Perspective Pictures and Visual Perception

Boris V. Rauschenbach

Abstract—The author analyzes the possibilities of developing a system of scientific perceptual perspective by means of interior pictures. Analysis shows that many different perspective pictures of one interior viewed from the same point may exist, all of them being equivalent (i.e. containing an equal sum of distortions of visual perception). The author believes that Renaissance perspective cannot set the aesthetic standard for a 'correct' perspective because it is only one of the possible variants.

I. INTRODUCTION

Articles by J. J. Gibson [1], H. C. Reggini [2], J. J. Ward [3] and G. Coppel [4] in *Leonardo* discuss the important problem of correlating visual perception and perspective systems. The apparent inability of Renaissance perspective to reproduce visual perception, shown in particular by V. Ronchi [5], prompted me to construct a system of 'perceptual perspective' based on visual perception regularity. This approach explains the idiosyncratic perspective pictures of different peoples and epochs [6–8] and demonstrates that an 'ideal perspective' system for reproducing images without distortions cannot exist. The choice of a correct variant must depend on the particular problem dealt with by the artist.

In aesthetic analysis, the 'oddities' of perspective constructions in a picture are often discussed by comparing them with analogous representations in Renaissance perspective. This approach is based on a fundamental methodological error. I argue that the geometrical 'oddities' of the picture should always be compared with visual perception rather than with any 'ideal' perspective representation. The representational artist will often reveal a demonstrable perceptual perspective bias. One should be able to find this variant and to explain why the artist prefers it.

II. 'IDEAL' REPRESENTATION OF AN INTERIOR

Mathematical calculations relevant to this section are given in the Appendix as well as in a previous article in *Leonardo* [6].

Suppose an artist wishes to represent a room 4 m wide, 6 m long and 3 m high. The plane of the picture is at a distance of 2 m from the room, and the artist's eye is at a distance of 2 m from the plane of the picture (i.e. 4 m from the room) at 1.5 m height and is directed along the axis of the room. Such a closely situated interior provides a sharper manifestation of the problems to be discussed below than more distant interiors, for which numerical values characterizing the degrees of distortion decrease.

The borders of the floor, the walls and the ceiling nearest to the artist constitute a rectangle with the ratios of sides 4:3. The artist wishes to represent this rectangle correctly and to have all other sizes conform to this rectangle. Figure 1 represents the floor, each wall and the ceiling (proceeding from the reference rectangle) as they are seen independently from each other by a viewer. The floor and the ceiling are marked by horizontal lines, the side walls by vertical shading, and the reference rectangle (the 'entrance' into the interior) and the back wall by heavy bars.

This geometrical scheme of the non-distorted visual perception of a room demonstrates why a distortion-free system for the representation of visual perception is impossible. By strictly following visual perception in the representation of different objects (here the floor, the walls, etc.), the artist is forced to overlap images or to 'break up' the picture; the resulting image cannot be regarded as an accurate representation of visual perception. In the same way, if a three-dimensional model of our visual perception of a room is flattened on the plane, the model will break up and sections will overlap.

An artist who represents an interior will distinguish elements he or she considers important and represent them in conformity with his or her visual perception; elements of subsidiary importance must then be distorted to avoid

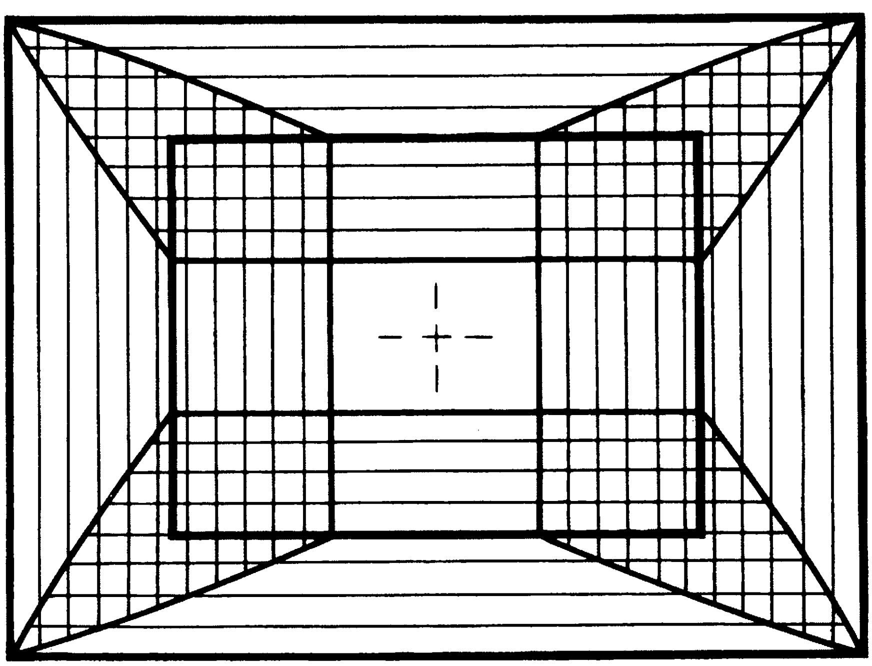

Fig. 1. A model of the non-distorted visual perception of an interior.

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overlapping or breaking up the elements of the picture. One can produce a correct floor representation and distort the walls in the picture or correctly represent the side walls and distort the floor, the ceiling and the back wall in the picture, among other possible distortions. The only thing that cannot be done is to show all the elements without distortion.

Before considering the different variants of representing an interior, I shall establish the criteria of correctness of such representation. An element is 'correct' if it reproduces visual perception without distortions, and an element is 'distorted' if it departs from the geometry of visual perception. The correctness of a representation can be measured by (1) the extent to which a correct scale co-ordination is observed, (2) the extent to which the reproduction of spatial depth is observed and (3) the extent to which proportion is observed. The degree of correctness of a picture's scale, depth and proportions may be different for different distances from the artist. Therefore the degree of correctness of a representation should be judged by the numerical value of the greatest distortions of scale, depth and proportion.

Figure 2 shows a schematic representation of an interior. The degree of scale correctness along the horizontal line is measured by a ratio of segments a and A. If the scale is inaccurate, the greatest value for this inaccuracy can be obtained by comparing the nearest and the back wall planes. The ratio of the lengths of these segments, (a/A), is one of the two possible ratios which is less than or equal to unity. If the ratio of the visible floor width on the back wall and the nearest planes is $(a/A)^* = 0.5$ and the artist represents them so that $(a/A) = 0.4$, then the degree of scale-correctness is $(a/A):(a/A^*) = 0.8$. If the strict representation of the nearest and the back wall planes is $(a/A):(a/A)^* = 1$, the distortion in the above example is $1.0 - 0.8 = 0.2$ or 20%.

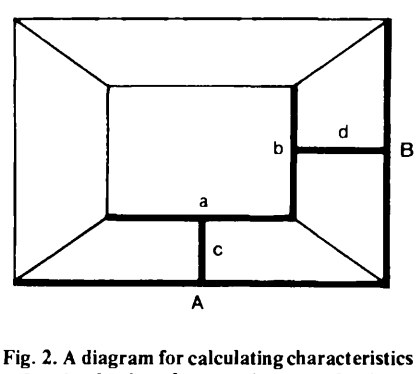

Fig. 2. A diagram for calculating characteristics of scale, depth and proportion reproduction.

The degree of scale correctness along the vertical direction is derived in the same way from the ratio (b/B), the degree of correctness in depth representation from (c/A) and the degree of correctness of the proportion of different planes from (A/B) and (a/b).

The degree of correctness of depth reproduction can be found in a great number of cases by comparing (c/A) both in the picture and in our visual perception since a viewer almost always judges the distance by a horizontal surface shift (in our case by the floor). Sometimes a viewer judges the distance by mentally moving not along the floor but along the wall, for example when the walls attract more attention than the floor. In this case the degree of correctness of depth reproduction is evaluated by values (d, B).

III. COMPARISON OF DIFFERENT VARIANTS OF INTERIOR PERCEPTUAL PERSPECTIVE

Figure 3 shows different variants of representing an interior without the overlapping shown in Fig. 1. Figure 3.1, a typical Renaissance perspective, perfectly reproduces proportion but is characterized by a great distortion in scale (see Table 1). Figure 3.2 retains a correct proportion reproduction and decreases a scale distortion only at the expense of depth reproduction (see Table 1, line 2). A strict reproduction of scale is shown in Fig. 3.3. Figure 3.4 reproduces the floor and the ceiling in conformity with visual perception, and Fig. 3.5 correctly depicts the side walls. Figure 6 distributes distortion values more evenly among scale, depth and proportion representation.

The sum of distortions in Figs 3.1 through 3.6 is roughly (±3%) equal to 55% revealing the operation of a 'law of distortion conservation'. Changing the variants of one's perspective can shift distortion from one element to another but it cannot decrease the sum of distortions. In any particular variant of perspective the sum of distortions will always equal 55% (see line 7, Table 1). The size of the representation affects these values according to an experimentally derived function (see Appendix), but however one calculates the magnitude of the effect of size constancy the analysis of ratios in Table 1 should remain valid.

Although Figs 3.1 through 3.6 reproduce the same interior from the same viewing point with an equal sum of distortions, each representation stresses different aspects of the interior. For example, Renaissance perspective necessitates strong scale distortion. An artist who makes the observance of relative scale the basis for reproducing objects will use the perspective given in line 3 of Table 1, which distorts only one of three elements (i.e. depth representation). Due to the law of distortion conservation, however, this depth representation will be quite significantly distorted. Researchers who habitually consider artistic construction according to the standard of Renaissance perspective are inclined to assert that this artist 'flattens' space

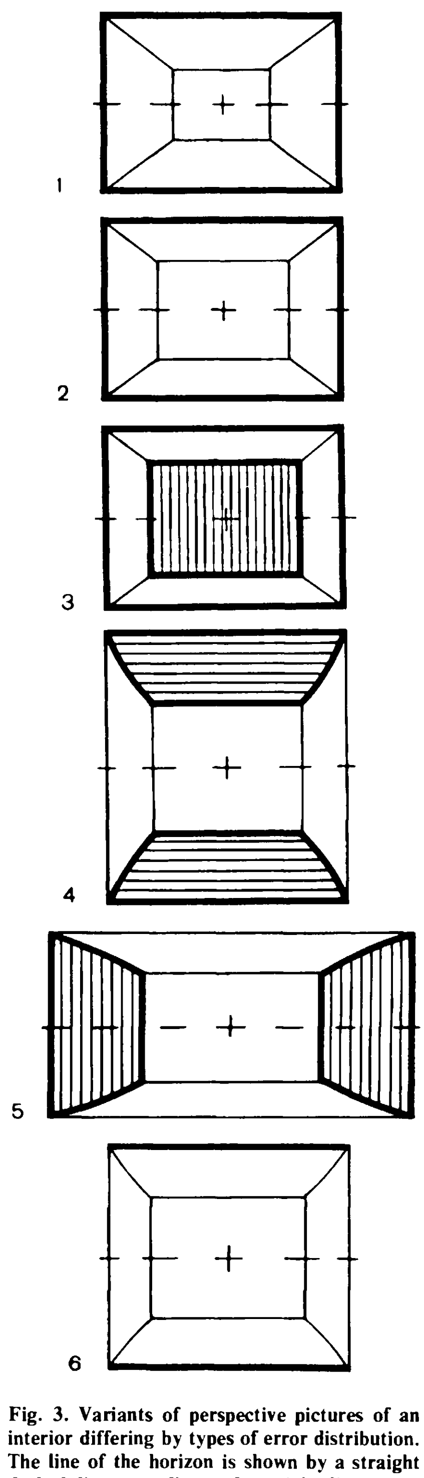

Fig. 3. Variants of perspective pictures of an interior differing by types of error distribution. The line of the horizon is shown by a straight dashed line, non-distorted straight lines by a heavy bar and planes by shaded areas.

Table 1. Table of distortion values in different variants of scientific perceptual perspective

	Variant of perspective	Distortions (%)						
		Scale		Depth		Proportion		Sum*
		(a,A)	(b,B)	(c,A)	(d,B)	(A,B)	(a,b)	Σ
1.	Renaissance	37	37	21	-	0	0	58%
2.	Proportion observance	22	22	33	-	0	0	55%
3.	Scale observance	0	0	52	-	0	0	52%
4.	Non-distorted floor reproduction	0	22	0	-	33	15	55%
5.	Non-distorted side wall reproduction	22	0	-	0	33	15	55%
6.	Equal distribution of errors	0	12	26	-	20	8	58%
7.	...	...		...		...		~55%

*Sum of the maximum values of scale and proportion distortions.

reproduction. It is important to understand that an artist may depart from Renaissance perspective to obtain more accurate scale reproduction so that the resulting flattening is only the inevitable result of this departure, not its primary motivation.

In the same way, one can represent scale and depth without distortion in reproducing the floor only by shifting the distortion to the reproduction of the side walls (see Table 1, line 4).

Figure 3.4 represents an interior depiction in which the floor is reproduced without distortion. From the standpoint of Renaissance perspective, one could say that the floor is represented from a high viewing point and the ceiling from a low viewing point since the corresponding 'vanishing points' (the crossed lines designated by *B* in Fig. 4.1) are higher or lower than the line of horizon. In fact there is only a single viewing point common to these figures, represented in Figs 4.1 through 4.3 as point *A*.

F. Novotny [9] and E. Loran [10] have analyzed Cezanne's spatial compositions by comparing them with photographs of the same motifs, i.e. by comparing with Renaissance perspective. Because Cezanne did not follow this variant of perceptual perspective, photographs should not be considered correct representations of his subject matter. Saying that Cezanne 'distorted' the rules of perspective of 'flattened space' reveals the critic's reliance on questionable assumptions about perception. If one uses the variant of perceptual perspective shown in Fig. 3.4 as a basis for studying Cezanne's landscape painting [6], it is easy to show that Cezanne did not violate any 'rules' of perspective. The features characteristic of Cezanne's work, namely the exaggeration of the background and the use of curved lines in representing straight ones, reflect his adherence to this particular perspective variant.

The problem of considerable distortions of proportion sometimes produces creative artistic solutions. An artist who cannot eliminate distortions may make up for this by not representing the most distorted elements. By omitting the ceiling and upper parts of the walls

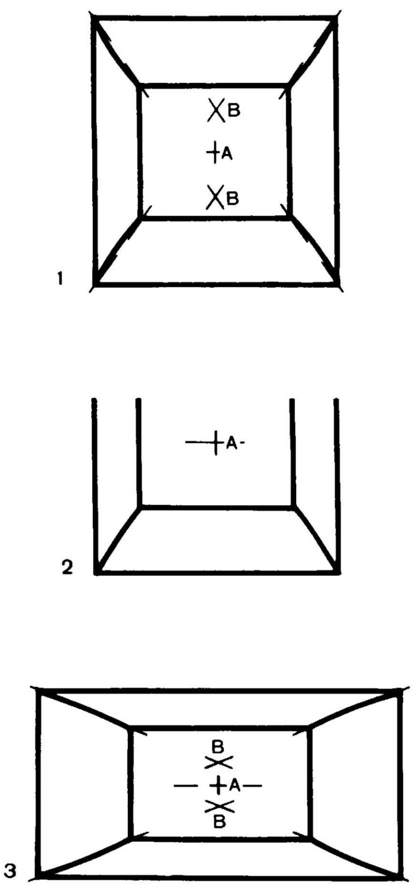

Fig. 4. An analysis of the non-distorted reproduction of horizontal planes and 'side walls'.

Fig. 5. Yu. Pimenov, *Before Going Out onto the Stage*, oil on canvas, 100 × 100 cm, 1965.

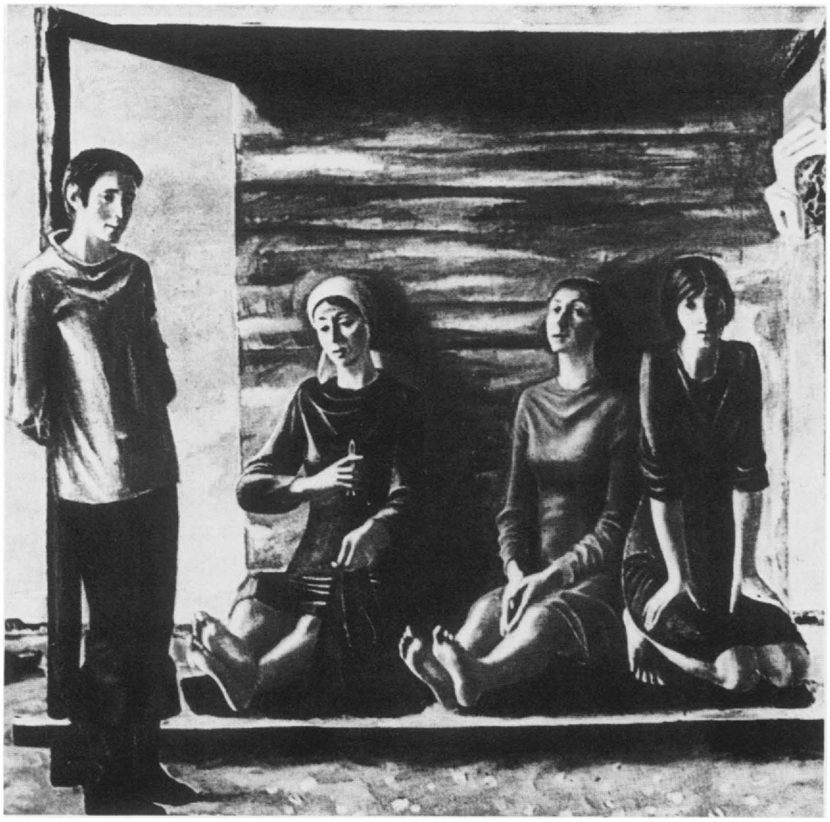

FIG. 6. O. Filatchev, *A Song*, oil on canvas, 170 × 180 cm, 1978.

(Fig. 4.2), an artist can mask geometrical distortions. The sum of the picture's actual distortions will then turn out to be less than 55%, although its perspective variant will still be characterized by a sum of distortions of 55%. In Yu. Pimenov's *Before Going Out Onto The Stage* (Fig. 5), for example, the imaginary vanishing point for the lines of wooden boards is much higher than the upper border of the picture while the horizon is only slightly higher than the actress's head (the upper part of the mirror), thus avoiding a necessarily distorted representation of the ceiling.

Filatchev's *A Song* (Fig. 6) follows Fig. 3.5 and Fig. 4.3 in faithfully reproducing depth by showing 'a side wall', or the space between the front column and the back wall, rather than the ceiling. In this case, the perspective variant helps produce a feeling of spaciousness appropriate for the artist's subject matter.

IV. CONCLUSIONS

The existence of many equitable variants of scientific perspective must be taken into account in attempting to understand the problems inherent in the geometrical reproduction of spatial planes. Because there cannot be an 'ideal' perspective, one should not judge different means of reproducing spatial relations by comparison with the particular mirror of the Renaissance perspective but rather with the scheme for visual perception described in Fig. 1.

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APPENDIX

Let $F_1(\bar{L})$ be a function defining the magnitude of the effect of size constancy (the ratio of a visible quantity and one corresponding to Renaissance perspective) depending on the dimensionless distance of $\bar{L} = L/L_0$, where L is the distance from the picture plane to the depicted object and L_0 is the distance from the same plane to the artist's eyes.

On the basis of the author's experiments one can take (for indoors):

$$F_1(\bar{L}) = (1 - Z^4)/(1 - Z) \text{ where } Z = \bar{L}/(1 + \bar{L}),$$

the accuracy of its definition being approximately equal to 5%. Some other kinds of function $F_1(\bar{L})$ can certainly be used in the calculations.

Introduce functions $F_2(\bar{L})$; $F_3(\bar{L})$ by defining them by means of equations

$$F_2(\bar{L}) = F_1(L)/(1 + \bar{L})$$

$$F_3(\bar{L}) = \int_0^{\bar{L}} \frac{F_1(\bar{L})}{(1 + \bar{L})^2} d\bar{L}.$$

The coordinates of a point in space are L, X, Z, where Z is the height above the ground plane and X is the lateral distance perpendicular to L and Z. On the picture plane the corresponding point in visual perception will be represented by

$$y^* = HF_3(\bar{L});\ x^* = XF_2(\bar{L});\ h^* = ZF_2(\bar{L})$$

where H is eye level. The magnitudes y and h are marked vertically on the picture plane and x is marked horizontally.

The analogous equations for Renaissance perspective will be:

$$y_1 = H\bar{L}/(1 + \bar{L});\ x_1 = X/(1 + \bar{L});\ h_1 = Z/(1 + \bar{L})$$

and different variants of perceptual perspective can be defined as

$$y_2 = Hf_L(\bar{L});\ x_2 = Xfx\ (\bar{L});\ h_2 = Zf_H\ (\bar{L})$$

where

$$f_H = K\ F_2\ (\bar{L}) + (1 - k)\ [F_3\ (\infty) - F_3\ (\bar{L})]$$

$$f_L = K\ [F_2\ (0) - F_2(\bar{L})] + (1 - K)\ F_3\ (\bar{L}).$$

Changing K changes perceptual perspective variant while $0 \leqslant k \leqslant 1$.

Function fx was chosen either as $fx\ (\bar{L}) = F_2\ (\bar{L})$ (1)

(i.e. in conformity with natural visual perception) or, for the proportion observance, as $fx\ (L) = fH\ (\bar{L})$. (2)

The data given in Table 1 correspond to:

Line 2 $k = 0; fx(L)$ using (2)
Line 3 $k = 1; fx(L)$ using (2) or (1)
Line 4 $k = 0; fx(L)$ using (1)
Line 6 $k = 0.5; fx(L)$ using (1).

Line 5 in Table 1 in fact reiterates Line 4.

Note: The notions and designations used in this appendix duplicate those appearing in a previous *Leonardo* article [6].