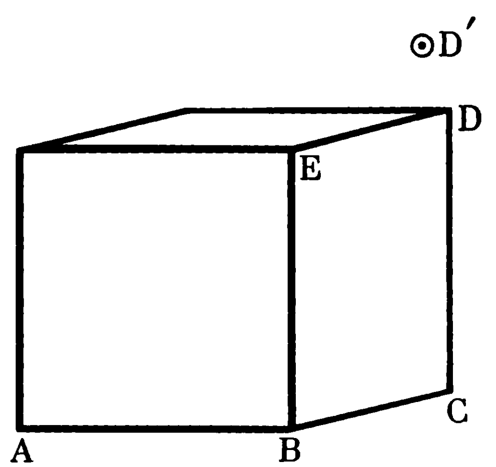

Fig. 2. The representation of a cube in a picture plane when two sides are in horizontal planes and two sides are in planes parallel to the picture plane.

perspective used by Cézanne in his landscapes. Assume that an artist is to depict a landscape in which vertical lines are intended to play a subordinate role. In this case the inevitable distortions would be assigned to representations of vertical lines of objects. It must be added here that in distant regions (regions near the horizon) vertical lines may be left undistorted, since Renaissance perspective is acceptably accurate in the depiction of distant objects. Hence, by shifting the inevitable distortions to the depiction of vertical lines in the foreground and middle ground, one can produce a perceptually accurate picture of, say, a landscape with mountains near the horizon.

III. A COMPARISON OF PERCEPTUAL AND RENAISSANCE PERSPECTIVES

The mathematical equations governing perceptual perspective are given in the Appendix. It is necessary to stress here that these equations permit precise constructions, just as those of Renaissance perspective do. Although an artist has the freedom to select the horizontal or the vertical geometrical lines of objects onto which the inevitable distortions will be shifted, after the choice has been made, the mathematical equations determine in a precise way how the geometrical elements are to be represented on the picture plane.

Mathematical expressions based on a function determined empirically in size constancy experiments enable one to calculate directly (without considerations of perspective) the visually apparent geometrical dimensions of an object being depicted. The geometrical dimensions of the same object in perceptual perspective and in Renaissance perspective permit a comparison to be made of what is defined by both perspective systems with what is perceived visually. The following is a demonstration of the method.

Assume a cube (Fig. 2) located in space before an artist who wishes to make a drawing of it on a piece of paper (the picture plane), using either Renaissance or perceptual perspective. It is convenient to characterize the depiction of the cube by nine parameters: three lengths of the edges *AB, BE* and *BC* (they are used to assess the correctness of the size of the representation of each), the ratios *AB/BE, BE/BC, BC/AB,* and the three ratios of opposite edges like *BE/CD* (these six ratios are used to assess the degree of similarity of the drawn cube to the perceived cube). Thus with nine evaluated parameters one can compare the two types of perspective for an object at an arbitrarily selected distance from the picture plane with nine similar parameters in visual perception. A practical evaluation can be made by considering three cubes on a horizontal plane at different distances from the artist—close, intermediate and distant. In this situation the above nine parameters can be determined for each cube in each perspective. Thus, a total of 27 quantities are available (three cubes) for characterizing each perspective in landscapes or the like.

In this demonstration Renaissance perspective and perceptual perspective (with distortions assigned to vertical lines) can be compared as follows: Out of 27 parameters in the Renaissance perspective 15 are distorted (more than one-half); in perceptual perspective nine are distorted (less than one-third) and those distorted involve vertical lines only (in the case of cubes). Perceptual perspective, therefore, contains about half the errors of Renaissance perspective (it should be remembered that there can be no perspective free of errors). It is surprising how many distortions of visual perception are involved in Renaissance perspective, which for centuries was regarded as an ideal, precise, scientific method of 3-dimensional representation of a picture plane.

For a better grasp of these ideas, Fig. 3 (top) is given to represent a typical landscape in Renaissance perspective, and Fig. 3 (bottom) is given to represent the same scene in perceptual perspective. In order to facilitate comparison, the direction of the reference line *AA* is drawn the same in both pictures.

First one should note that both representations present well (though in different ways) both depth and the 3-dimensional modelling of forms, and they present perspective from the same viewing point. In judging their differences one notices that in perceptual perspective the sizes of distant objects (e.g. the mountains) are larger in comparison to those in Renaissance perspective and the sizes of near objects (e.g. the flower bed) are smaller. In the picture plane the distance from the line *AA* to the base of the mountains is larger, while the distance from the line *AA* to the picture's lower border (depth of the foreground) is smaller. The mountains

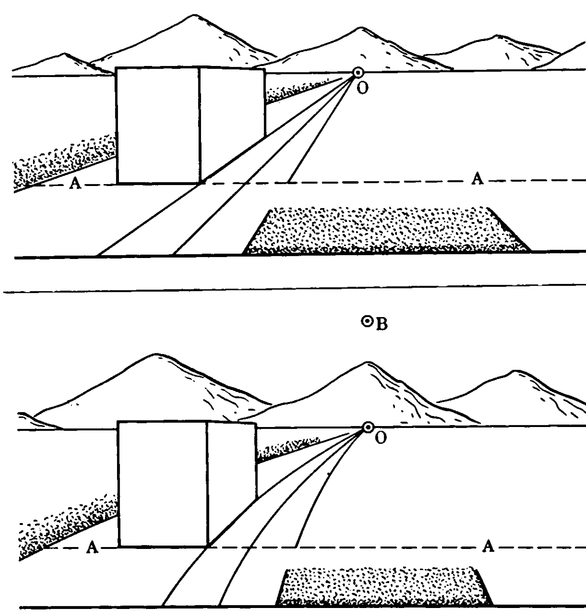

Fig. 3. (Top) *A typical landscape in Renaissance perspective.* (Bottom) *A typical landscape in perceptual perspective.*

are broader. In fact, the left and right portions of the landscape cannot be accommodated in the picture. In addition, lines that are straight physically (the sides of the road leading to the mountains and the hedgerows behind the house) are represented as straight in depictions based on Renaissance perspective, but in perceptual perspective straight lines are shown curved, except those that are vertical (e.g. the vertical sides of a cube), horizontal (parallel to line *AA*) or perpendicular to line *AA* and passing through point *O*.

It is interesting to note that the above differences concern lines that, in accordance with the choice made, are presented in perceptual perspective without distortion. It follows that, in comparison with what one will see in a photograph, for example, distant mountains will be enlarged, a near-by flower bed will be diminished, many of the straight lines will be slightly curving, etc. And artists, unwilling to submit to the dictates of Renaissance perspective, when it leads to a distorted representation of a scene, but freely following visual perception of the scene in depicting both a straight road vanishing at the foot of distant mountains and objects in the foreground and at intermediate distances, will produce a picture with much the same geometrical characteristics of the scene diagrammed in Fig. 3 (bottom).

IV. PERSPECTIVE IN CEZANNE'S LANDSCAPES

Those who have studied Cézanne's landscapes have emphasized that he did not employ Renaissance perspective. A detailed study of this matter by comparing his pictures with photographs of his motifs was made by F. Novotny [8] and by E. Loran [9]. On the basis of their findings one can say that the geometrical 'oddities' of Cézanne's landscapes were intended to convey what is perceived visually. Clearly he followed his own perception rather than academic rules.

Novotny, in summarizing his study, stated: 'In many cases changes are made of relative sizes, and in such a way that large foreground objectives are conveyed in a somewhat reduced size, while background objects, on the contrary, are enlarged in the picture' [8, p. 39]. The same conclusion may be drawn when comparing the representations of a typical landscape in Fig. 3. Moreover, the numerical assessment of the degree of these reductions and enlargements in Cézanne's landscapes demonstrates that they have approximately the same magnitude as I have demonstrated here using perceptual perspective. Cézanne's pictorial perspective has undoubtedly caused Novotny to ponder its significance. He aptly noted the 'portrait' fidelity of Cézanne's landscapes. But Novotny's suggestion that Cézánne's perspective constructions were made 'in favour of colour' [8, p. 5] is not convincing to me. Here I tend to agree with Loran, who wrote: 'Cézanne's comment that "nature is more in depth than in surface" suggests that his major concern was with the spatial elements rather than with decorative pattern' [9, p. 31]. It is precisely the striving towards 'portrait' fidelity in landscapes, i.e. the strict adherence to visual perception, that has drawn attention to the unwanted distortions of Renaissance perspective.

That Cézanne resorted to a kind of perceptual perspective is indicated not only by the general character of the 'oddities' but also by details of the representations. Thus, diminishing convergence in depiction of the borders of the avenue, noted by Novotny [8, p. 37] and by Loran [9, p. 50], is expected in the framework of perceptual perspective. One can see this by comparing the representations of the flower bed in Fig. 3, where they are depicted in the form of trapezoids. In perceptual perspective the base angles of the trapezoid are closer to 90° than they are in the case of the Renaissance perspective.

Some peculiarities experienced by many viewers in perceiving a Cézanne landscape arise from the fact that they have been profoundly influenced by the tradition of Renaissance perspective. For example, many studies devoted to Cézanne's work stress his desire to paint from unnaturally high points of view. However such a conclusion may be erroneous.

The typical landscape in Fig. 3 is represented from a viewing point that is at the level of the horizon. However, extending the lateral sides of the flowerbed trapezoids for determining the vanishing point and, consequently, the height of the viewing point yields different results. In Fig. 3 (top) it falls on point *O* at the horizon, and in Fig. 3 (bottom) on point *B* above the horizon. Hence, one may conclude erroneously that the representation in Fig. 3 (bottom) has been painted from an unnaturally high eye level. The fault is that in Fig. 3 (bottom) the lateral sides of the flower-bed trapezoid were extended along straight lines meeting at point *B*. They should have been extended in curved lines to reach the vanishing point on the horizon line. This can be seen in the representation of the road in Fig. 3 (bottom). Its vanishing point is obviously point *O*. However, if the stretch of road in the foreground is continued along straight lines, the corresponding vanishing point would be at *B*.

Viewers who are more accustomed to Renaissance perspective will believe Cézanne's perspective to be one in which the depiction of depth is reduced. I feel that the contrary is closer to the truth: Cézanne represented in paint his perception of space, whereas Renaissance perspective exaggerates depth by enlarging representations of objects in the foreground and by reducing them in the background. Many viewers today tend to accept these perspective exaggerations in visual artworks; undoubtedly the fact that photographic images are thought to be reliable records of what is seen is a contributing factor. A detailed quantitative comparison of Cézanne's perspective in his landscapes with perceptual perspective could be made. This, in particular, would help one to understand Cézanne's curved-space effects.

These ideas suggest that Cézanne was not a destroyer of Renaissance perspective, which was (and is) often thought to be 'scientific'; he was an artist who made the next advance in the development of a perspective that may well be considered scientific. (However, I do not wish to imply that he followed, even intuitively, precise mathematical equations.)

An understanding of the general approach by artists to the representation of objects in space is often essential for analyzing their work. I have indicated above that, after examining Cézanne landscapes in terms of Renaissance perspective, some people have stated wrongly that Cézanne heightened eye levels. It is equally wrong to make critical judgements about the parallel and inverted perspectives of antiquity and of the Middle Ages by proceeding from the dogmas of Renaissance perspective.

Renaissance perspective may be regarded as a variant of perceptual perspective. It can be proved that if the inevitable distortions in a scene are assigned such that straight lines on objects are represented in a picture by straight lines, one is dealing with Renaissance perspective. Renaissance perspective was introduced earlier than other variants of perceptual perspective, probably because it is the simplest (though far from the best) variant of perceptual perspective.

V. APPENDIX

Consider a suspended straight wire of length s parallel to a picture plane on which there is a representation of the wire. If the wire is moved into picture plane, the length of its representation equals the length of the wire. When the wire is moved away from the picture plane and the wire is maintained parallel to the picture plane, the representation of the wire in the picture plane is decreased in length. At a distance L from the picture plane, the length of the wire's representation in Renaissance perspective is s_1 and its perceived length is s_2 and $s_2 = s_1 F_1(\bar{L})$, where $F_1(\bar{L})$ is a function defining the magnitude of the effect of size constancy. This function depends on the distance L of the wire (length s) from the picture plane. For convenience, this distance is given in a nondimensional form, $\bar{L} = \frac{L}{L_0}$ where L_0 is, say, the distance between the plane of a picture on an easel and an artist's eyes.

I shall first present the equations for finding a point as represented in visual perception in the picture plane corresponding to a point in the scene being depicted. Using the equations one can represent in the picture plane any point in space and, in principle, depict any object. But, as stated above, objects depicted in this way lack commutability.

The coordinates of a point in space that I employ are: L, the distance from the picture plane; Z, the height above the ground plane; X, the lateral distance perpendicular to L and Z. Then, on the picture plane the corresponding point in visual perception will be represented by:

$$y^* = H F_3(\bar{L});\ x^* = X F_2(\bar{L});\ h^* = Z F_2(\bar{L}) \qquad (1)$$

where H is the eye level. This set (1) of equations represents perceptual perspective generally.

For Renaissance perspective the corresponding point on the picture plane will be represented by:

$$y_1 = H \frac{\bar{L}}{1+\bar{L}};\ x_1 = X \frac{1}{1+\bar{L}};\ h_1 = Z \frac{1}{1+\bar{L}}. \qquad (2)$$

Here the magnitudes y and h are marked off on the picture plane vertically and x, horizontally. Equation set (2) is one variant of the general set (1). It is the variant in which lines in space are represented by straight lines on the picture plane (Fig. 3).

The second variant of interest here is the one in which distortions are assigned only to vertical geometrical elements, as in Fig. 3 (bottom). For this particular perceptual perspective the corresponding point in the picture plane will be represented by:

$$\begin{gathered} y_2 = y^*\ ;\ x_2 = x^*\ ; \\ h_2 = Z\,[(F_3(\infty) - F_3(\bar{L})]. \end{gathered} \qquad (3)$$

This set (3) of equations shows that the horizontal distances within the space being depicted are without distortion of visual perception, but the vertical distance above the ground plane is altered ($h_2 \neq h^*$).

The functions $F_2(\bar{L})$ and $F_3(\bar{L})$ are related to $F_1(\bar{L})$ as follows:

$$F_2(\bar{L}) = \frac{F_1(L)}{l + \bar{L}}$$

$$F_3(\bar{L}) = \int_O^{\bar{L}} \frac{F_1(\bar{L})}{(1+\bar{L})^2} d\bar{L}\,.$$

If it is assumed that $F_1(\bar{L}) = 1$, i.e. the influence of the brain (size constancy) is disregarded, then the equations for perceptual perspective (1) and (3) will reduce to those for Renaissance perspective (2).

In order to use expressions (1) and (3), one must know the function $F_1(\bar{L})$. I have determined the function experimentally in measurements of size constancy. I indicate here only the general properties of this function: $F_1(O) = 1$; it is continuous, monotonically increasing with a finite limit; in the ideal case one has

$$\frac{dF_1(\bar{L})}{d\bar{L}(O)} = 1.$$

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